Geometrical optics

Textbook: Chapter 34

- Electromagnetic Oscillations & Alternating Current
- Maxwell’s Equations & Magnetism of Matter
- Electromagnetic Waves
- **Images**
- Interference
- Diffraction

IMAGES

Textbook: Chapter 34

- PLANE MIRRORS
- SPHERICAL MIRRORS
- SPHERICAL REFRACTIVE SURFACES
- THIN LENSES
- OPTICAL INSTRUMENTS

Images: Thorlabs.com

IMAGES

In this chapter key points are formulas to find images, magnification, and particular rays

Spherical refractive surfaces (slides 20-24) and demonstration of the thin lenses formula (slides 31-35) are here for your personal knowledge but will not be asked during the test

Note on this chapter:

We will use **geometrical optics** to understand light propagation through **optical components** to form **images**

e.g. image of an object magnified by a lens, of a planet seen through a telescope, of a bacteria seen through a microscope … or simply the formation of an image on the retina

Beams of light are represented by an **infinite number of rays**
There is **no interaction between rays** → Rays can be studied independently

Wave propagation is thus simplified as geometrical problems through this approximation and **laws to construct images** are extracted

Objects are any **source** of rays (objects that emit or reflect rays)

Real image

source

Images can be **real** or **virtual**

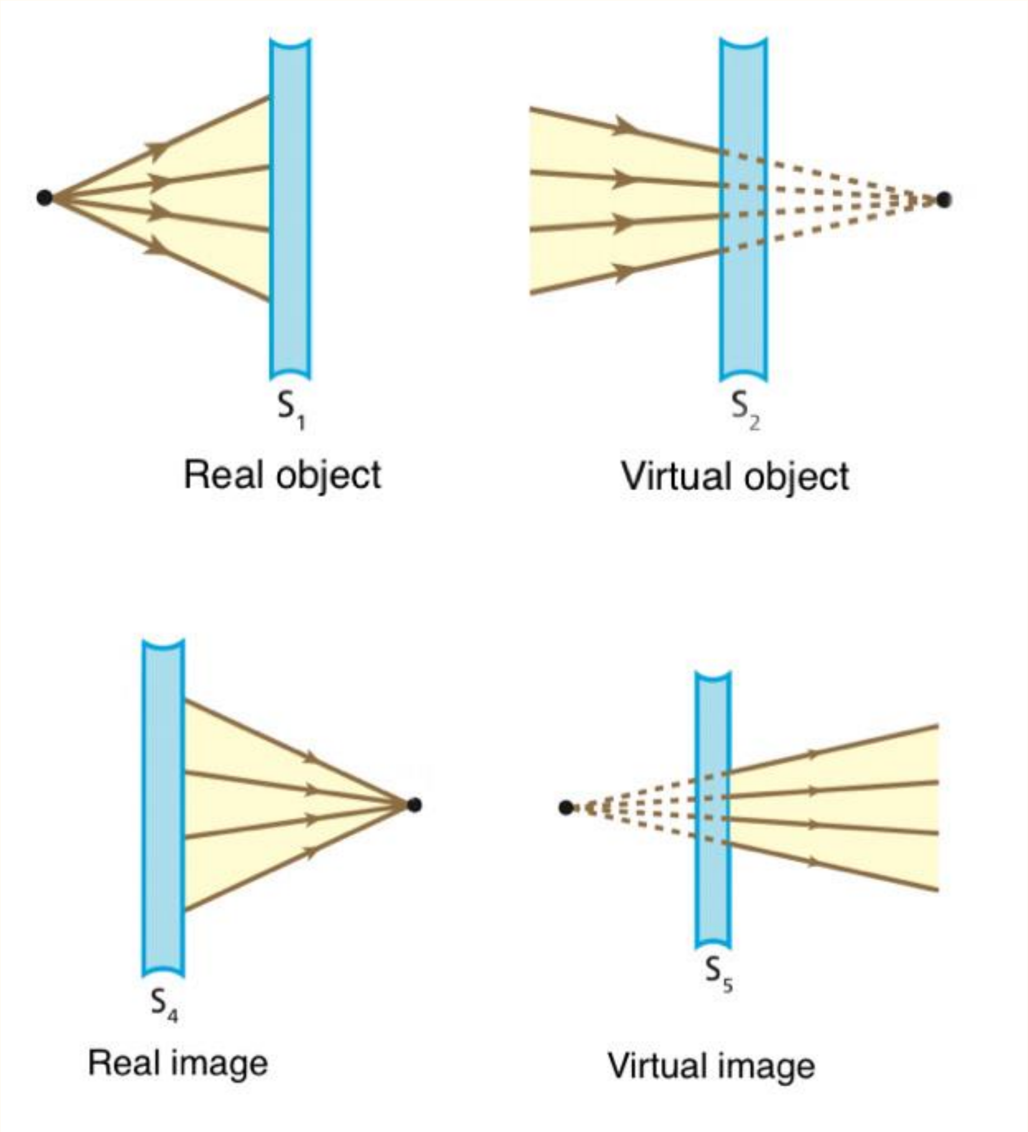
Real images: Real images are formed at the **intersection of real rays**
Real rays **converge** to the image
→ Can be formed on a screen

Virtual images: Virtual images are formed at the **intersection of virtual rays**
Real rays **diverge**
→ Cannot be formed on a screen but can be seen *through* a component

Virtual image

source

Images (real or virtual) of a component can be **objects** (real or virtual) for **another component** in an optical system



S_1
Real object
S_2
Virtual object
S_4
Real image
S_5
Virtual image

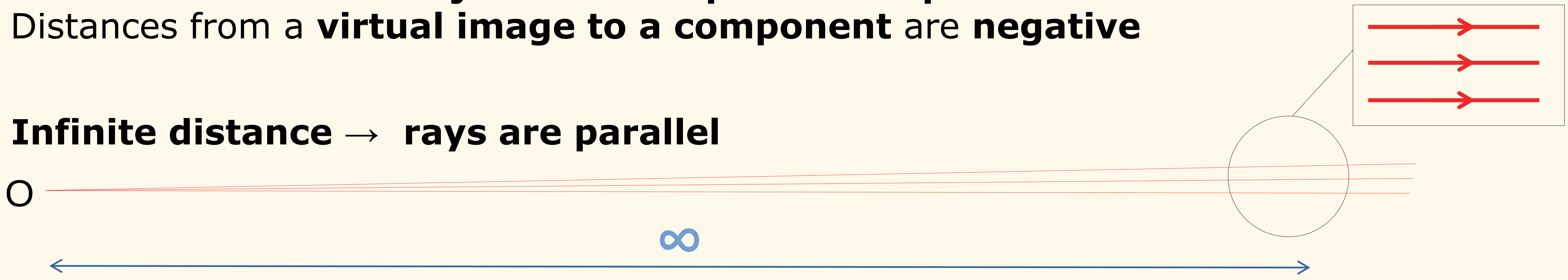
Real rays: **Continuous line + arrow** for direction of propagation

Virtual rays: **Dotted line → Extension of a real ray**

In geometrical optics, we use **algebraic distances**
→ distances that can be **positive or negative**

Distances from a **real object to a component** are **positive**
Distances from a **virtual image to a component** are **negative**

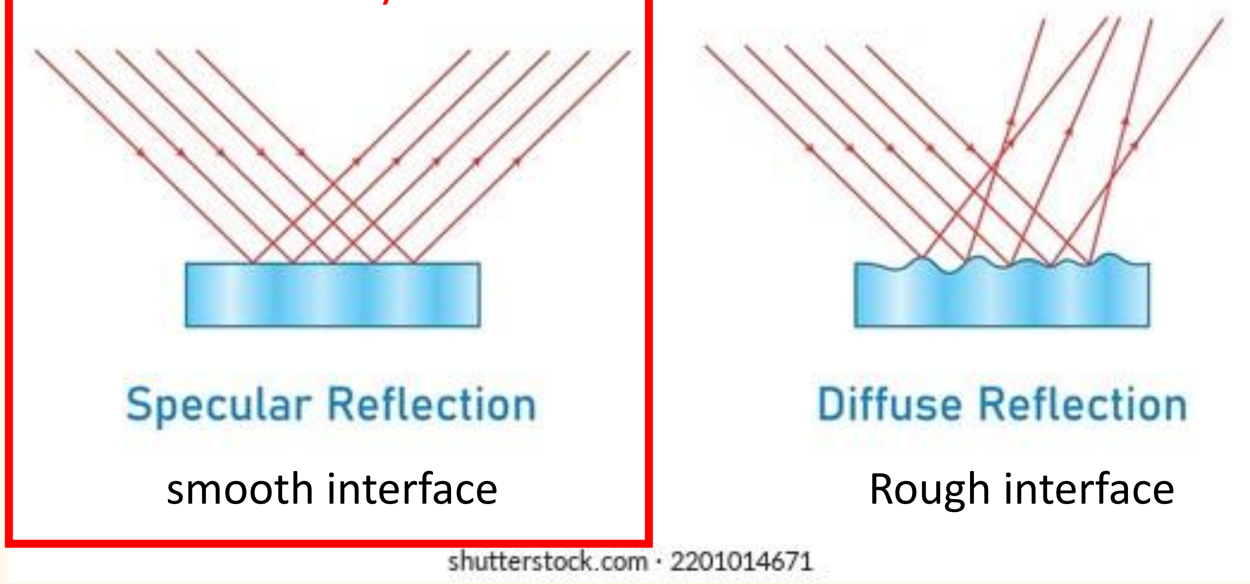
Infinite distance → rays are parallel



LAW OF REFLECTION

Plane mirrors are plane **reflective** surfaces

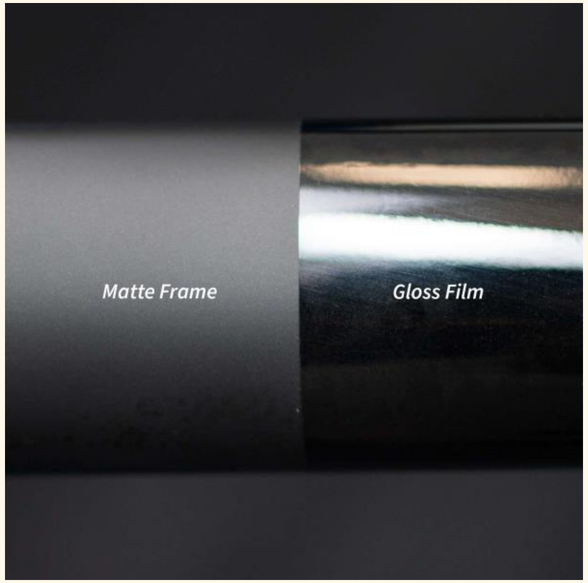
We will focuss only on this case



smooth interface

Rough interface

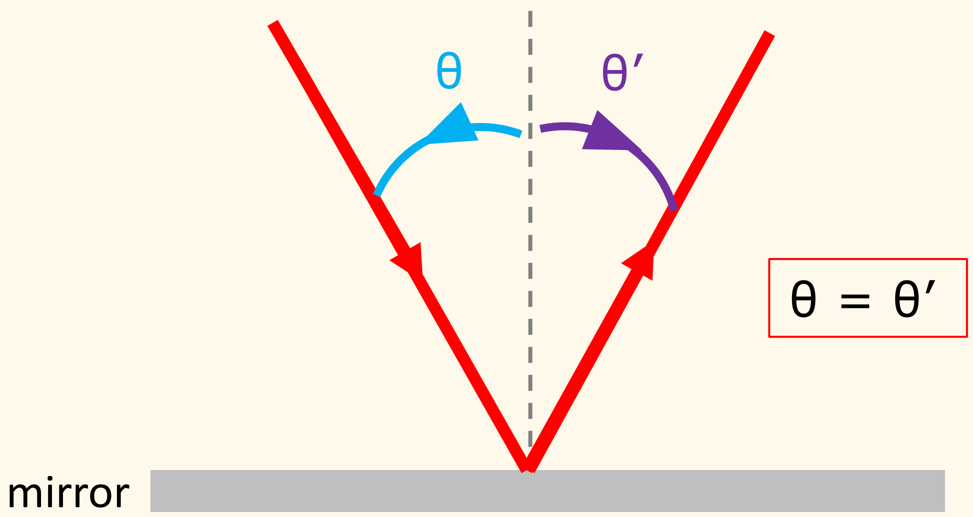
glossy paint vs matte paint



Notes : Two convention coexist in physics

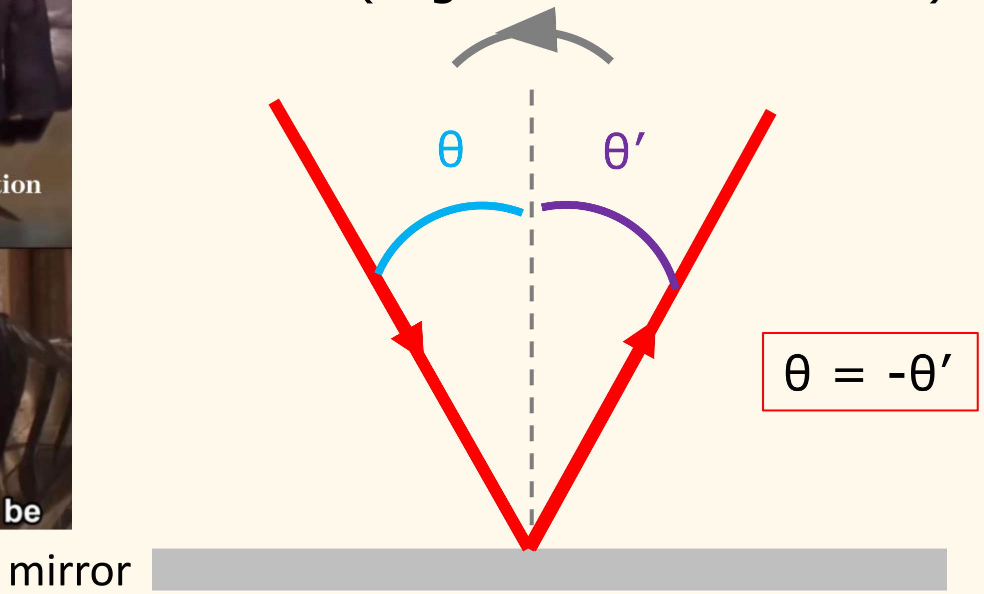
We’ll choose this convention

Angles are oriented from the normal to the rays



→ Easier but you have to remember the direction of the rays

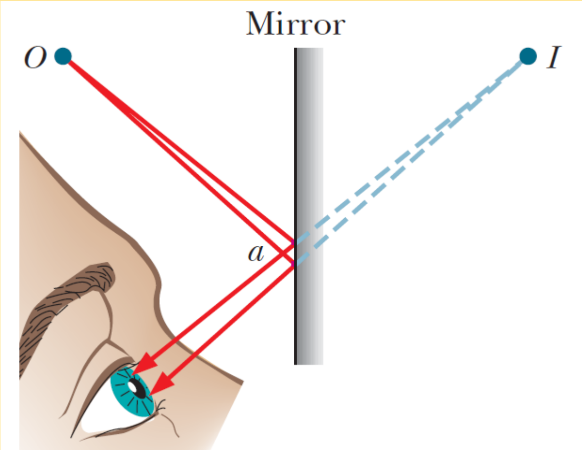
Angles are always oriented counter clockwise (trigonometric direction)



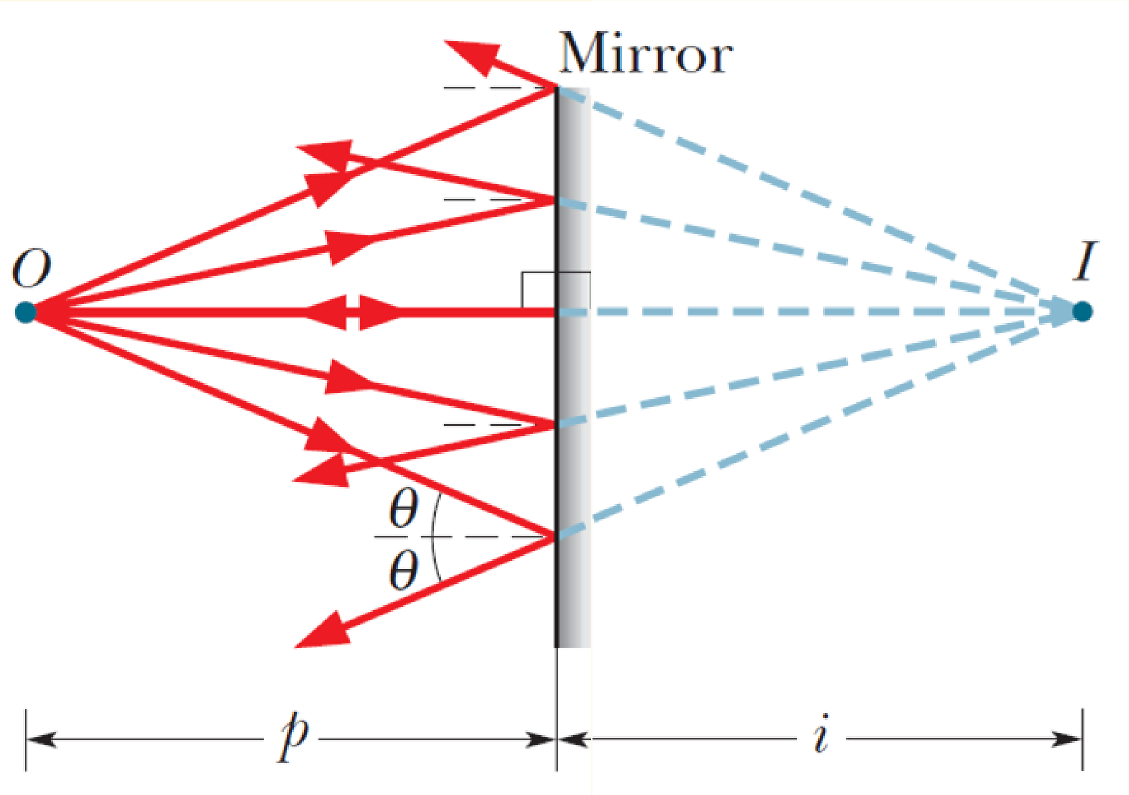
→ More accurate but it introduces in minus signe in the law of reflexion

Plane mirrors are plane **reflective** surfaces

Law of reflection: θ = θ′



Virtual image behind the mirror



O: Point object → source of rays

I: Point image → intersection of rays

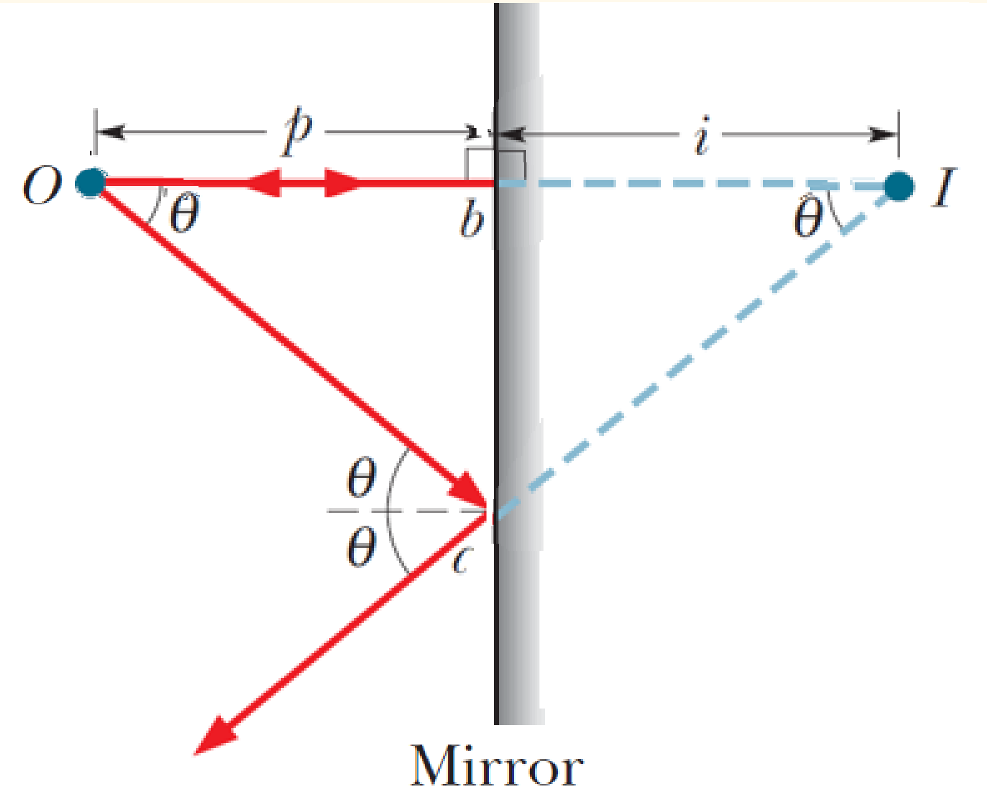
p: object distance > 0

i: image distance < 0

Only 2 rays to construct the image **of a point**:

