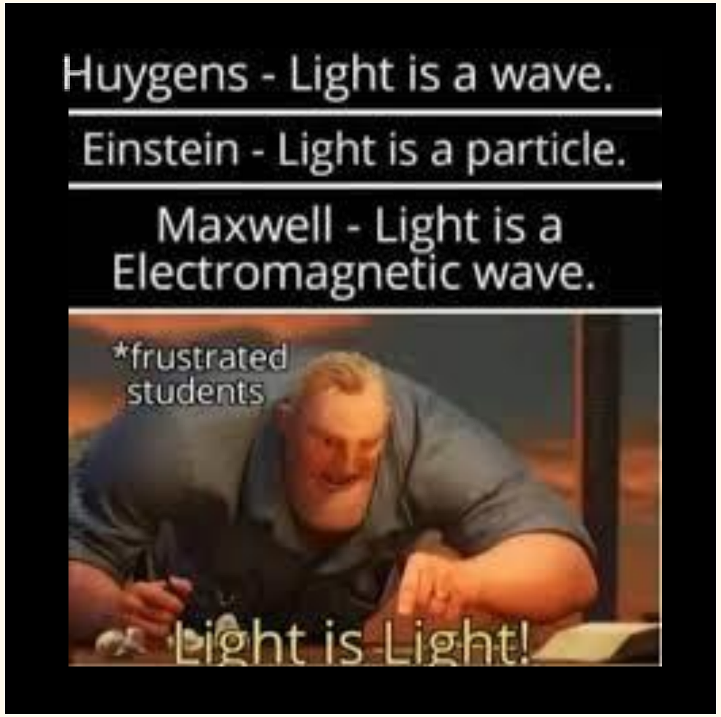
ELECTROMAGNETIC WAVES

CHAPTER 33



- Electromagnetic Oscillations & Alternating Current
- Maxwell’s Equations & Magnetism of Matter
- **Electromagnetic Waves**
- Images
- Interference
- Diffraction

Videos links:

How Did We Figure Out What Light Is? (history of light)

Light Is Waves: Crash Course Physics #39

Spectra Interference: Crash Course Physics #40

The origin of Electromagnetic waves, and why they behave as they do

Understanding Electromagnetic Radiation!

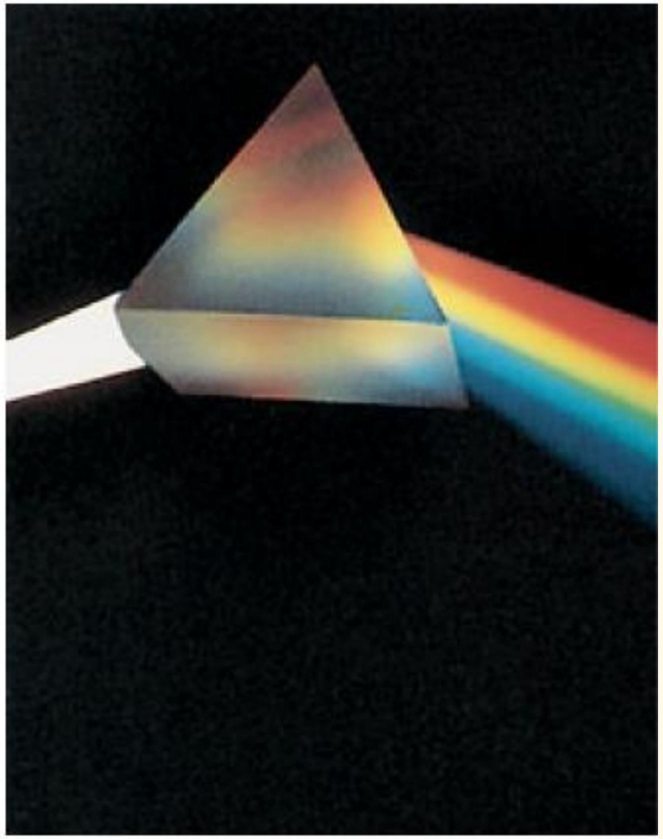
But why would light "slow down"?

ELECTROMAGNETIC WAVES

Textbook: Chapter 33

- EM SPECTRUM AND TRAVELING EM WAVES
- ENERGY TRANSPORT AND POYNTING VECTOR
- RADIATION PRESSURE
- POLARIZATION
- REFLECTION AND REFRACTION
- TOTAL INTERNAL REFLECTION

Note: EM: Electromagnetic



Courtesy Bausch & Lomb

EM SPECTRUM AND TRAVELING EM WAVES

James Clark Maxwell

→ **Light = Traveling EM wave**

EM wave:

Electric Field $\vec{E}$

+

Magnetic Field $\vec{B}$

Optics is Electromagnetism

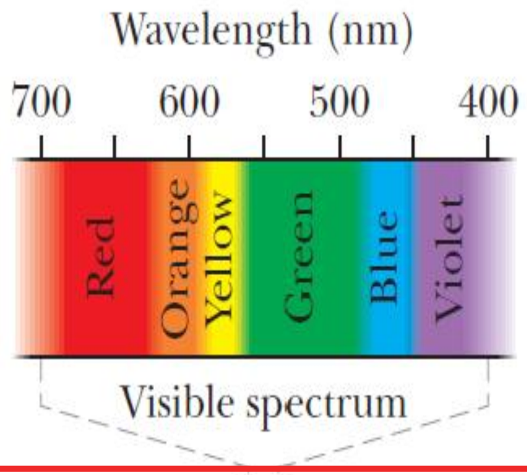
Examples of EM waves

- Visible light
- Infrared light
- UV light
- μ waves
- Radio waves
- …

EM can be sorted into domains regarding their frequency

All EM waves travel in the **free space** *(vacuum)* at speed **c**

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.10^8 \text{ m/s}$$



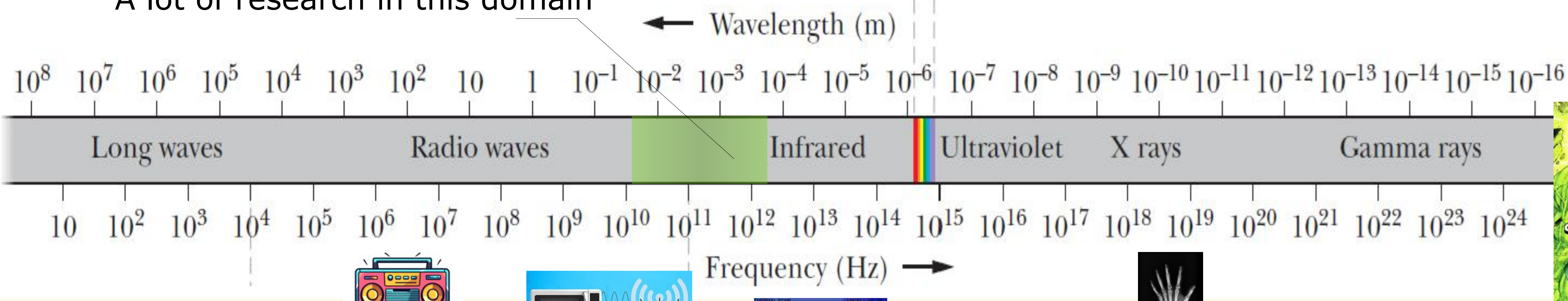
Visible light