- 1 ray at normal incidence

- 1 ray oriented with an angle θ



Triangles abO & abI are congruent

→ Ob = Ib

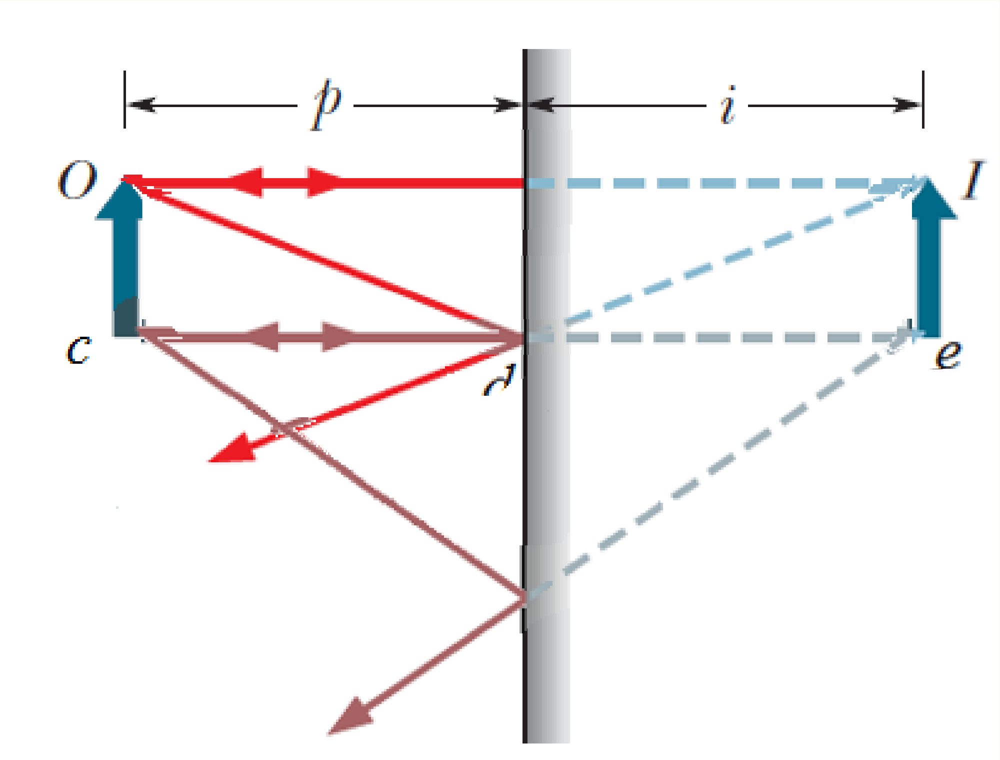
In algebraic distances:

$$p = -i$$

Plane mirrors form virtual images at equal distance of the object

Objects: Represented by arrows to show size and orientation

→ need 2 points



The image has the same size and the same orientation than the object

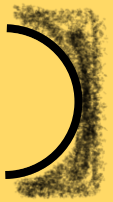
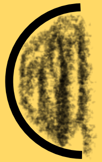
(congruence of Ocd & deI)

$$p = -i$$

Plane mirrors form virtual images at equal distance of the object

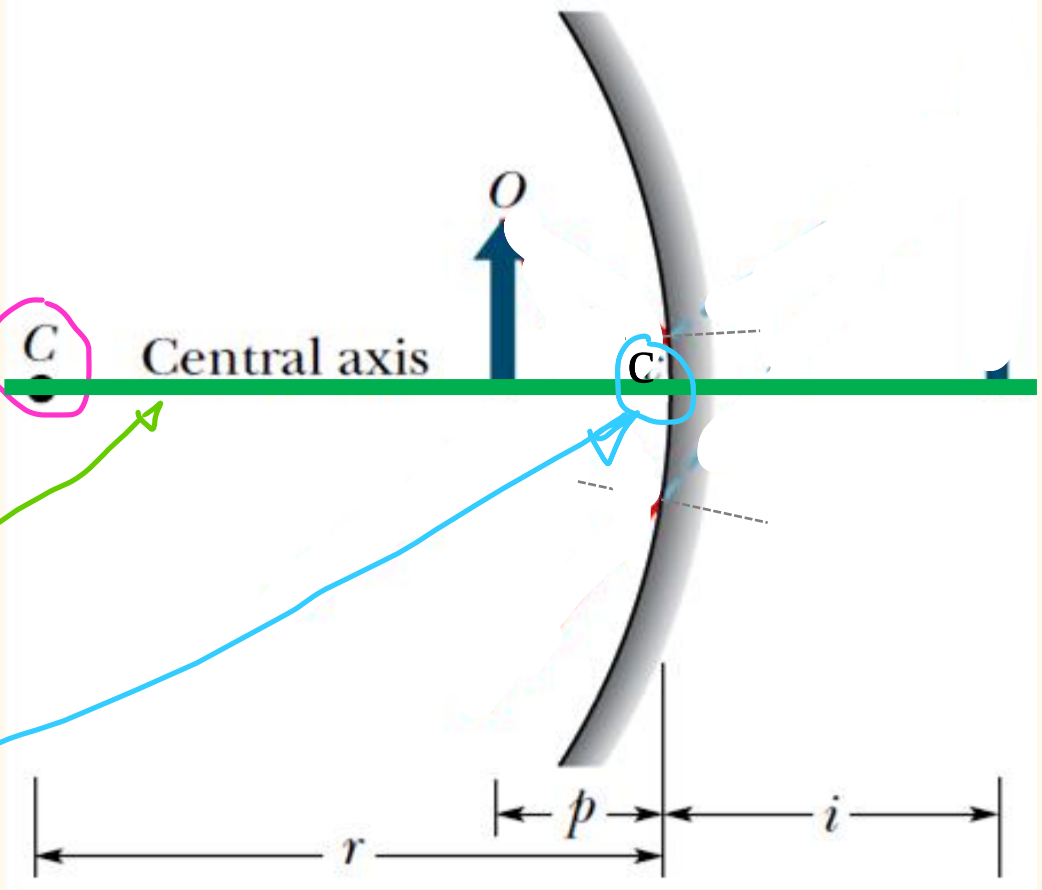
SPHERICAL MIRRORS

Spherical mirrors are curved **reflective** surfaces

Concave  or **Convex** 

We define:

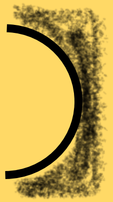
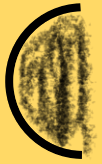
- The **center of curvature C**
The mirror is a portion of a sphere of center C and radius r
- The **central axis**
- The **center of the mirror c**



Concave mirror: $r > 0$
Convex mirror: $r < 0$

SPHERICAL MIRRORS

Spherical mirrors are curved **reflective** surfaces

Concave  or **Convex** 

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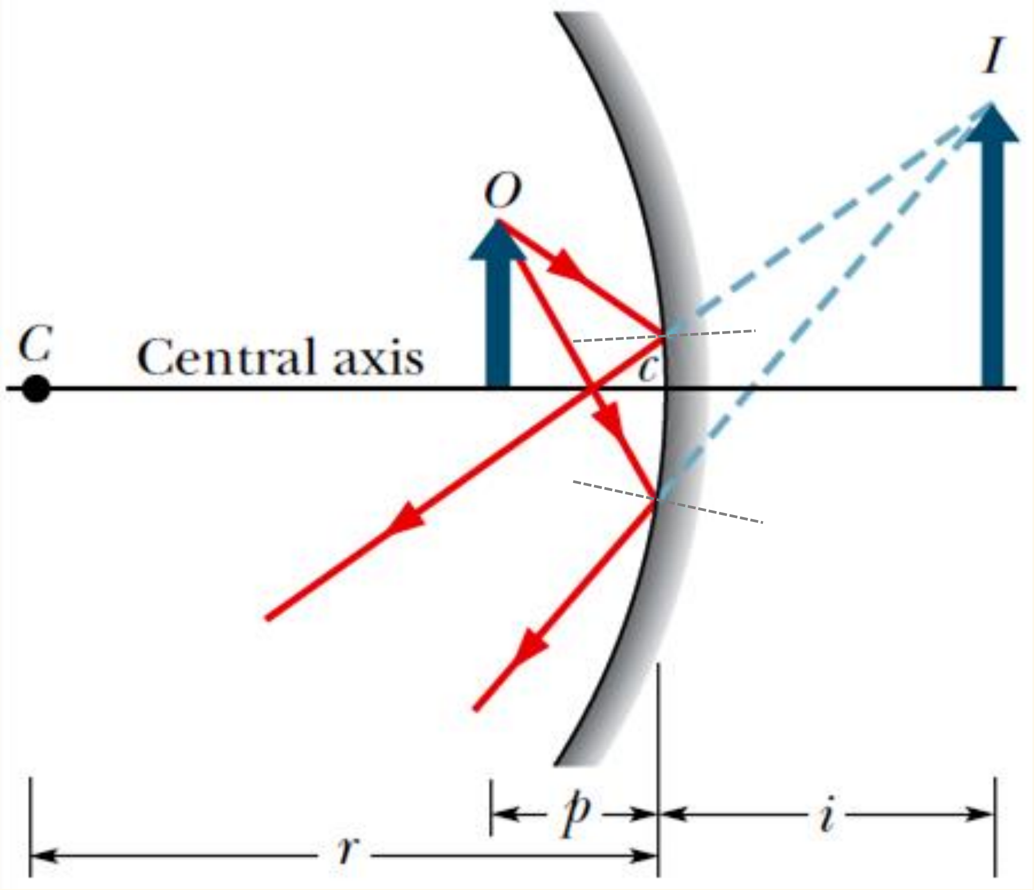
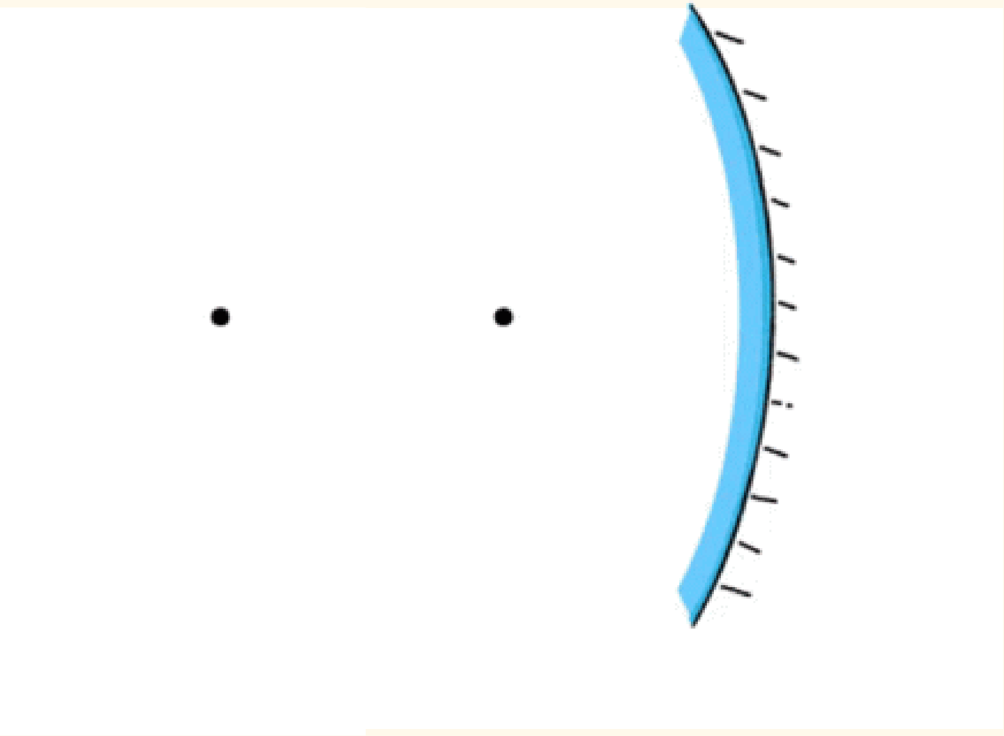


Concave mirror: r > 0
Convex mirror: r < 0

SPHERICAL MIRRORS

Rays **parallel to the central axis** (from an object at infinite distance) intersect at the **focal point F** of the spherical mirror
→ Fc = **focal length f**



Concave (or converging) mirror

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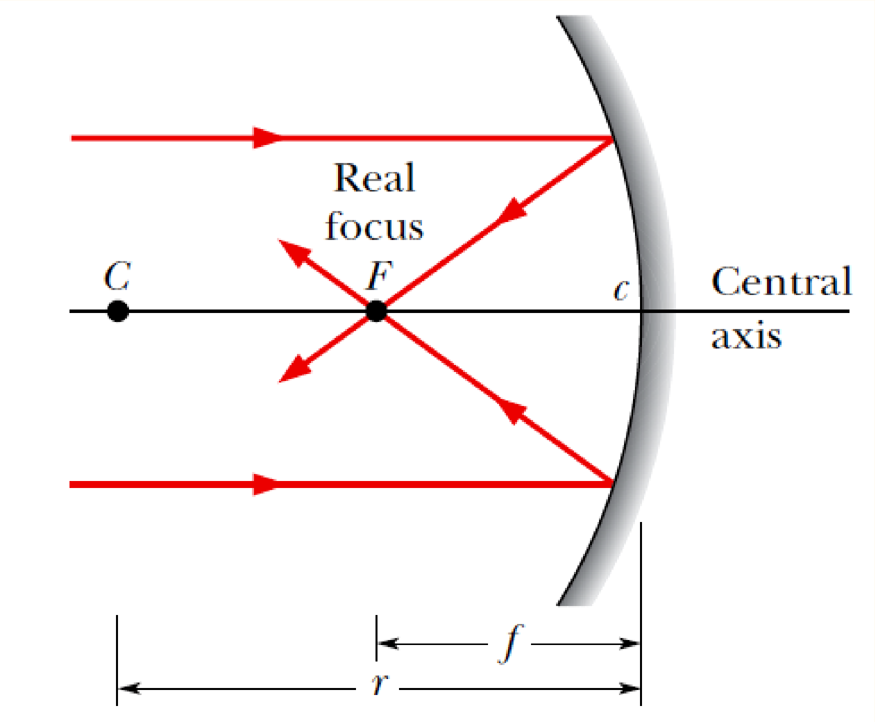

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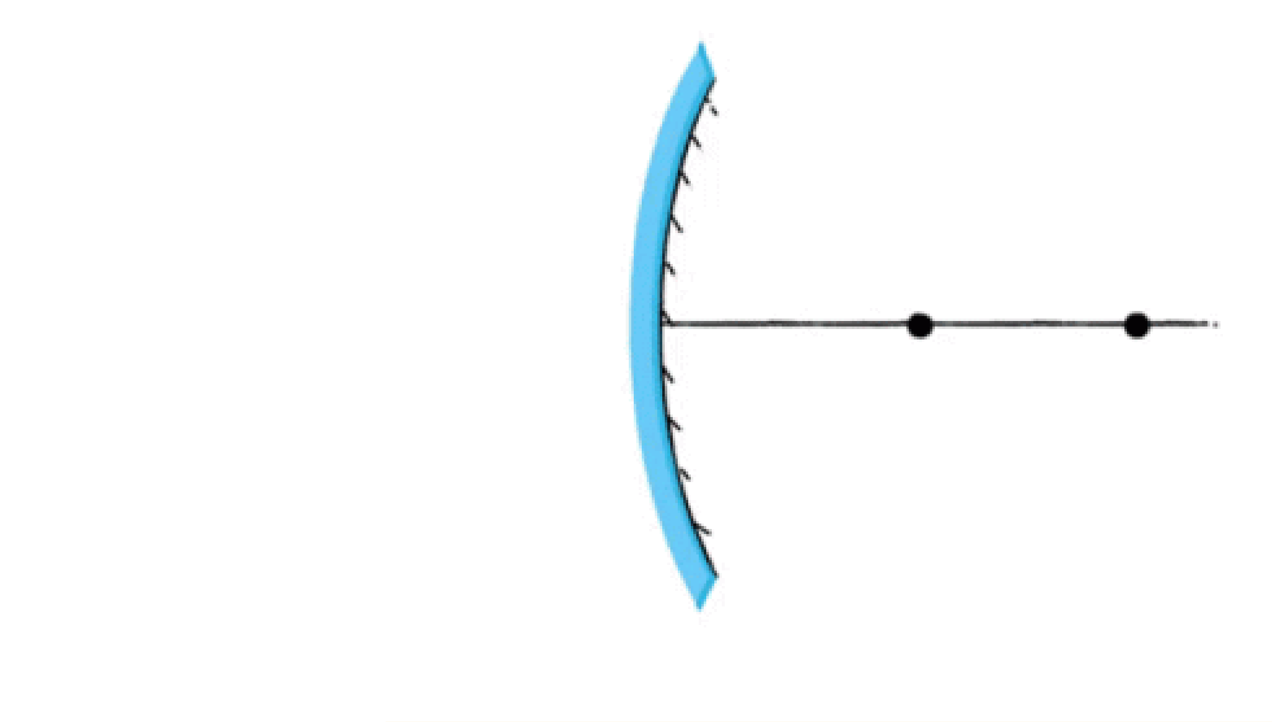

Convex (or diverging) mirror

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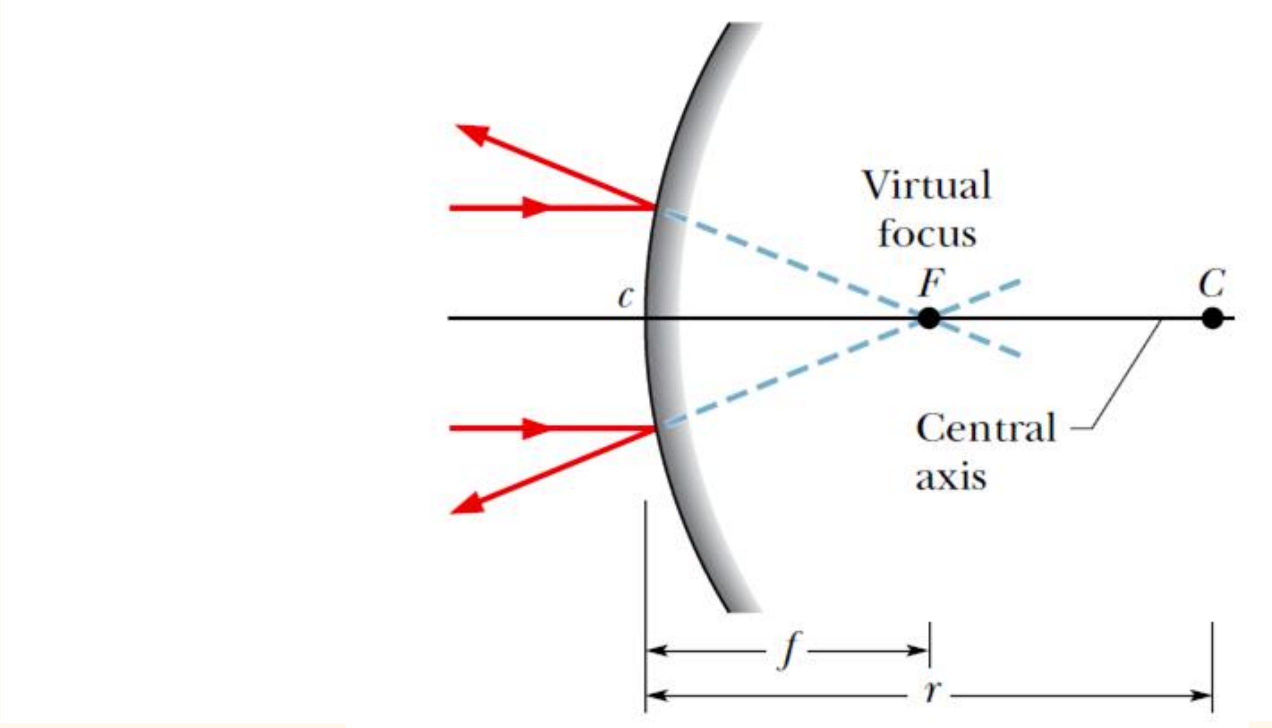

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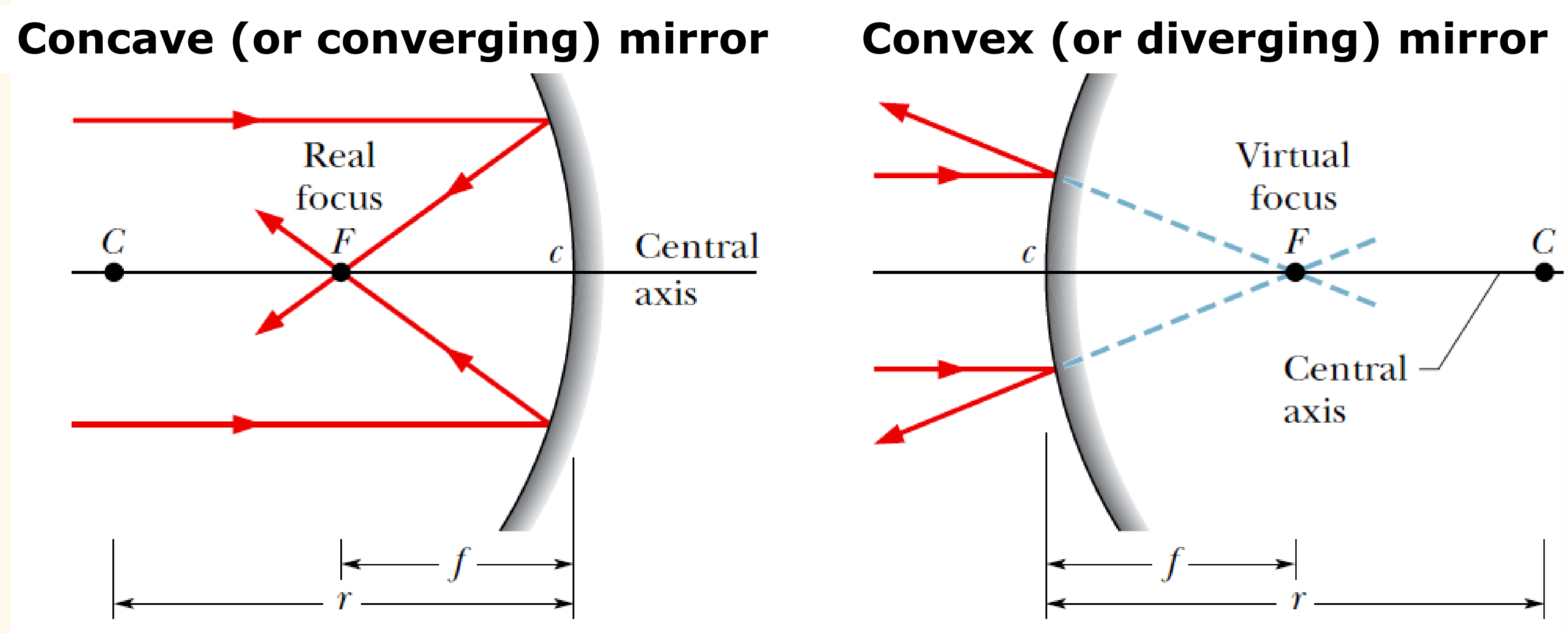


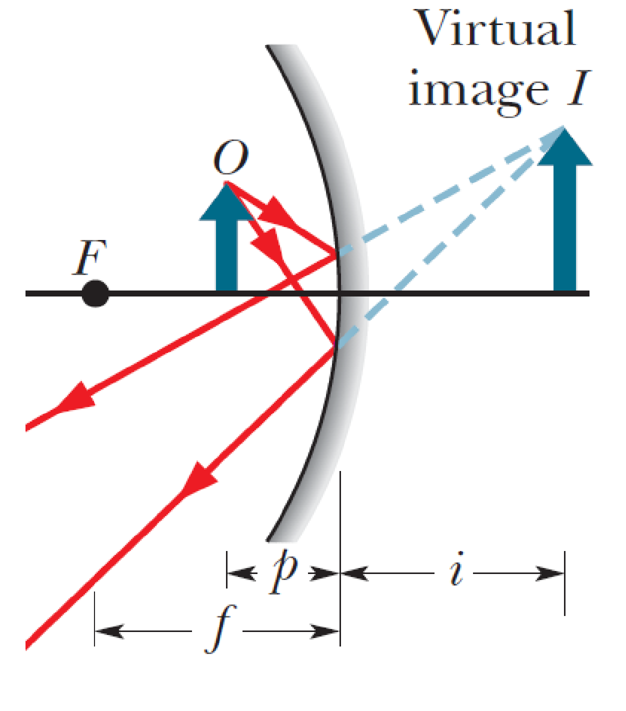
$$f = \frac{r}{2}$$

Concave mirror
→ $f > 0$
F real

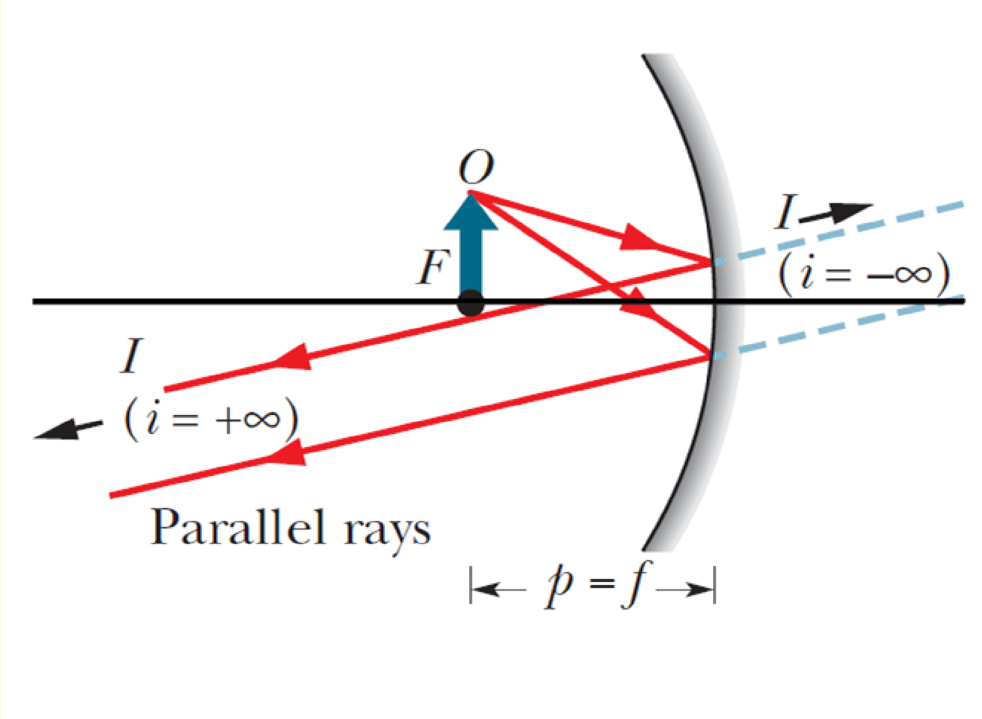
Convex mirror
→ $f < 0$
F virtual

Relation between object distance p and image distance i ?

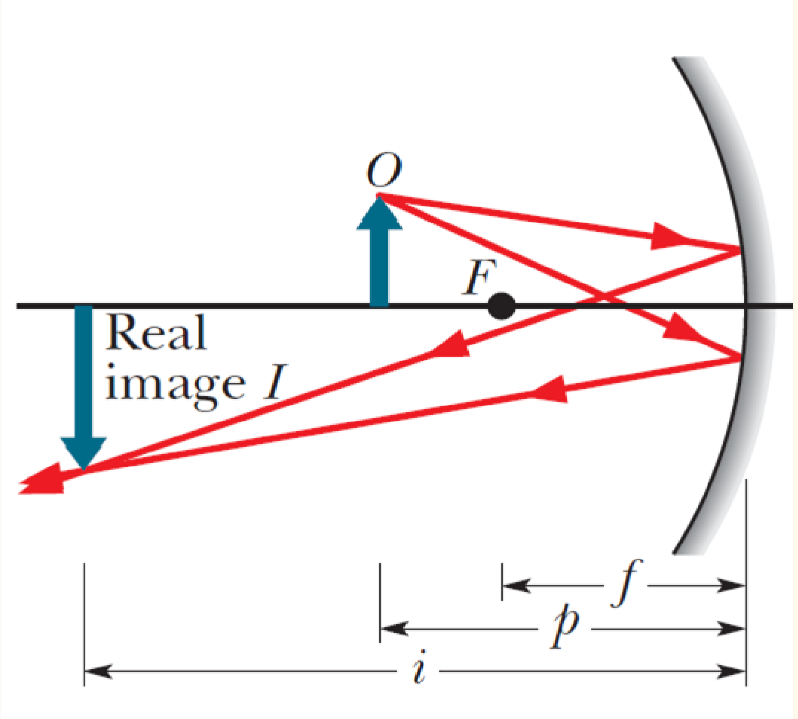
Concave mirrors



$p < f$
Virtual image
not reversed
$i < 0$



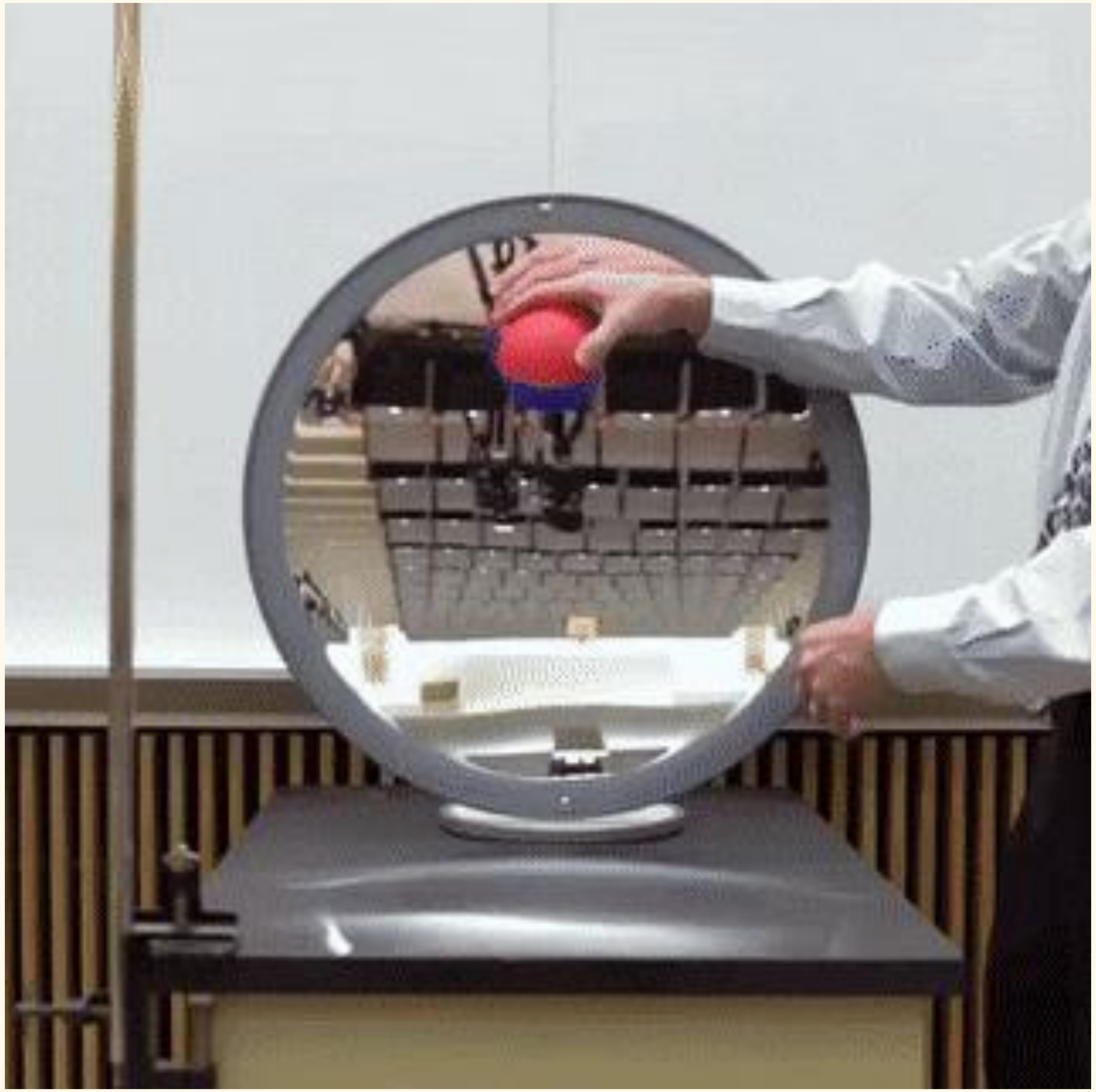
$p = f$
ambiguous image
i at $\pm\infty$



$p > f$
Real image
reversed
$i > 0$

SPHERICAL MIRRORS

Concave mirrors



Generalization for spherical mirrors

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

Will be demonstrated soon (optionnal)

It can be demonstrated that

$$m = -\frac{i}{p}$$

Magnification

h and h′ respective size of the object and the image

$$|\boldsymbol{m}| = \frac{\boldsymbol{h'}}{\boldsymbol{h}}$$ Lateral magnification

By convention:

m > 0 if not reversed

m < 0 if reversed

Proof of the spherical mirror formula:

In OaC: $\alpha + \theta + (180° - \beta) = 180°$

$\alpha + \theta = \beta$

In OaI: $\alpha + 2\theta + (180° - \gamma) = 180°$

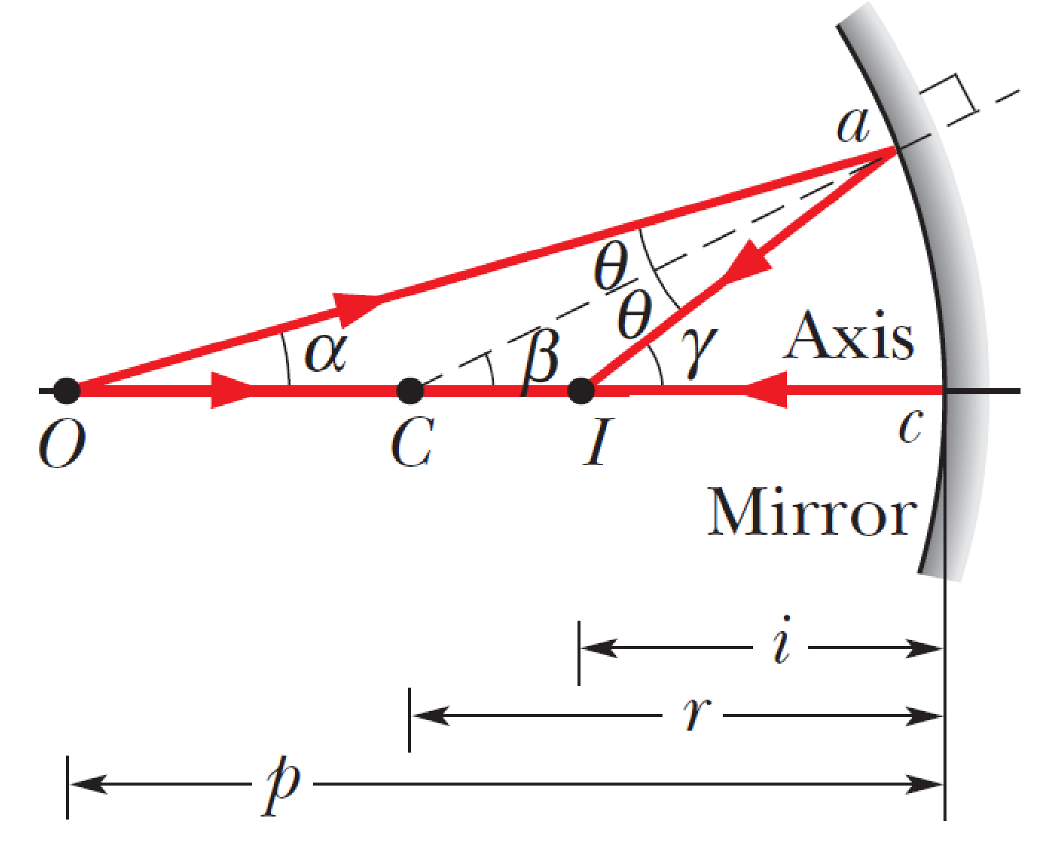
$\alpha + 2\theta = \gamma$

Thus, $\gamma = \alpha + 2(\beta - \alpha) = 2\beta - \alpha$

$\gamma + \alpha = 2\beta$

Writing the angles in radians:

$$\alpha \simeq \frac{\overset{\frown}{ac}}{Oc} = \frac{\overset{\frown}{ac}}{p} \qquad \beta = \frac{\overset{\frown}{ac}}{Cc} = \frac{\overset{\frown}{ac}}{r} \qquad \gamma \simeq \frac{\overset{\frown}{ac}}{Ic} = \frac{\overset{\frown}{ac}}{i}$$



Note: $\overset{\frown}{ac}$ is an arc length, approximations valid for small angles α and γ

Proof of the spherical mirror formula:

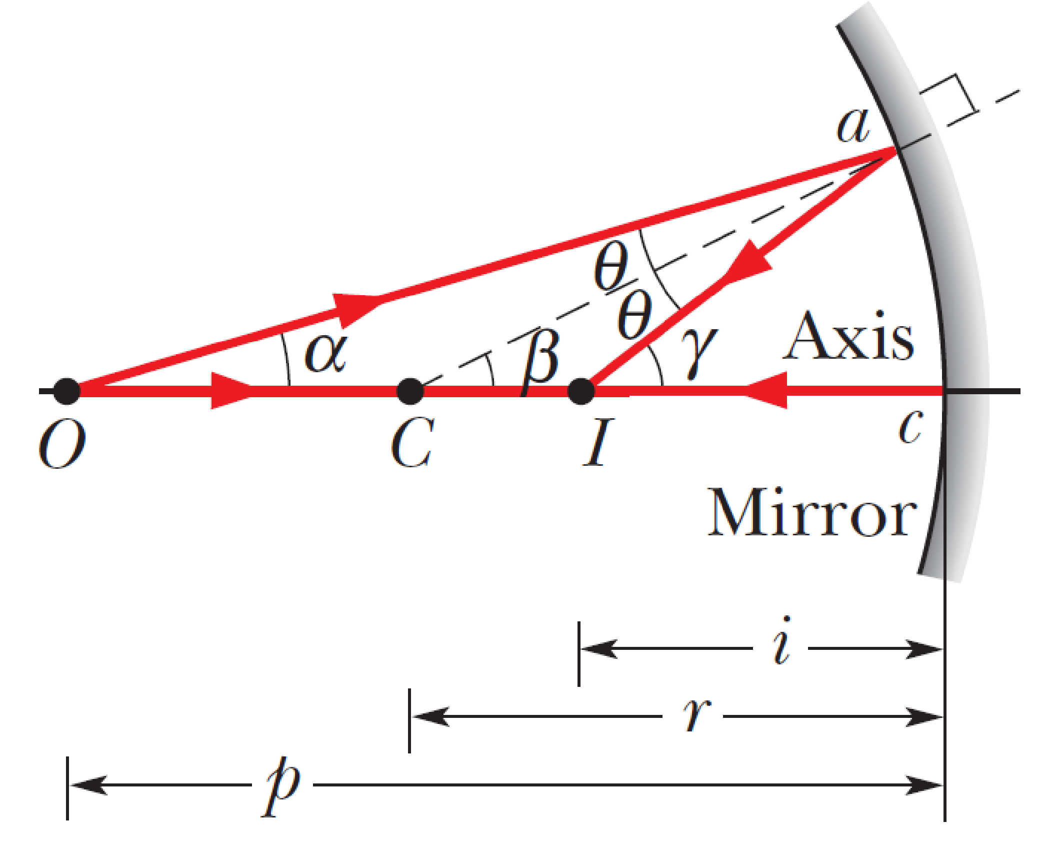
We have: $\gamma + \alpha = 2\beta$

$\alpha \simeq \frac{\overset{\frown}{ac}}{Oc} = \frac{\overset{\frown}{ac}}{\mathrm{p}}$ $\quad \beta = \frac{\overset{\frown}{ac}}{Cc} = \frac{\overset{\frown}{ac}}{\mathrm{r}}$ $\quad \gamma \simeq \frac{\overset{\frown}{ac}}{Ic} = \frac{\overset{\frown}{ac}}{\mathrm{i}}$

So: $\frac{\overset{\frown}{ac}}{i} + \frac{\overset{\frown}{ac}}{p} = 2\frac{\overset{\frown}{ac}}{r}$

With $f = r/2$

$$\frac{1}{i} + \frac{1}{p} = \frac{1}{f}$$



→ Geometrical consequence of this formula:
Existence of particular rays

Geometrical construction of images with **particular rays**

Incident rays **parallel to the central axis** reflect **passing through F**

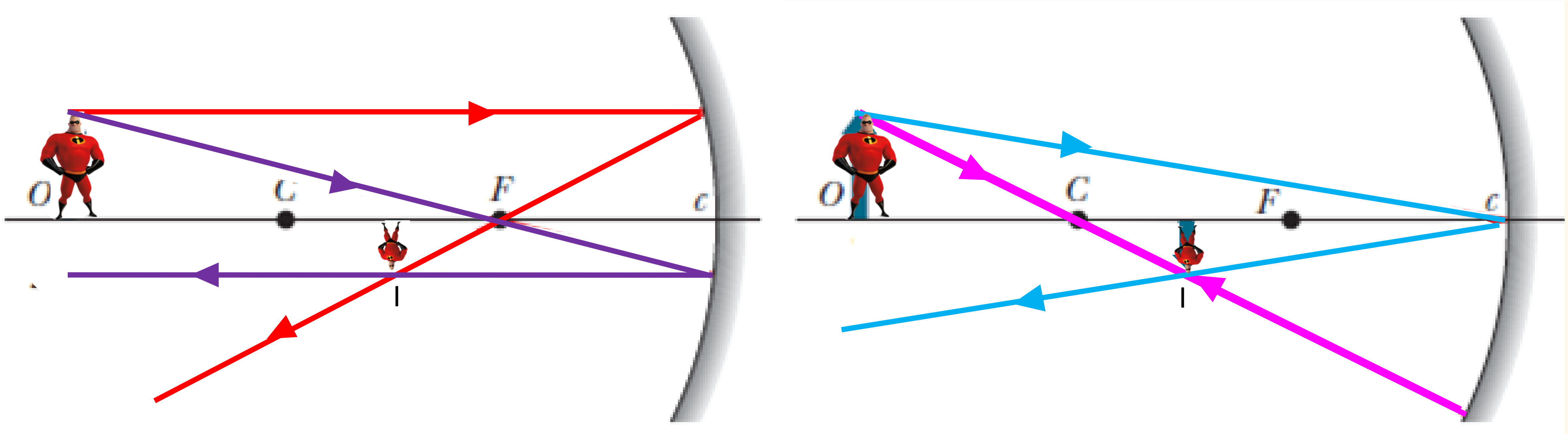
Incident rays passing **through F** reflect **parallel to the central axis**

Incident rays passing **through C** reflect on the **same path**

Incident rays passing **through c** reflect **symmetrically**

→ Using (at least) the intersection of **2 of these rays** to construct the image

Geometrical construction of images with **particular rays**



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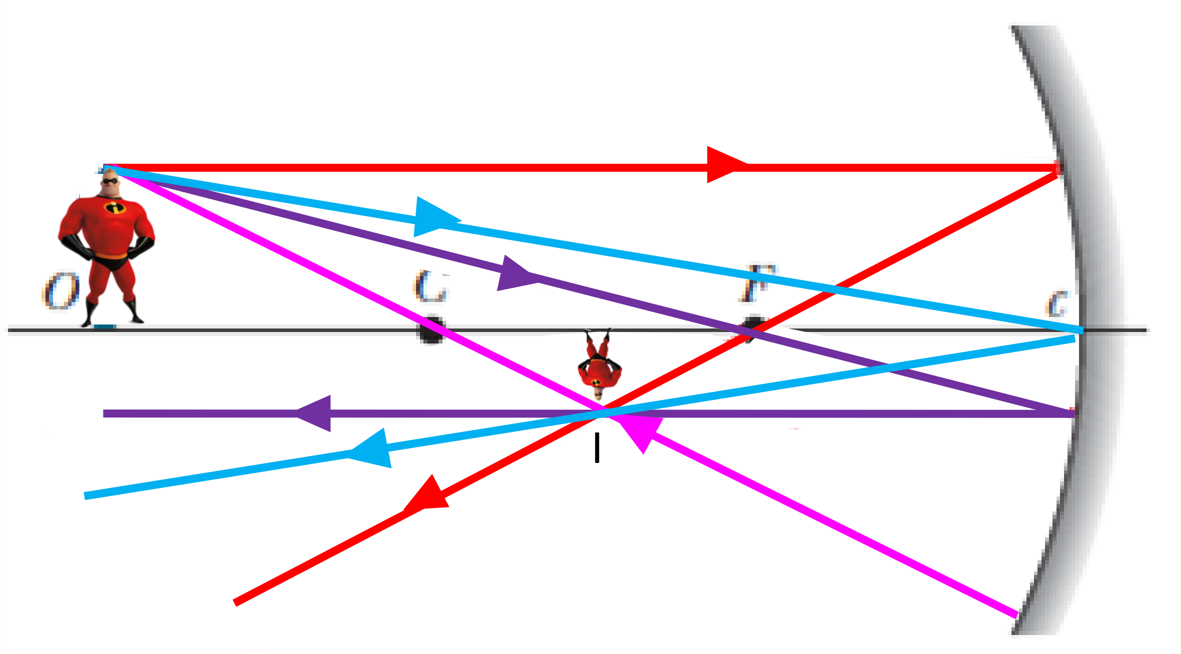
Geometrical construction of images with **particular rays**



$$\text{——} = \frac{h'}{h} = m$$ Lateral magnification

Incident rays **parallel to the central axis** reflect **passing through F**

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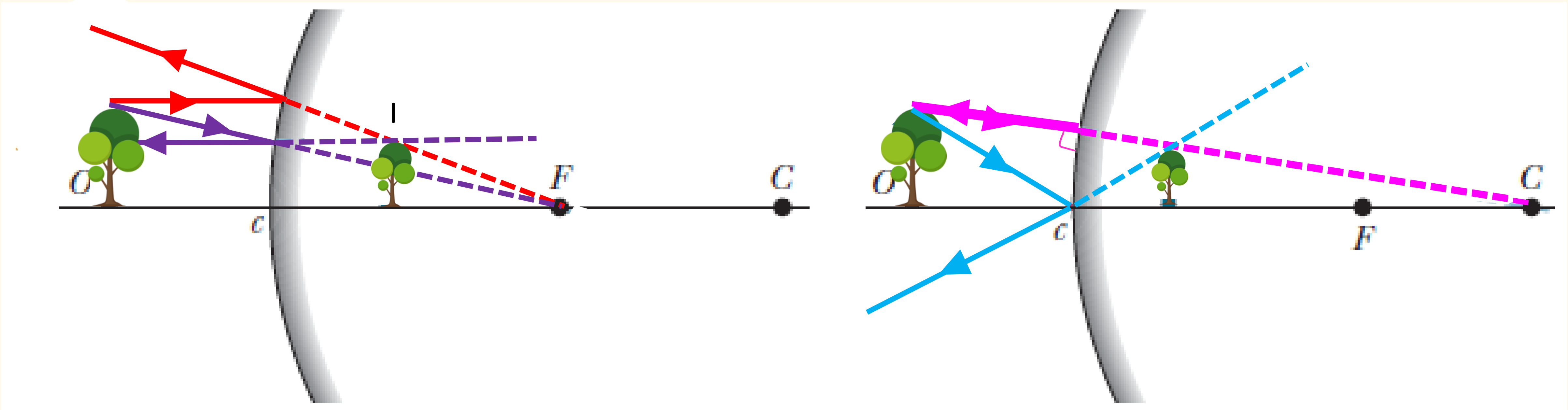
Geometrical construction of images with **particular rays**



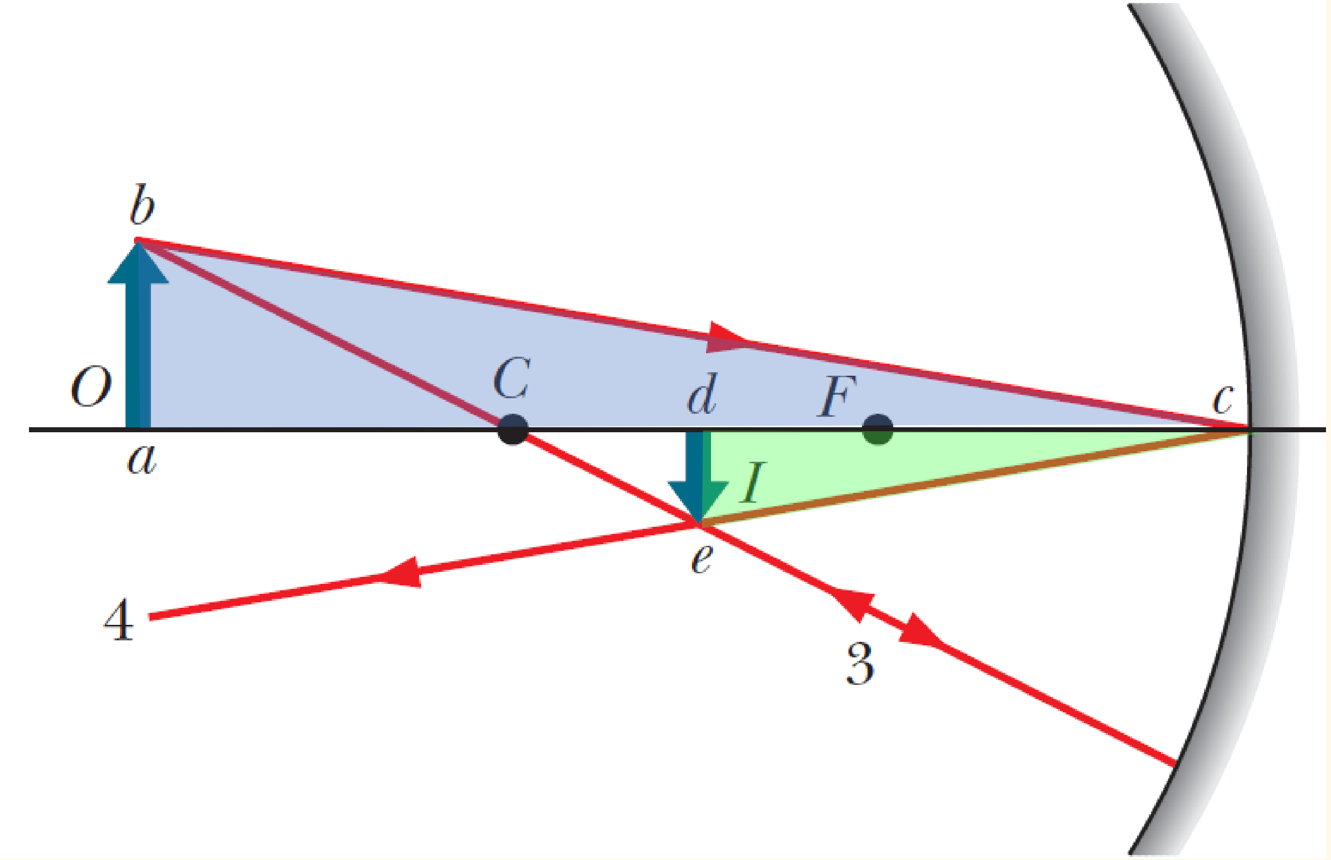
Incident rays **parallel to the central axis** reflect **passing through F**

Incident rays passing **through F** reflect **parallel to the central axis**

Incident rays passing **through C** reflect on the **same path**

Incident rays passing **through c** reflect **symmetrically**

Demonstration of the lateral magnification

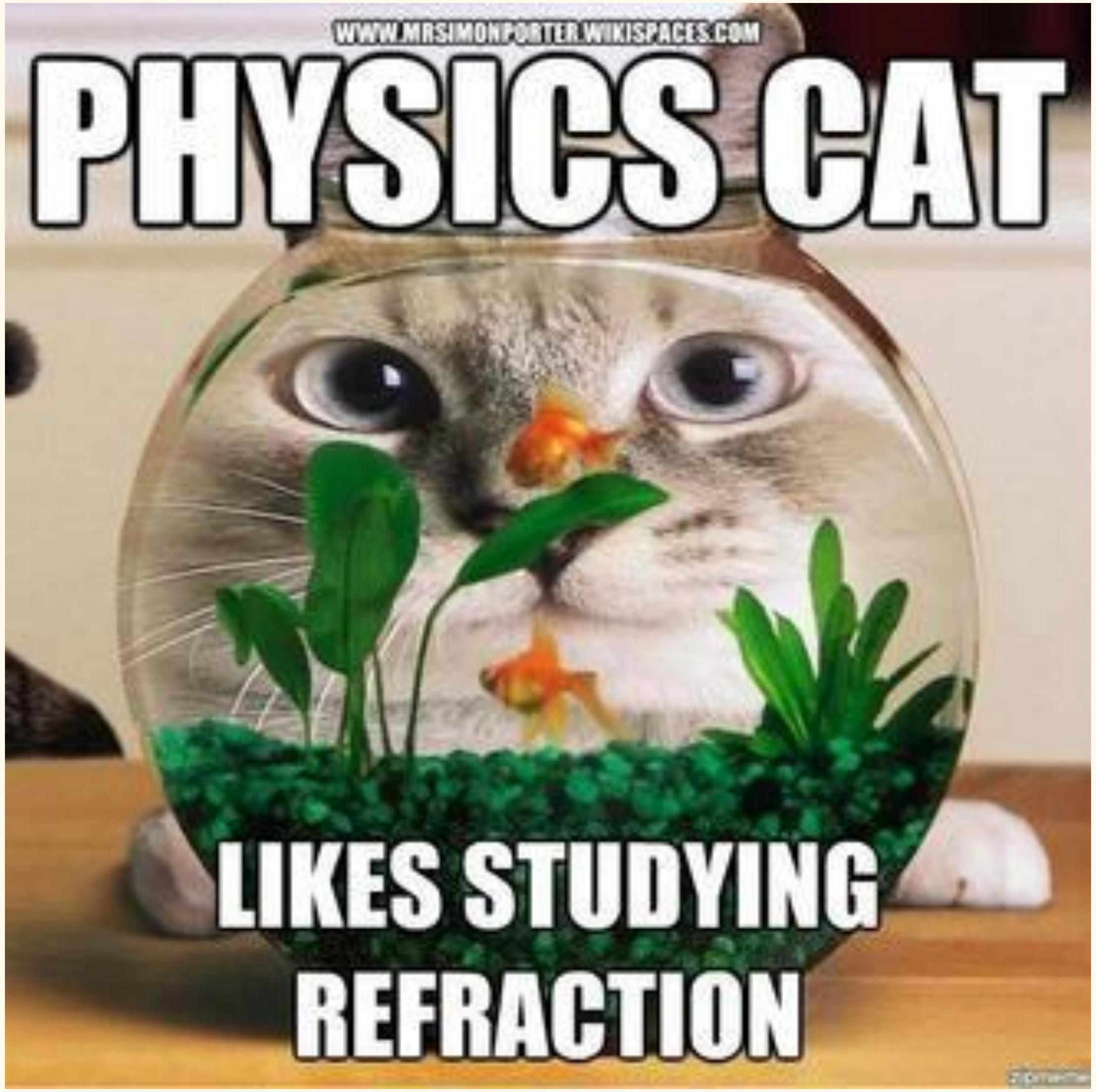


bac and *edc* are congruent

$$\frac{ab}{ac} = \frac{de}{dc}$$

With algebraic distances:

$$\frac{h}{p} = -\frac{h'}{i} \longrightarrow \frac{-i}{p} = \frac{h'}{h} = m$$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REFRACTION

SPHERICAL REFRACTIVE SURFACES

Concave or convex spherical
volumes of refractive index $\mathbf{n_2}$
External medium of index $\mathbf{n_1}$

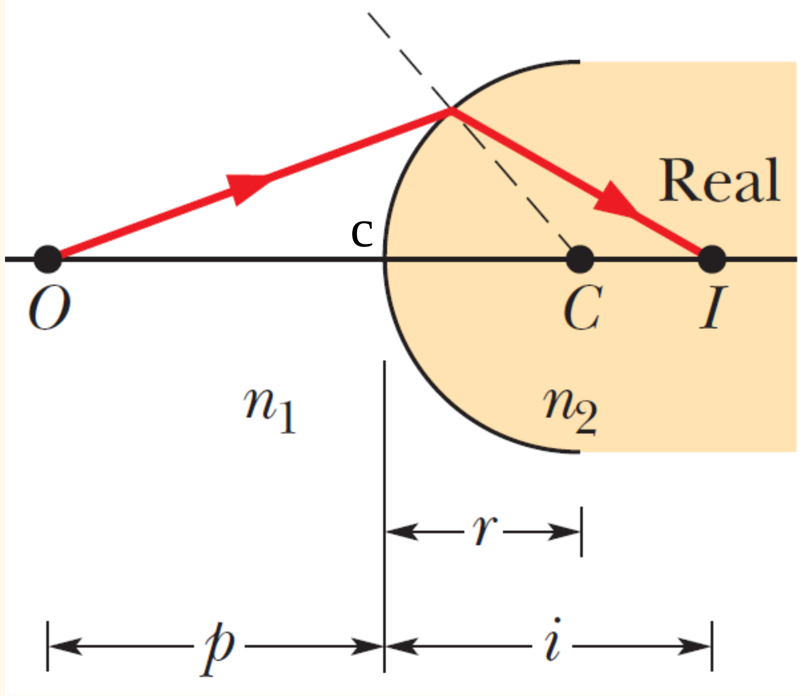
Like spherical mirror, we define:

- Center of curvature C
- Center c
- Central axis
- Radius r

But unlike spherical mirrors

r < 0 for concave

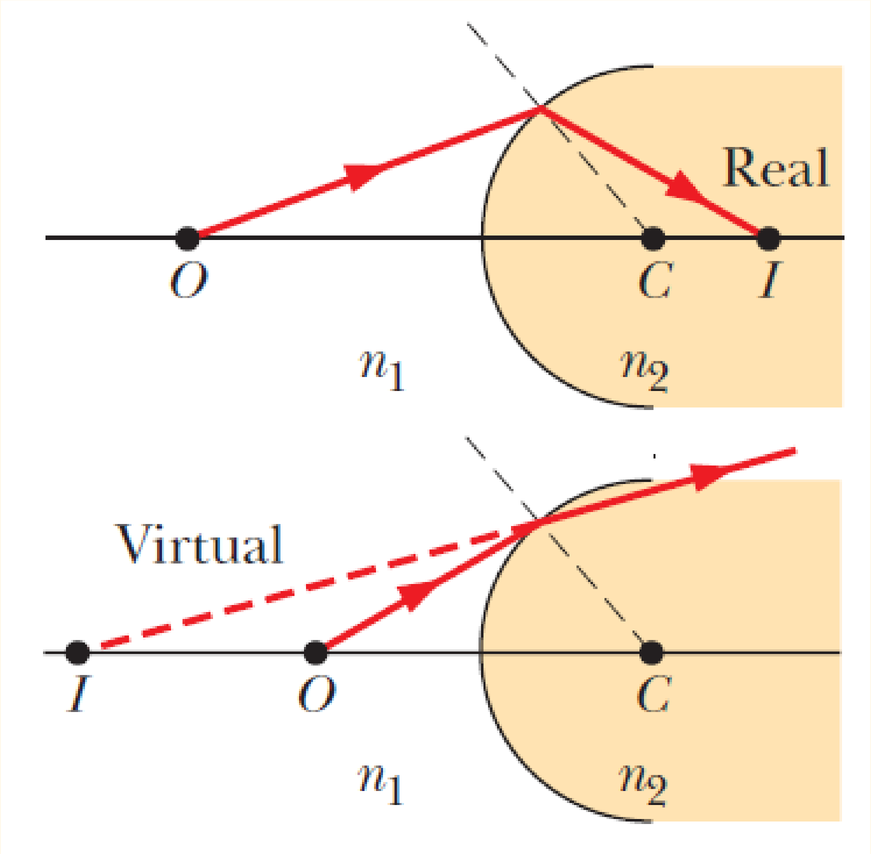
r > 0 for convex



Incident rays are refracted at the interface

SPHERICAL REFRACTIVE SURFACES - **OPTIONNAL**

Images can be real or virtual



General formula for spherical refractive surfaces

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$$

Will be demonstrated soon (optionnal)

But unlike spherical mirrors

r < 0 for concave

r > 0 for convex

Proof of the spherical refractive surface formula:

In OaC: $\alpha + \beta + (180° - \theta_1) = 180°$

$\alpha + \beta = \theta_1$

In OaI: $\alpha + \gamma + ((180° - \theta_1) + \theta_2) = 180°$

$\alpha + \gamma + \theta_2 = \theta_1$

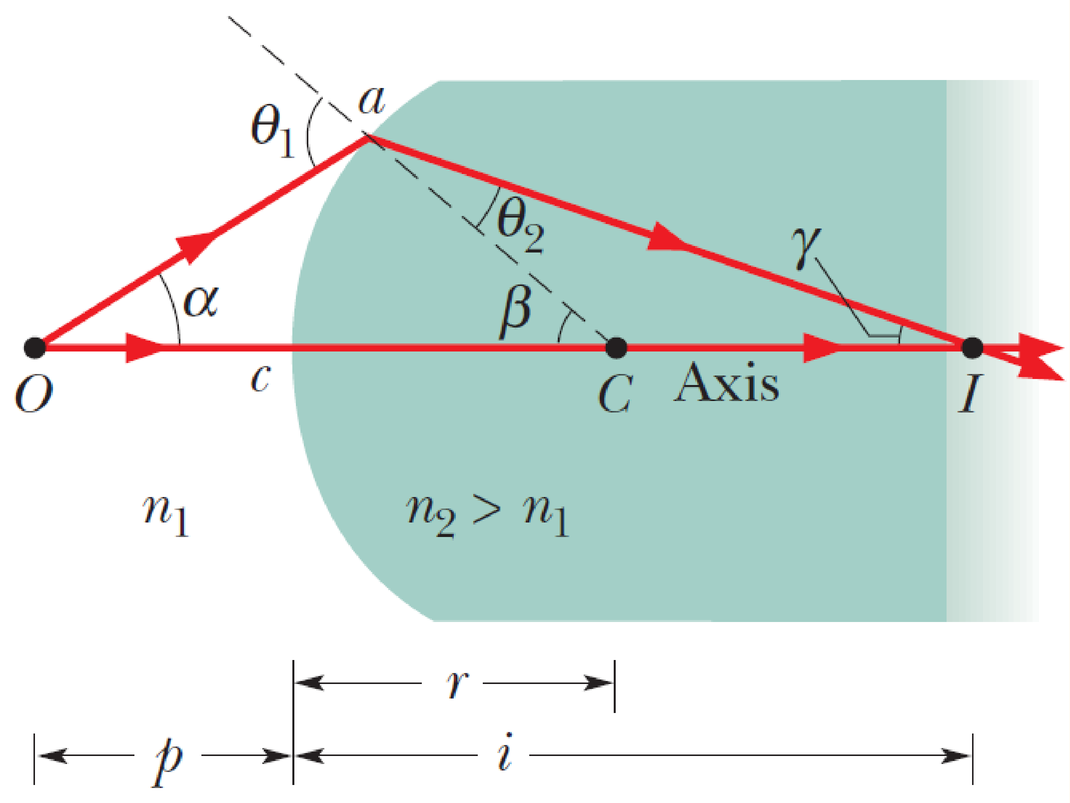
Thus: $\alpha + \gamma + \theta_2 = \alpha + \beta$

$\gamma + \theta_2 = \beta$

S-D law of refraction: $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$

For small angles $\sin(x) \simeq x$

So we can write: $n_1\theta_1 = n_2\theta_2$



Valid for ray close to the axis or for small curvatures (Paraxial approximation)

Proof of the spherical refractive surface formula:

We have: $\alpha + \beta = \theta_1$ $\quad n_1\theta_1 = n_2\theta_2$

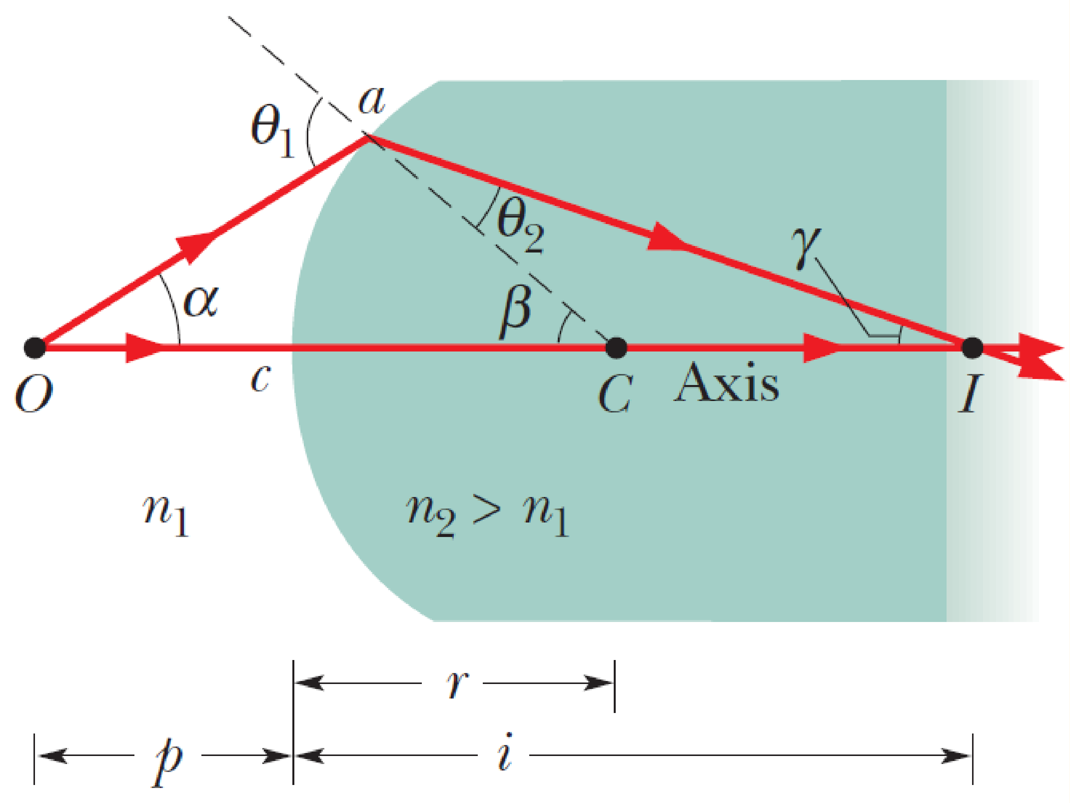
$\gamma + \theta_2 = \beta$

So: $\gamma + \frac{n_1}{n_2}\theta_1 = \beta$

$$\gamma + \frac{n_1}{n_2}(\alpha + \beta) = \beta$$

$$\gamma + \frac{n_1}{n_2}\alpha = \left(1 - \frac{n_1}{n_2}\right)\beta$$

$$n_2\gamma + n_1\alpha = (n_2 - n_1)\beta$$



Proof of the spherical refractive surface formula:

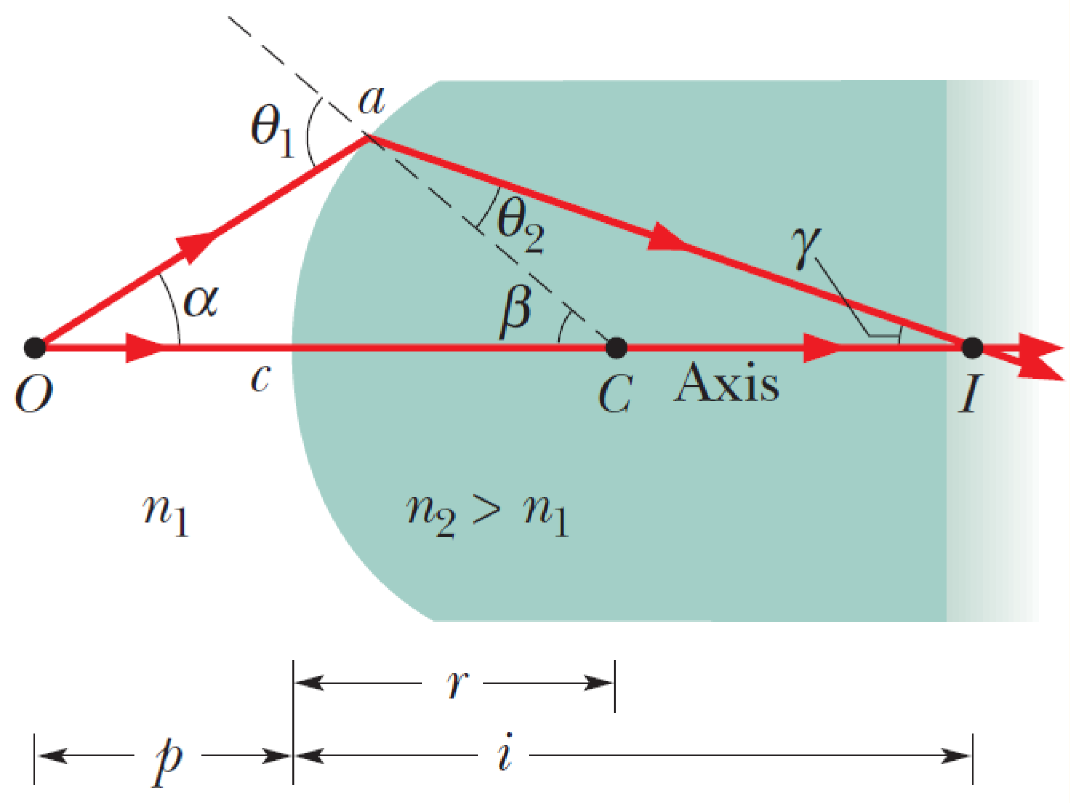
We have: $n_2\gamma + n_1\alpha = (n_2 - n_1)\beta$

Writing the angles in radians:

$$\alpha \simeq \frac{\overset{\frown}{ac}}{Oc} = \frac{\overset{\frown}{ac}}{p} \qquad \beta = \frac{\overset{\frown}{ac}}{Cc} = \frac{\overset{\frown}{ac}}{r} \qquad \gamma \simeq \frac{\overset{\frown}{ac}}{Ic} = \frac{\overset{\frown}{ac}}{i}$$

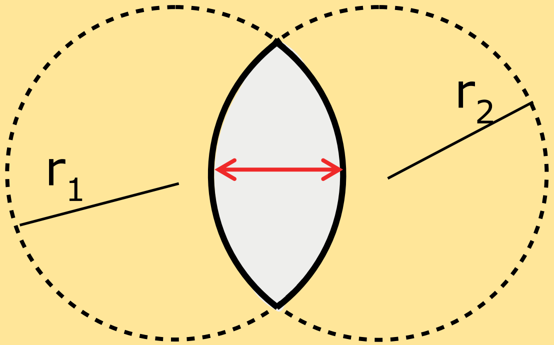
So: $n_2\frac{\overset{\frown}{ac}}{i} + n_1\frac{\overset{\frown}{ac}}{p} = (n_2 - n_1)\frac{\overset{\frown}{ac}}{r}$

$$\frac{n_2}{i} + \frac{n_1}{p} = \frac{n_2 - n_1}{r}$$



Lens: Optical component made of **two refracting surfaces**

→ We will study the case where surfaces = spheres of radius r_1 and r_2



Thin lens approximation:

Thickest part << r_1, r_2, p, i

In the following, we assume that we are in the case of **Paraxial Approximation:**

Rays are **close** to the central axis and form **small angles** with the central axis

For the sake of clarity distances and angles are exaggerated in the figures

Within paraxial approximation, for thin lenses we have a **General formula**

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}$$

(The same than for spherical mirrors)

Focal length f is defined by the lens maker's equation

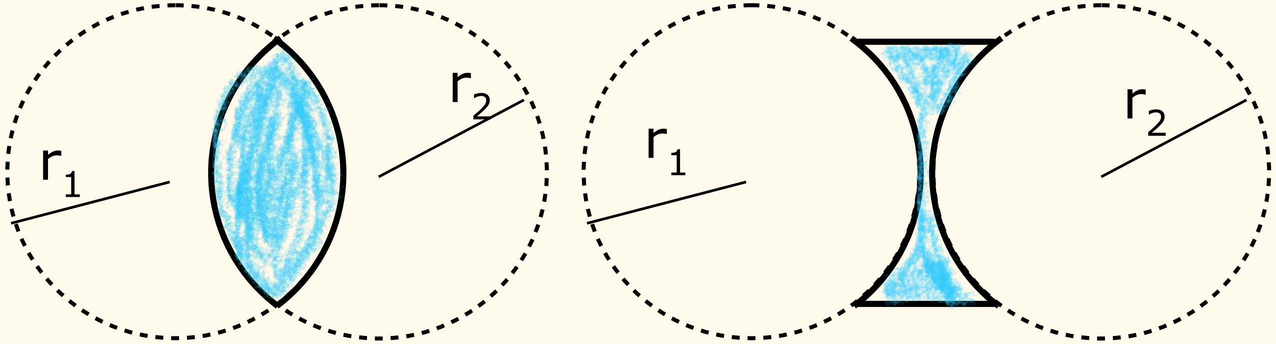
$$\frac{1}{f} = \left(\frac{n_{lens}}{n_{medium}} - 1\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

r_1, r_2: radius of curvature of the surfaces

n_{lens}, n_{medium} : refractive index of the lens and the surrounding medium

Sign convention on r for refractive surfaces

Convex lens | **Concave lens**

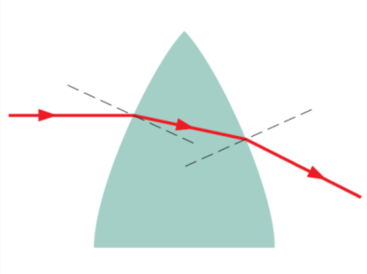


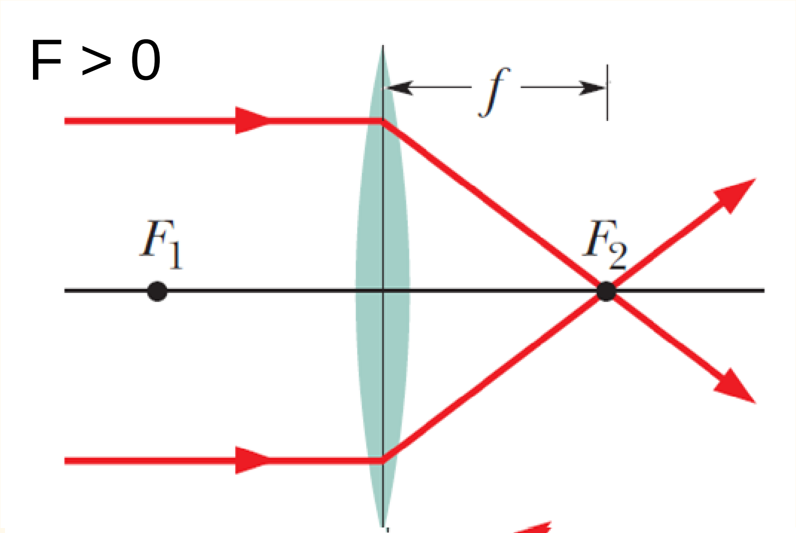
$r_1 > 0$ & $r_2 < 0$ → **f > 0**

$r_1 < 0$ & $r_2 > 0$ → **f < 0**

THIN LENSES

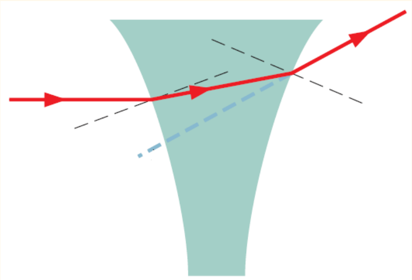
2 refractions

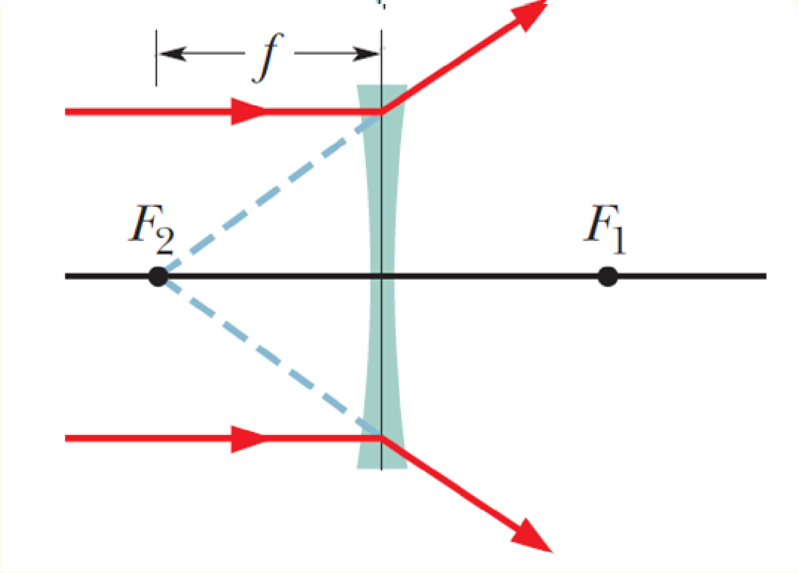




Lenses **focus light**
→ Rays bend at the interfaces

Rays parallel to the central axis converge → **Converging lens**
f > 0

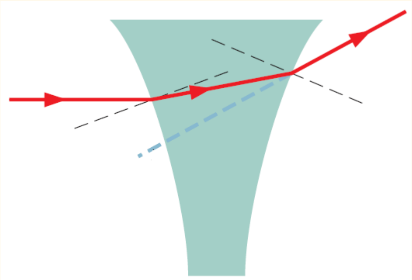


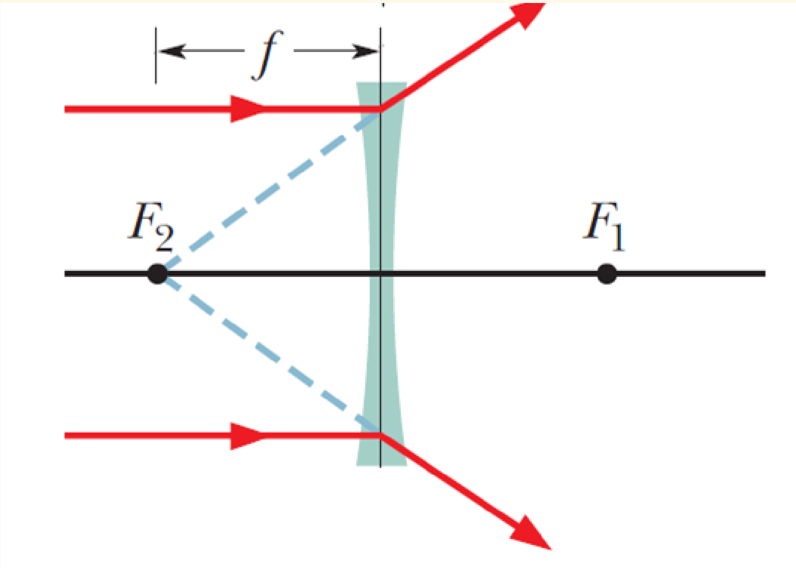


Rays parallel to the central axis diverge → **Diverging lens**
f < 0

Rays converge at the **real focal point F_2** at distance **f** of the center
another real focal point **F_1** is at **-f**

Note:
For diverging lenses the extension of the rays converge to F_2even if the real rays diverge





Images for thin lenses:

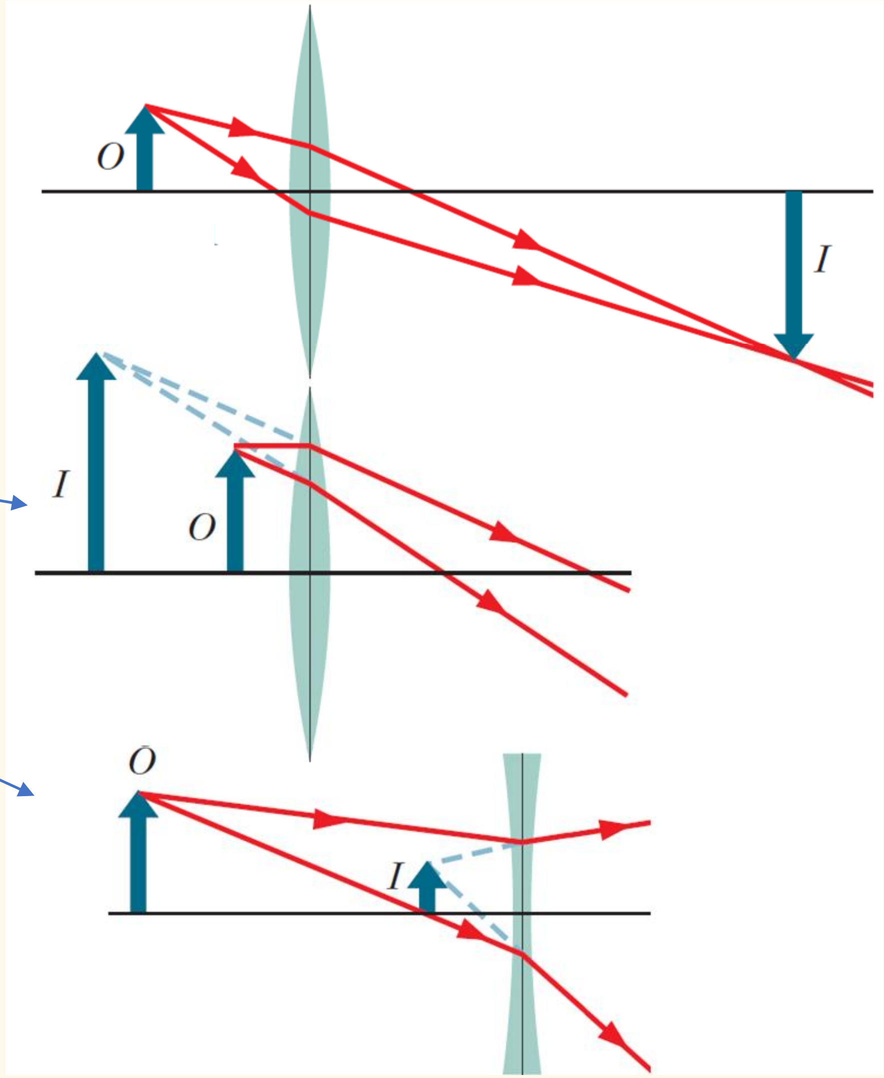
$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i} \longrightarrow i = \frac{p.f}{p - f}$$

Converging lenses: $f > 0 \rightarrow i >$ or < 0
Real or virtual images

Diverging lenses: $f < 0 \rightarrow i$ always < 0
Virtual images

Magnification: $m = -\frac{i}{p}$ (The same than for spherical mirrors)

Note: For lenses, real (resp. virtual) images are on the opposite (resp. same) side than the object



Images for thin lenses:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i} \longrightarrow i = \frac{p.f}{p - f}$$

Important convention about the sign of p, i and f

$f > 0$ for convex lens

$f < 0$ for concave lens

$p > 0$ if object to the left of lens

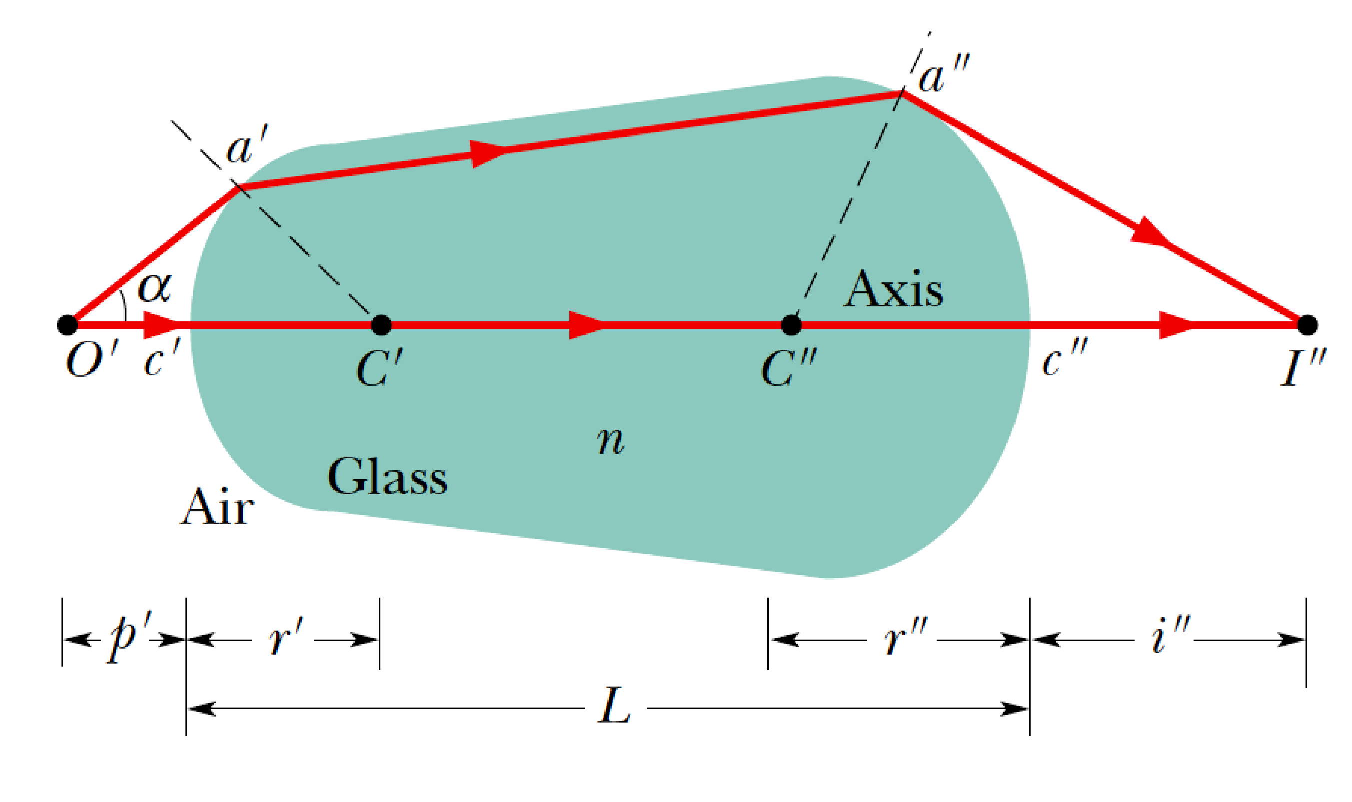
$p < 0$ if object to the right of lens

$i < 0$ if image to the left of lens

$i > 0$ if image to the right of lens

Proof of the thin lenses formula:

We first assume than the lens is not thin → 2 spherical surfaces
- Radii r′ and r′′
- Center of curvature C′ and C′′
- Center c′ and c′′ spaced by L



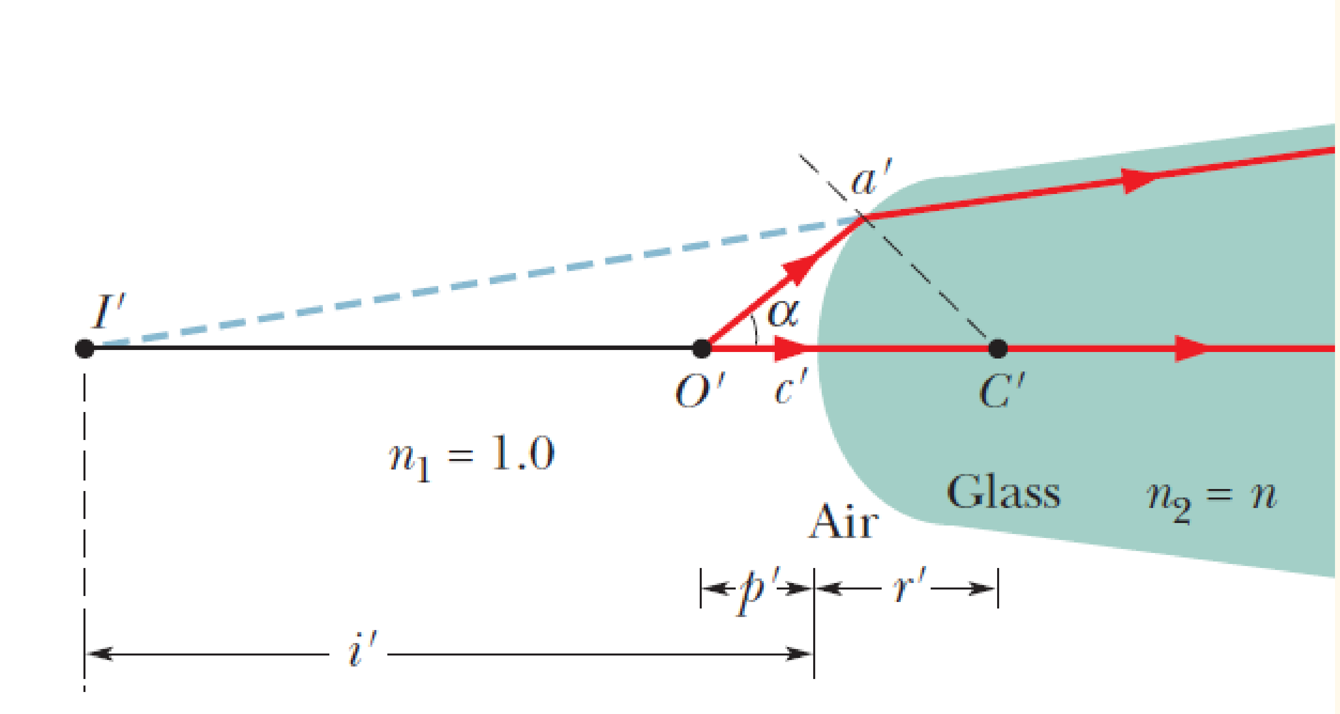
Rays from O′ on the central axis undergo 2 refraction to form the image I′′

We treat the 2 surfaces separately

Proof of the thin lenses formula:

First surface:
I' is the virtual image of O'
Generalized formula for spherical refractive surfaces ($n_1 = 1$, $n_2 = n$)

$$\frac{1}{p'} + \frac{n}{i'} = \frac{n-1}{r'}$$



Note that the distances are algebraic here: $i' < 0$
We rewrite the formula explicitly:

$$\frac{1}{p'} - \frac{n}{j'} = \frac{n-1}{r'}$$

With $j' = -i'$

Proof of the thin lenses formula:

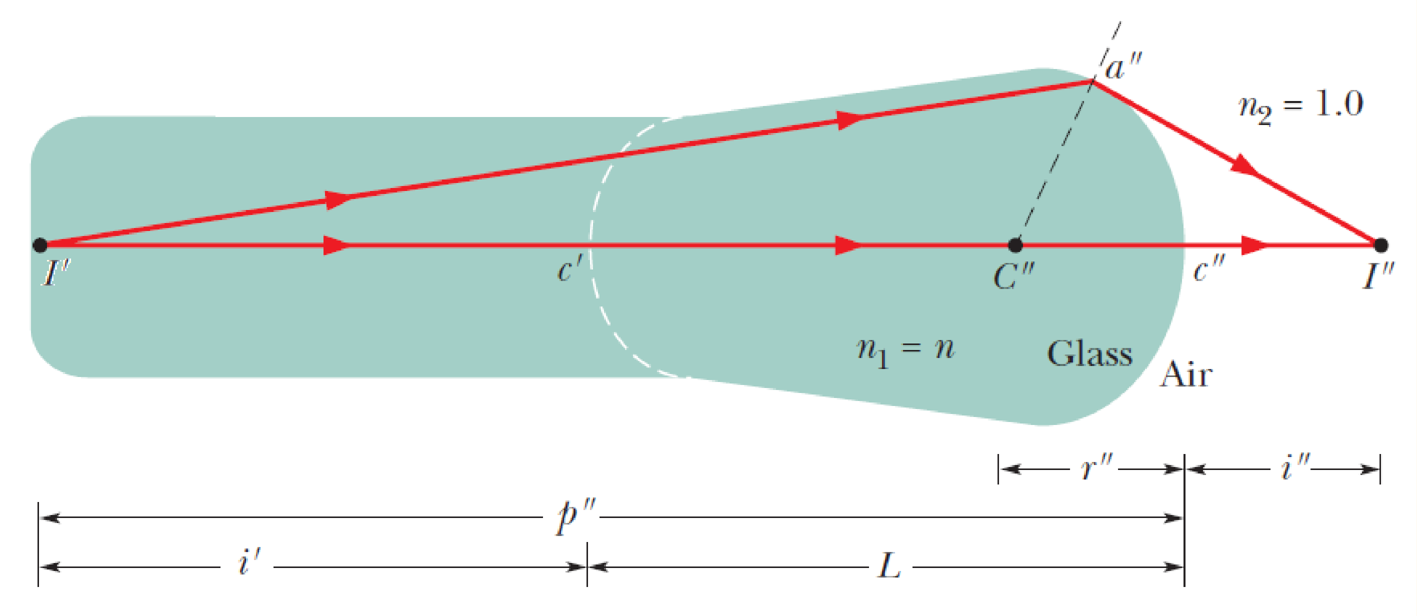
Second surface:

I" is the image of *I'*

I' is a real object for the 2nd surface

Spaced by L+j' of c'

Generalized formula for spherical refractive surfaces ($n_1 = n$, $n_2 = 1$)



$$\frac{n}{L + j'} + \frac{1}{i''} = \frac{1 - n}{r''}$$

We now assume that the lens is thin L <<

$$\frac{n}{j'} + \frac{1}{i''} = \frac{1 - n}{r''}$$

Proof of the thin lenses formula:

We have: $\frac{1}{p'} - \frac{n}{j'} = \frac{n-1}{r'}$ and $\frac{n}{j'} + \frac{1}{i''} = \frac{1-n}{r''}$

Thus,

$$\frac{1}{p'} - \left(\frac{1-n}{r''} - \frac{1}{i''}\right) = \frac{n-1}{r'}$$

$$\frac{1}{p'} + \frac{n-1}{r''} + \frac{1}{i''} = \frac{n-1}{r'}$$

$$\frac{1}{p'} + \frac{1}{i''} = (n-1)\left(\frac{1}{r'} - \frac{1}{r''}\right)$$

For the final steps, we will rename p' as p (position of the object) and I'' as I (position of the image) to use the previous notations, the last expression is rewritten:

$$\frac{1}{p} + \frac{1}{i} = (n-1)\left(\frac{1}{r'} - \frac{1}{r''}\right)$$

Proof of the thin lenses formula:

We have: $\frac{1}{p} + \frac{1}{i} = (n-1)\left(\frac{1}{r'} - \frac{1}{r''}\right)$

Lens maker's formula: $\frac{1}{f} = \left(\frac{n_{lens}}{n_{medium}} - 1\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Here $n_{lens} = n$, $n_{medium} = 1$, $r_1 = r'$ and $r_2 = r''$ so $\frac{1}{f} = (n-1)\left(\frac{1}{r'} - \frac{1}{r''}\right)$

Finally: $\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$

Geometrical construction of images with **particular rays**

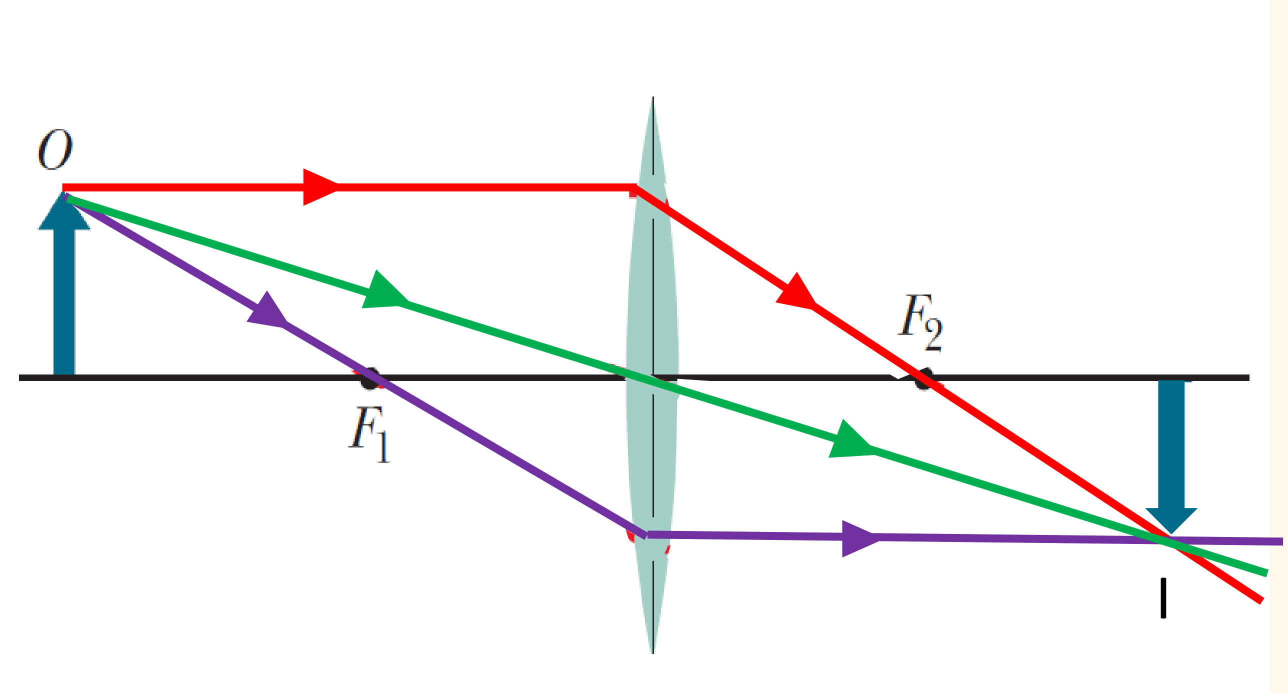
Incident rays **parallel to the central axis** will **pass through F_2**

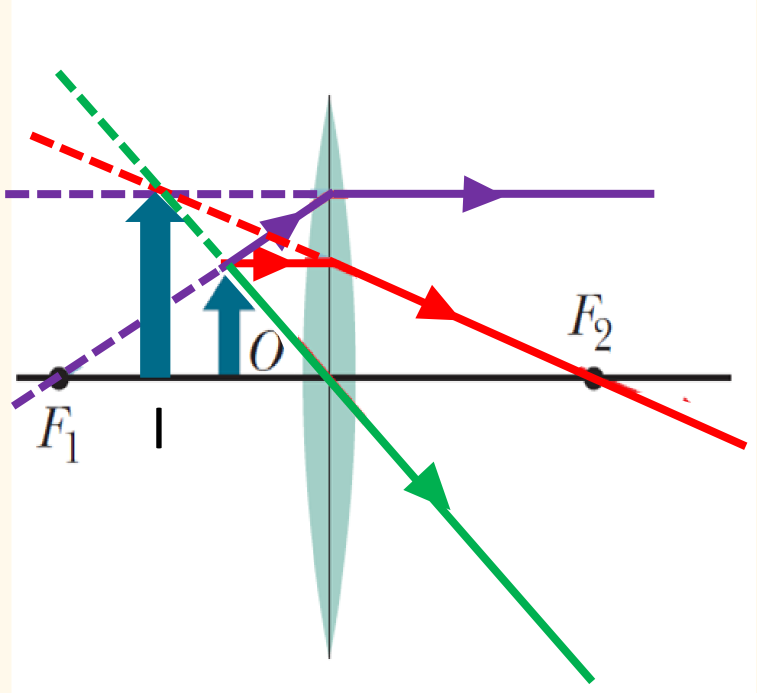
Incident rays passing **through F_1** will be **parallel to the central axis**

Incident rays passing **through the center** are **not deviated**

→ Using (at least) the intersection of **2 of these rays** to construct the image

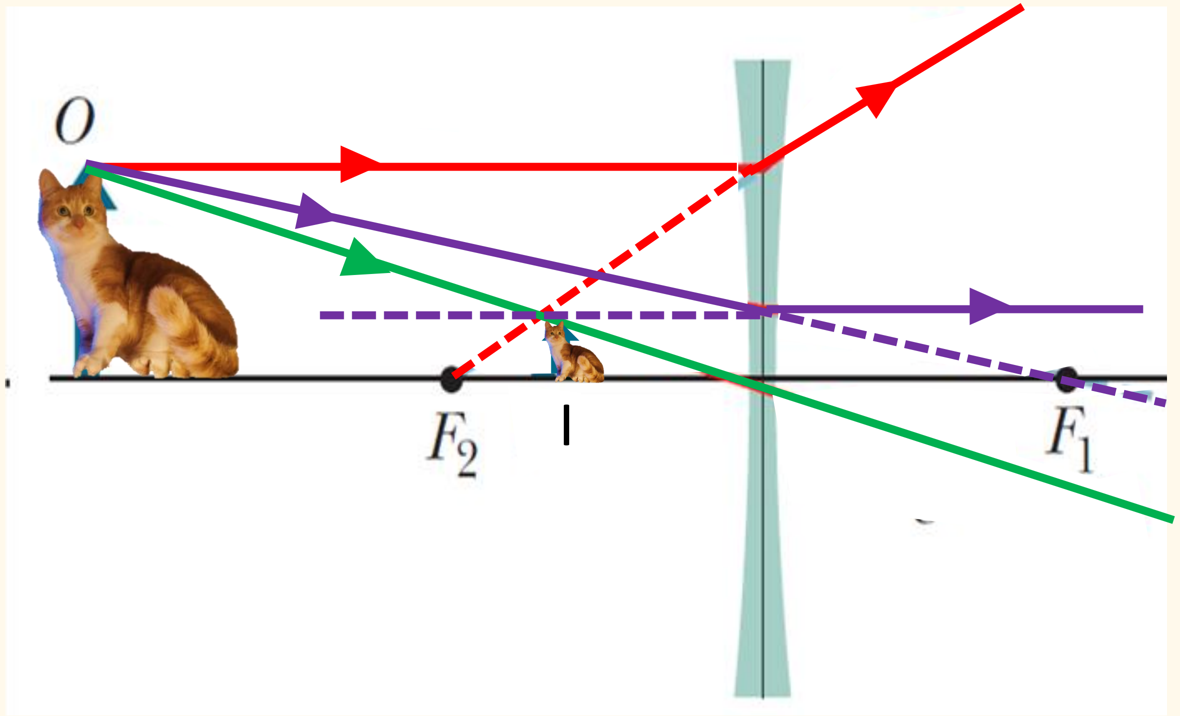
Geometrical construction of images with **particular rays**





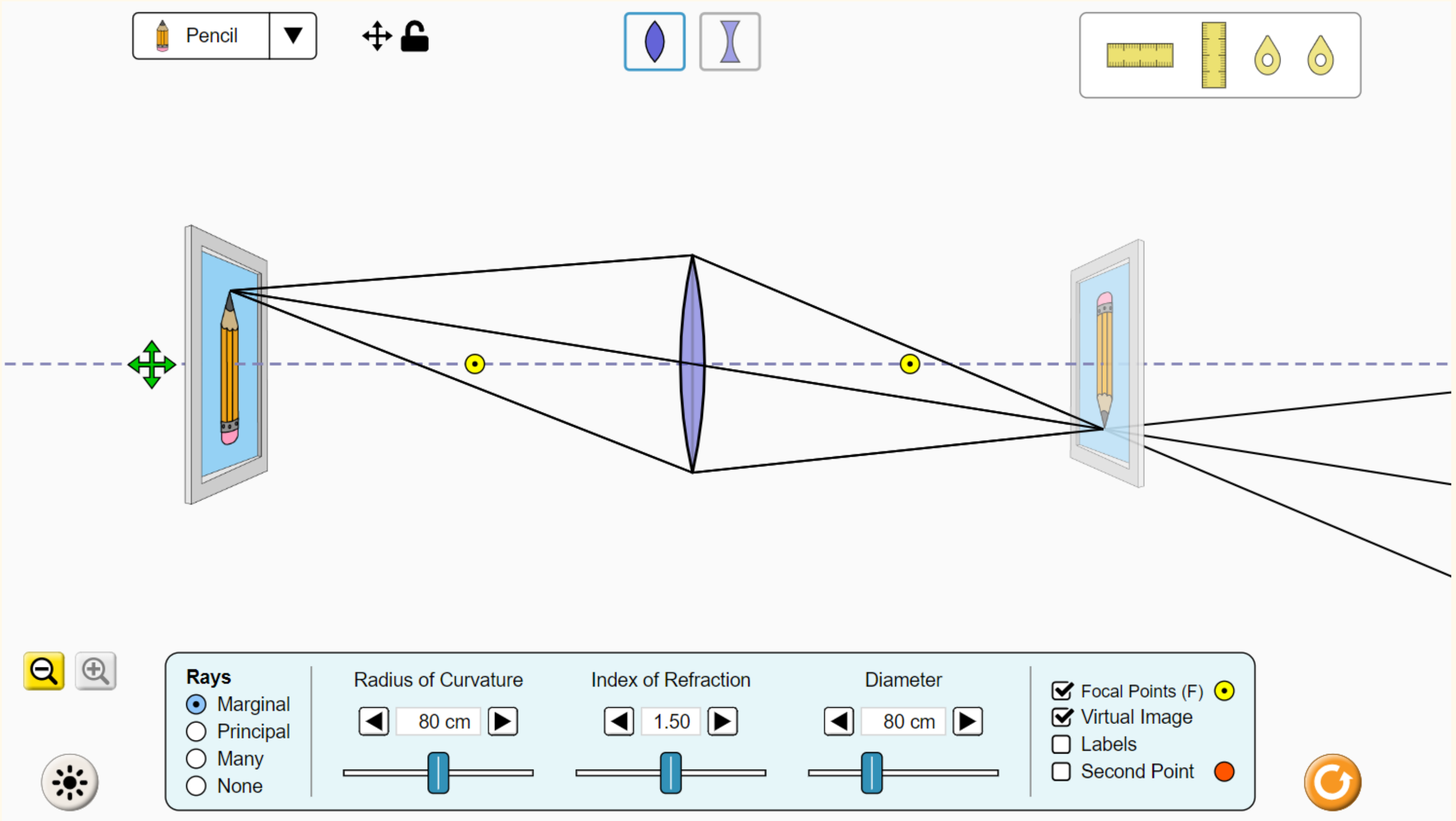
Incident rays parallel to the central axis will pass through F_2
Incident rays passing through F_1 will be parallel to the central axis
Incident rays passing through the center are not deviated

Geometrical construction of images with **particular rays**



Incident rays parallel to the central axis will pass through F_2
Incident rays passing through F_1 will be parallel to the central axis
Incident rays passing through the center are not deviated

https://phet.colorado.edu/sims/html/geometric-optics/latest/geometric-optics_all.html?

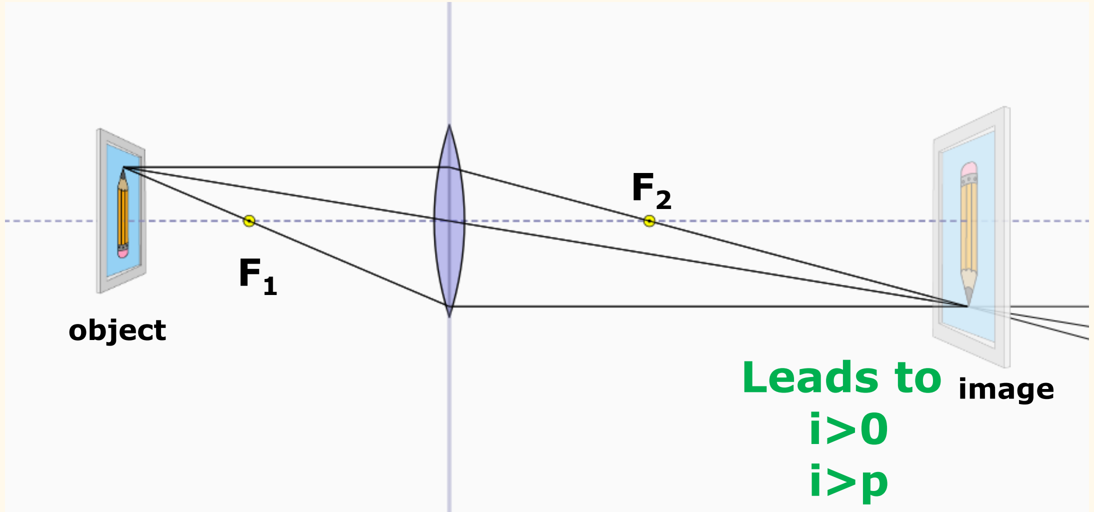


THIN LENSES - summary

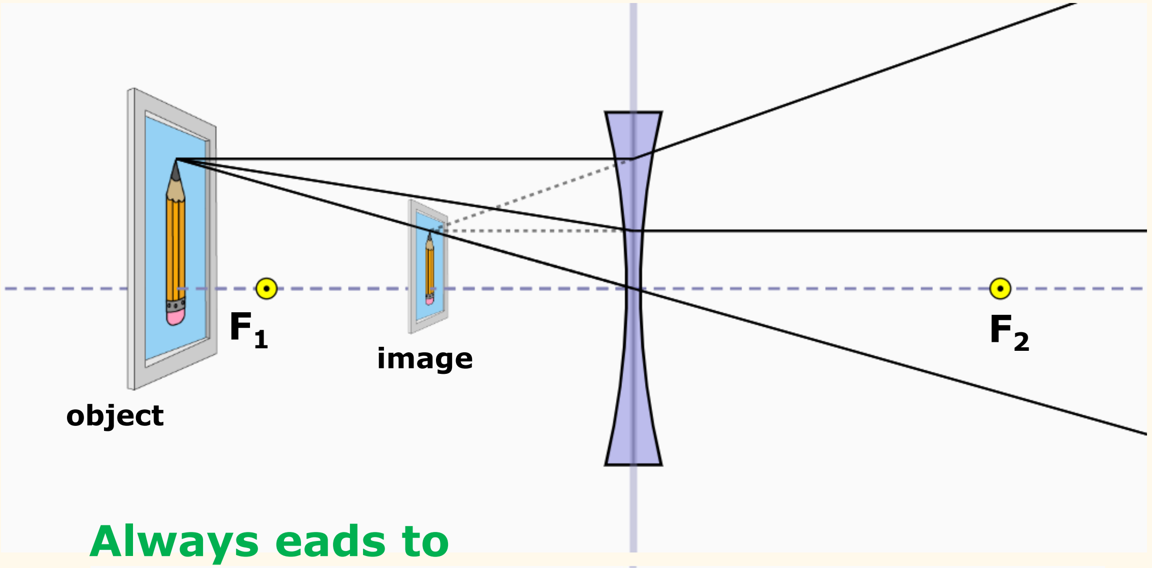
converging

diverging

$p>f$

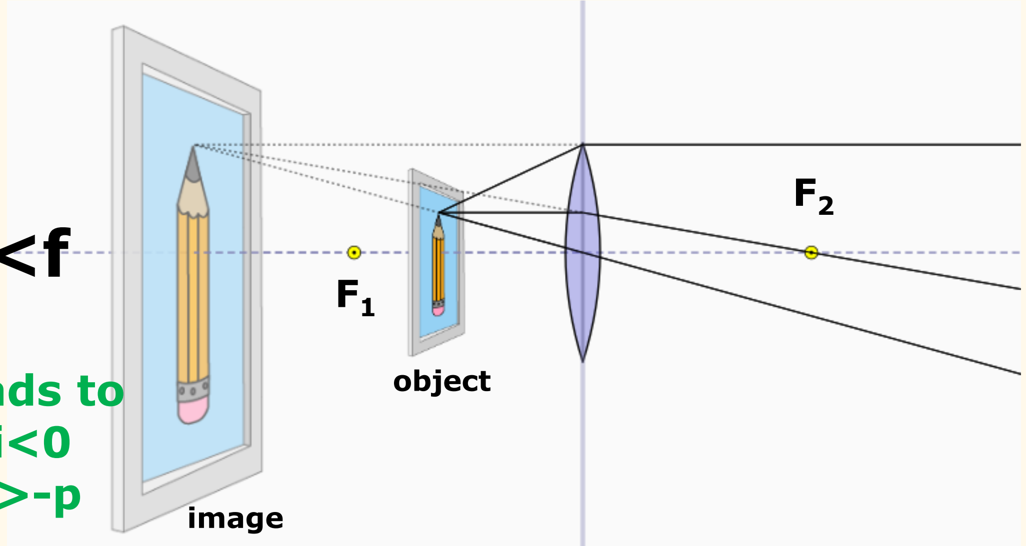


Leads to
$i>0$
$i>p$

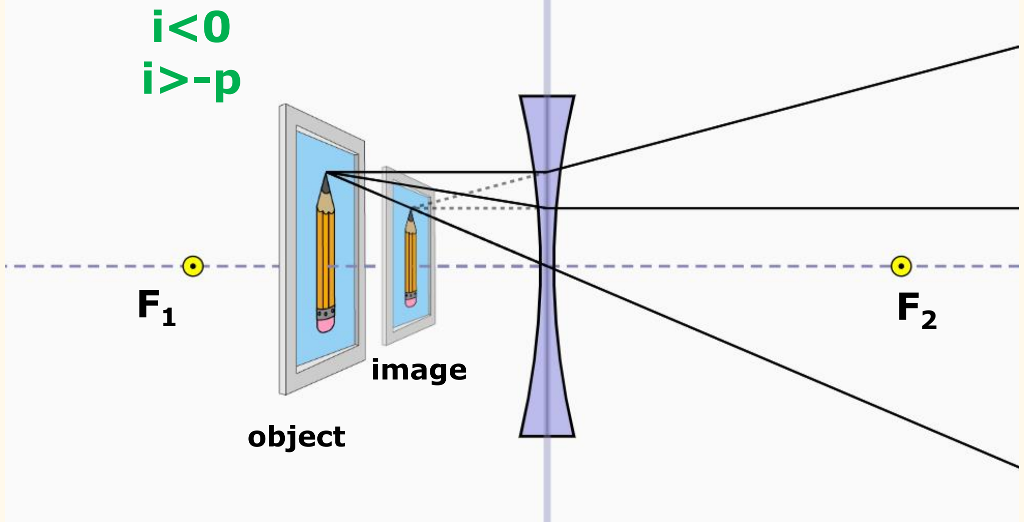


Always eads to
$i<0$
$i>-p$

$p<f$



Leads to
$i<0$
$i>-p$

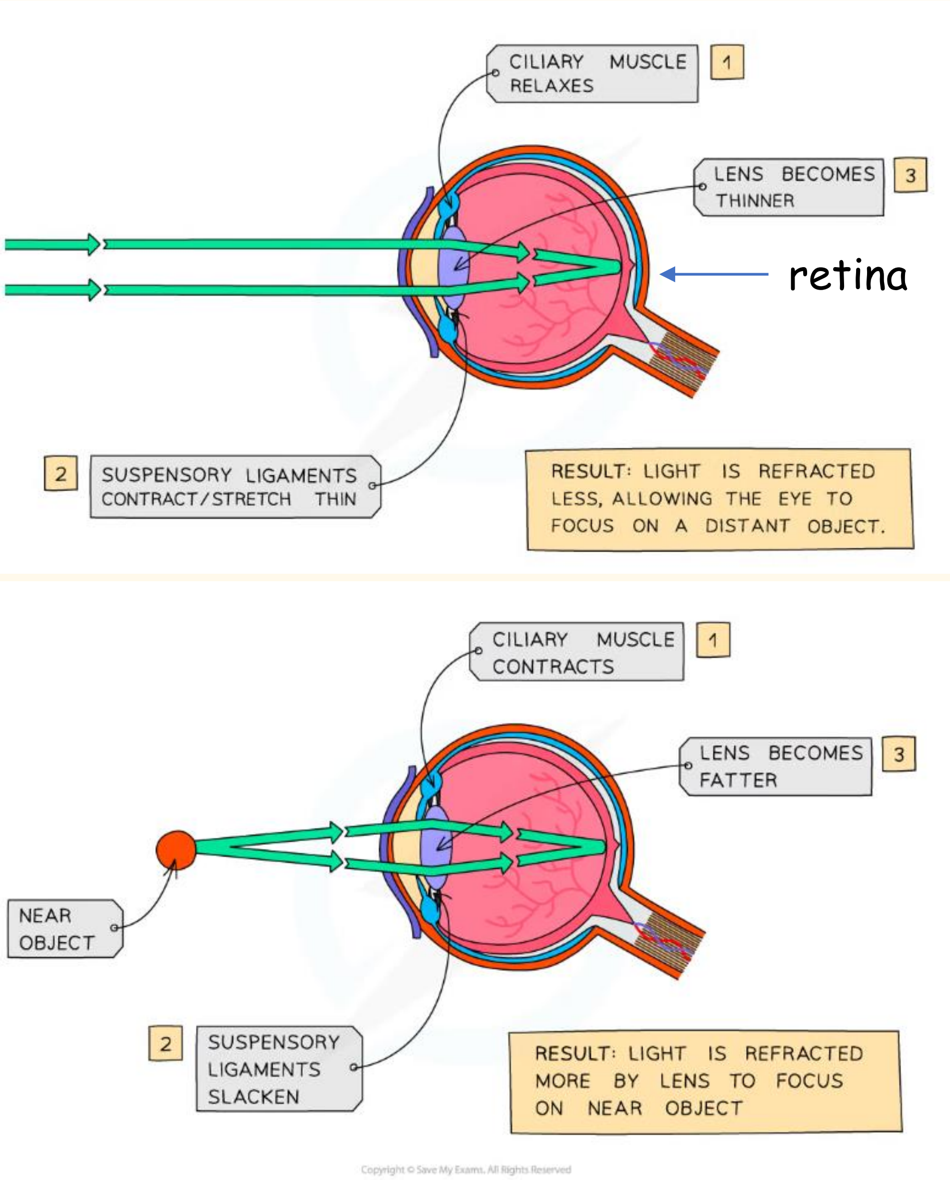


Without instruments: using eyes

→ **Form an image on the retina**

Clear image are formed for objects from **∞ distance** to the **near point P_n at ~ 25 cm**

Below this point the human lenses cannot contract enough to focus light on the retina

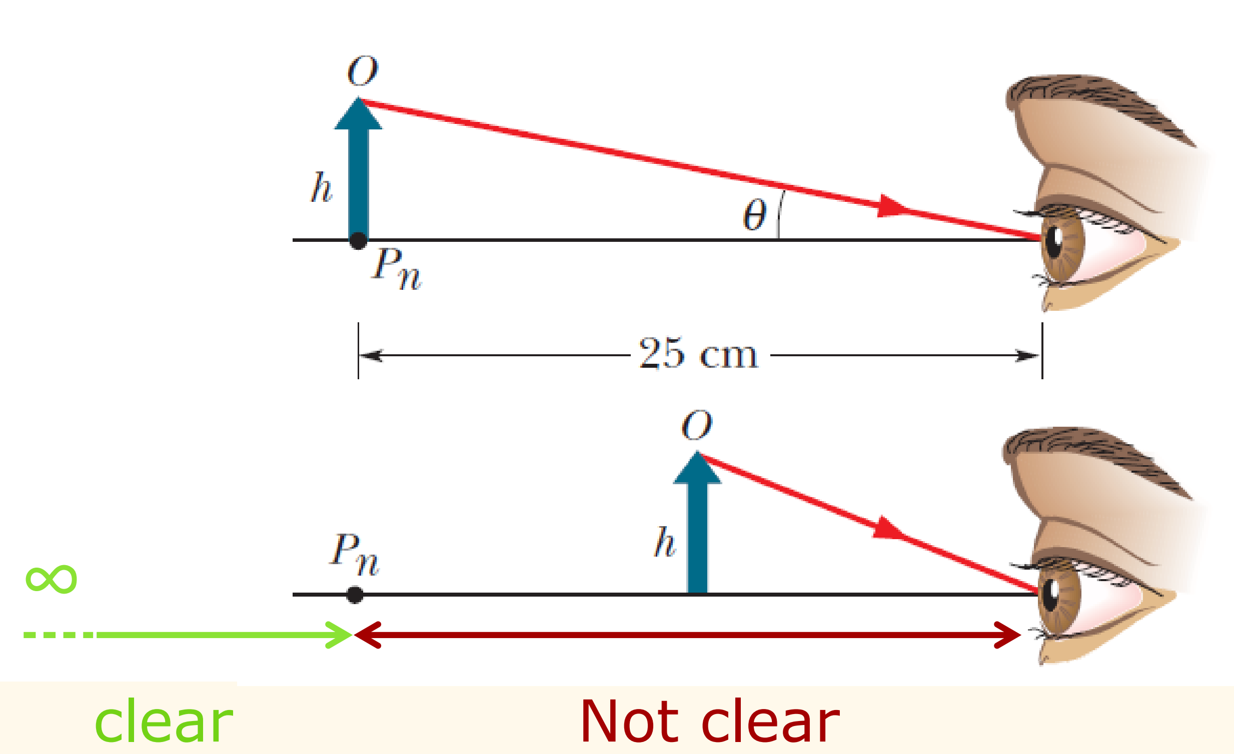


Without instruments: using eyes

→ **Form an image on the retina**

Clear image are formed for objects from **∞ distance** to the **near point P_n at ~ 25 cm**

Size of the image on the retina depends of the **angle θ** formed by the object in the field of view



Larger image on the retina
→ **increasing θ** bringing the object closer
But the image is **not clear beyond P_n**
→ **Need instruments**

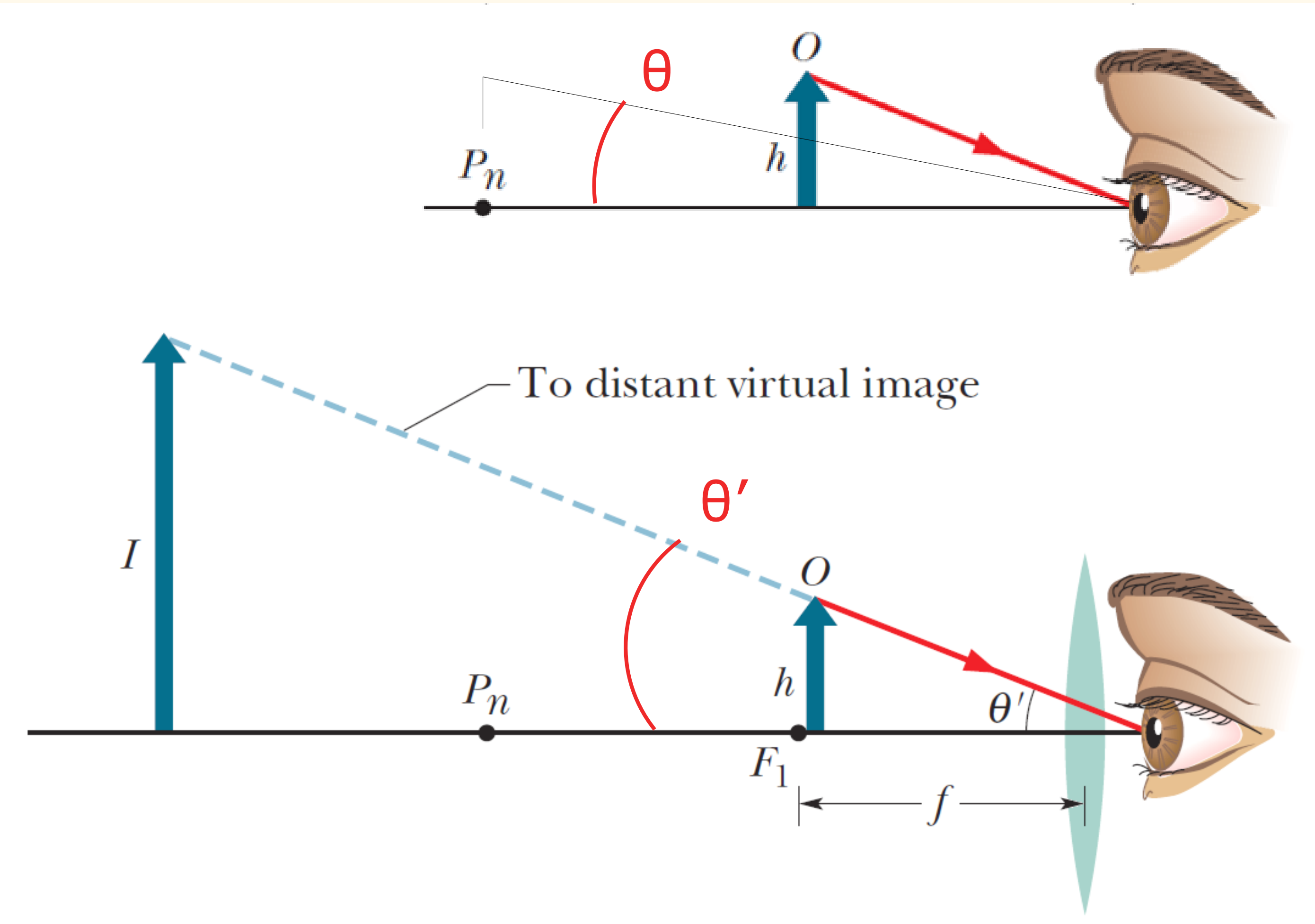
Magnifying glass

A **converging lens** with F_1 just before the object produces an **enlarged virtual image** I that is an object for the eye
→ I located before P_n
→ **Clear image** on the retina

Angular magnification m_θ

$$m_\theta = \frac{\theta'}{\theta}$$

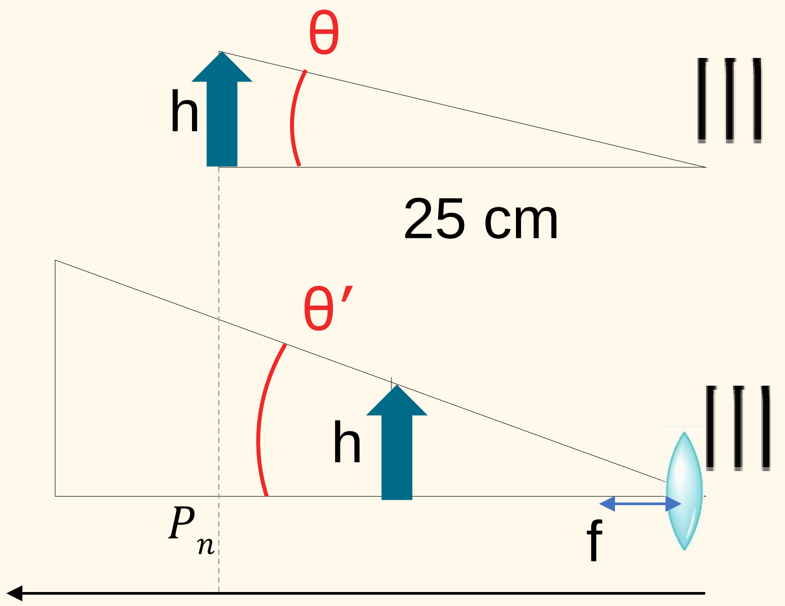
Note: $m_\theta \neq m$
m is lateral magnification



Note: θ is defined for an object at P_n

Angular magnification m_θ

$$m_\theta = \frac{\theta'}{\theta}$$



Paraxial approximation: θ and θ' <<
→ for x << $\sin(x) \simeq x$

$\sin(\theta) = h$ / 25cm

$\sin(\theta') \simeq h / f$ (object close to F_1)

$$\frac{\theta'}{\theta} \simeq \frac{\sin(\theta')}{\sin(\theta)} = \frac{h}{f}\frac{25\ cm}{h} = \frac{25\ cm}{f}$$

$$\boldsymbol{m_\theta = \frac{25\ cm}{f}}$$

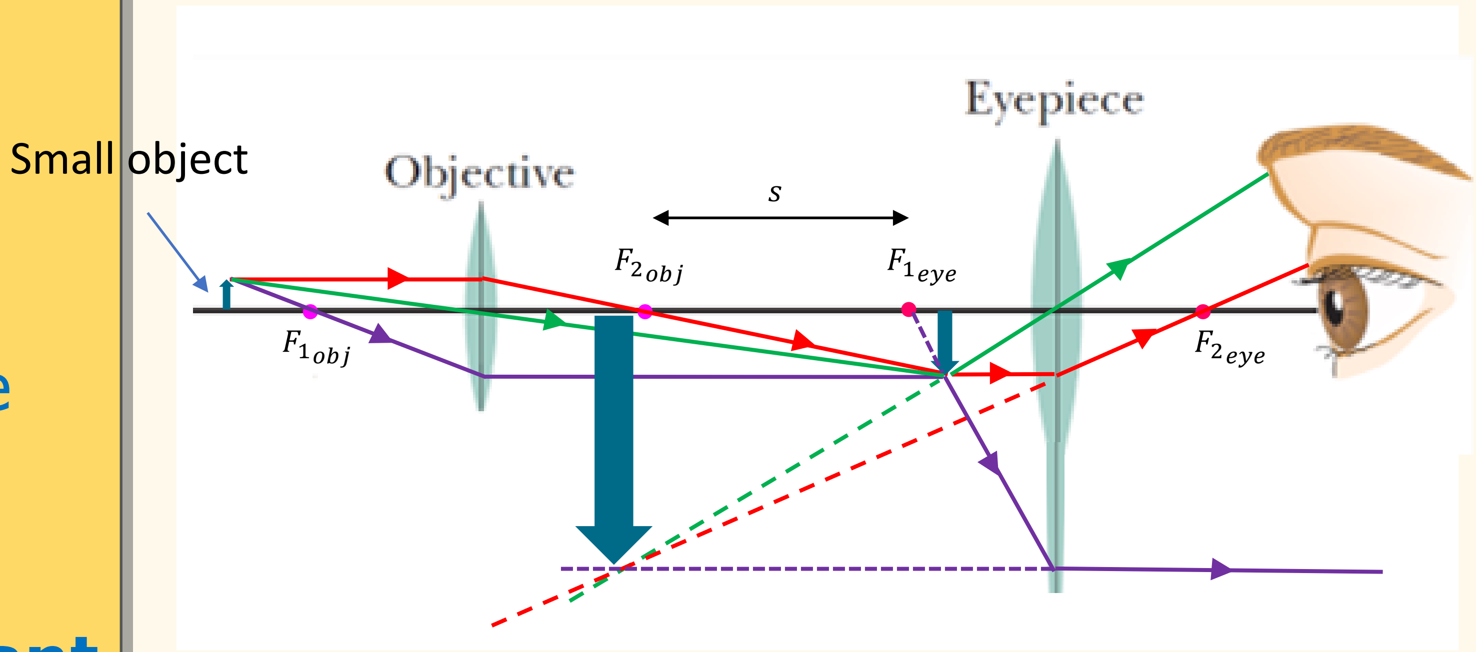
Microscope

The **compound microscope**:
has **2 converging lenses:**

→ **Objective** & **Eyepiece**

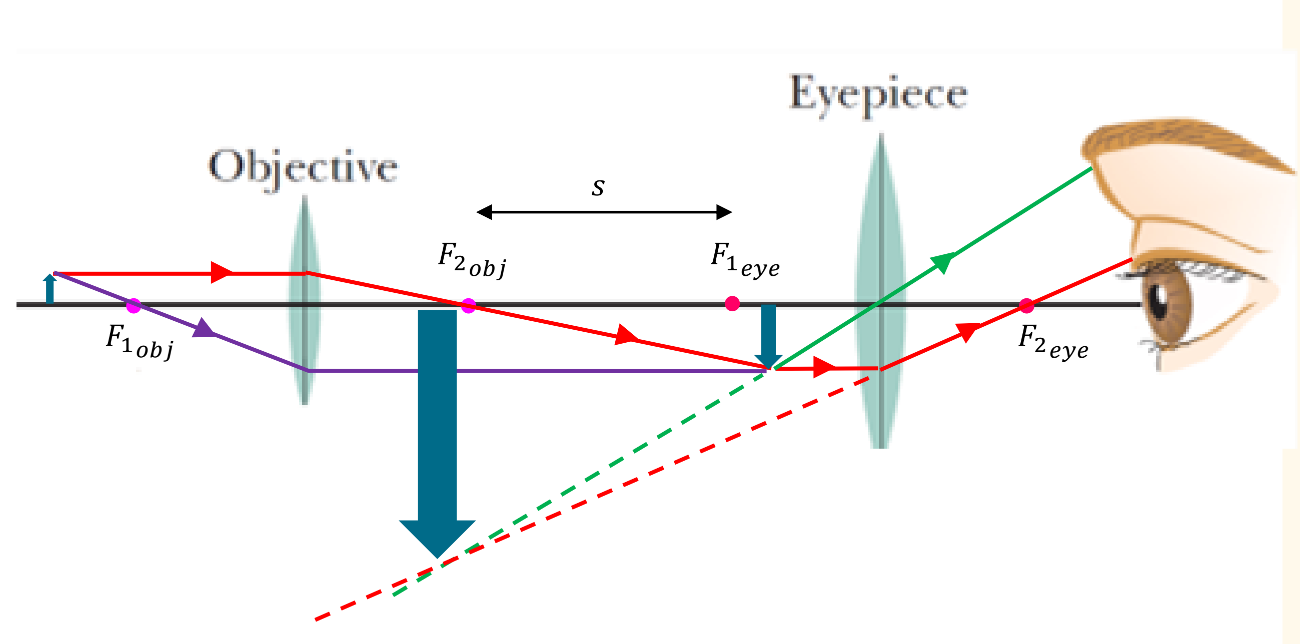
The image of the object by the objective is an object for the eyepiece.

Eyepiece forms a **virtual distant enlarged image** of this object (image) for the eye



Tube length s can be adjusted

Next: calculation of the magnification



Distance between the object & the objective ≃ f_{obj}

Distance between the 1st image & the objective ≃ s

Distance between the 1st image & the eyepiece ≃ f_{ey}

Lateral magnification of the objective (m):

$$m = -s/f_{obj}$$

Angular magnification of the objective (m_θ):

$$m_\theta = 25cm/f_{ey}$$

Magnification of the instrument (M):

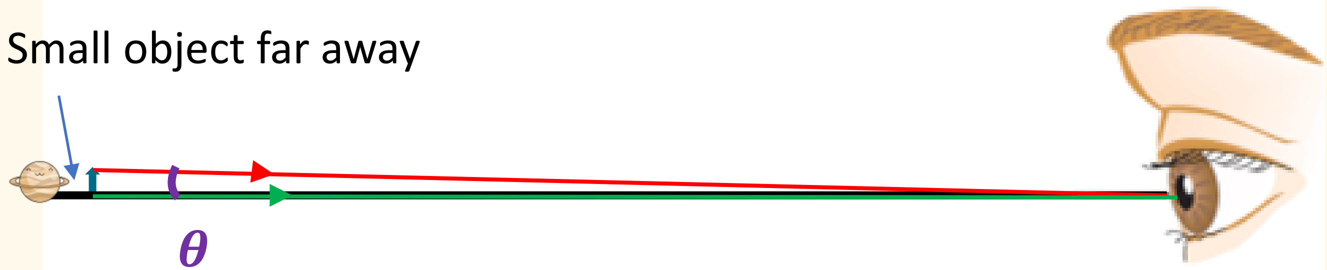
$$M = m\, m_\theta$$

$$M = -\frac{s}{f_{obj}} \frac{25cm}{f_{ey}}$$

OPTICAL INSTRUMENTS

Telescope

Size of the image on the retina depends of the **angle θ** formed by the object in the field of view

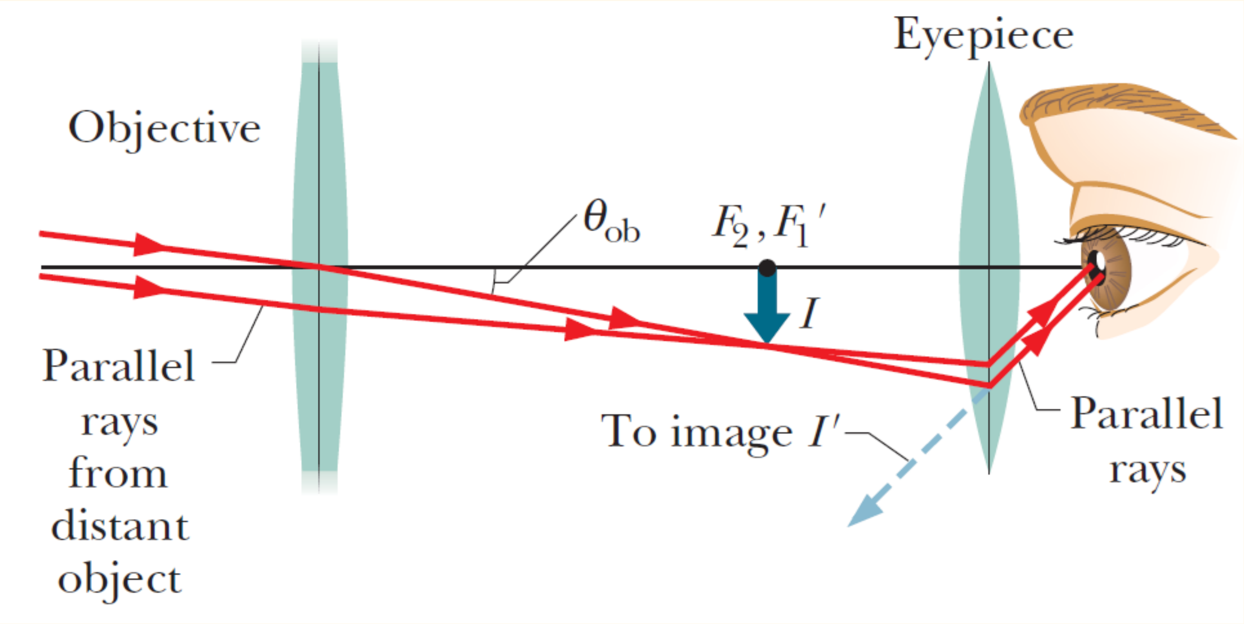


The refracting telescope has **2 converging lenses:**

→ **Objective** & **Eyepiece**

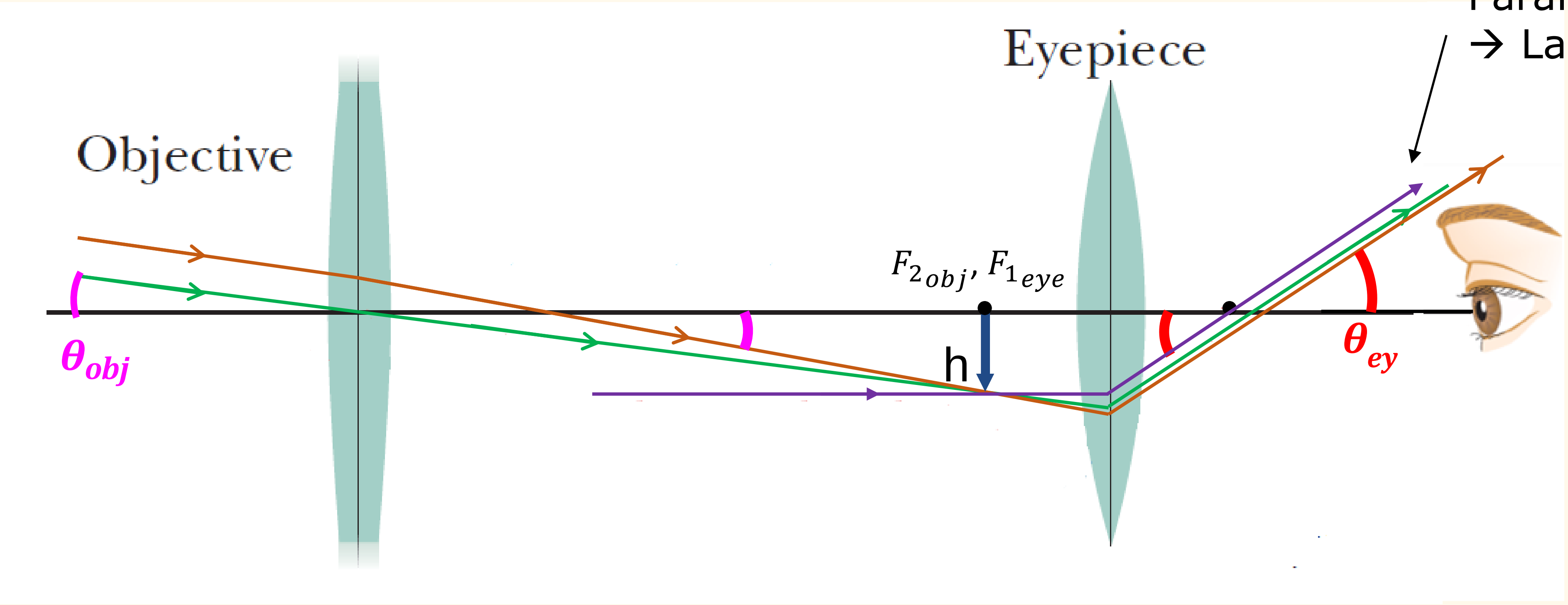
The image of the object by the objective is an object for the eyepiece.

Eyepiece forms a image of this object (image) at **infinite distance** for the eye



Object are not close and small but at ∞ distance and large

Next: calculation of the magnification



Parallels but with higher angle
→ Larger image

Note: minus sign because the image is reversed

$F_{2\ obj}$ and $F_{1\ ey}$ coincide
Object seen without (resp. with) the instrument: θ_{obj} (θ_{ey})
Paraxial approximation: $\theta_{obj} = h / f_{obj}$ and $\theta_{ey} = h / f_{ey}$

Angular Magnification of the instrument (m_θ):

$$m_\theta = \frac{\theta_{ey}}{\theta_{obj}} = -\frac{f_{ey}}{f_{obj}}$$

Images of plane mirrors $p = -i$

Images and lateral magnification of spherical mirrors $\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$ $m = -\frac{i}{p}$

Particular rays of spherical mirrors

Images and lateral magnification of thin lenses $\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$ $m = -\frac{i}{p}$

Particular rays of thin lenses

Angular magnification of a lens $m_\theta = \frac{25\ cm}{f}$

Magnification of a compound microscope $M = m\, m_\theta = -\frac{s}{f_{obj}} \frac{25cm}{f_{ey}}$

Magnification of a refracting telescope $m_\theta = \frac{\theta_{ey}}{\theta_{obj}} = -\frac{f_{ey}}{f_{obj}}$

READING ASSIGNMENT

Chapter 35 of the textbook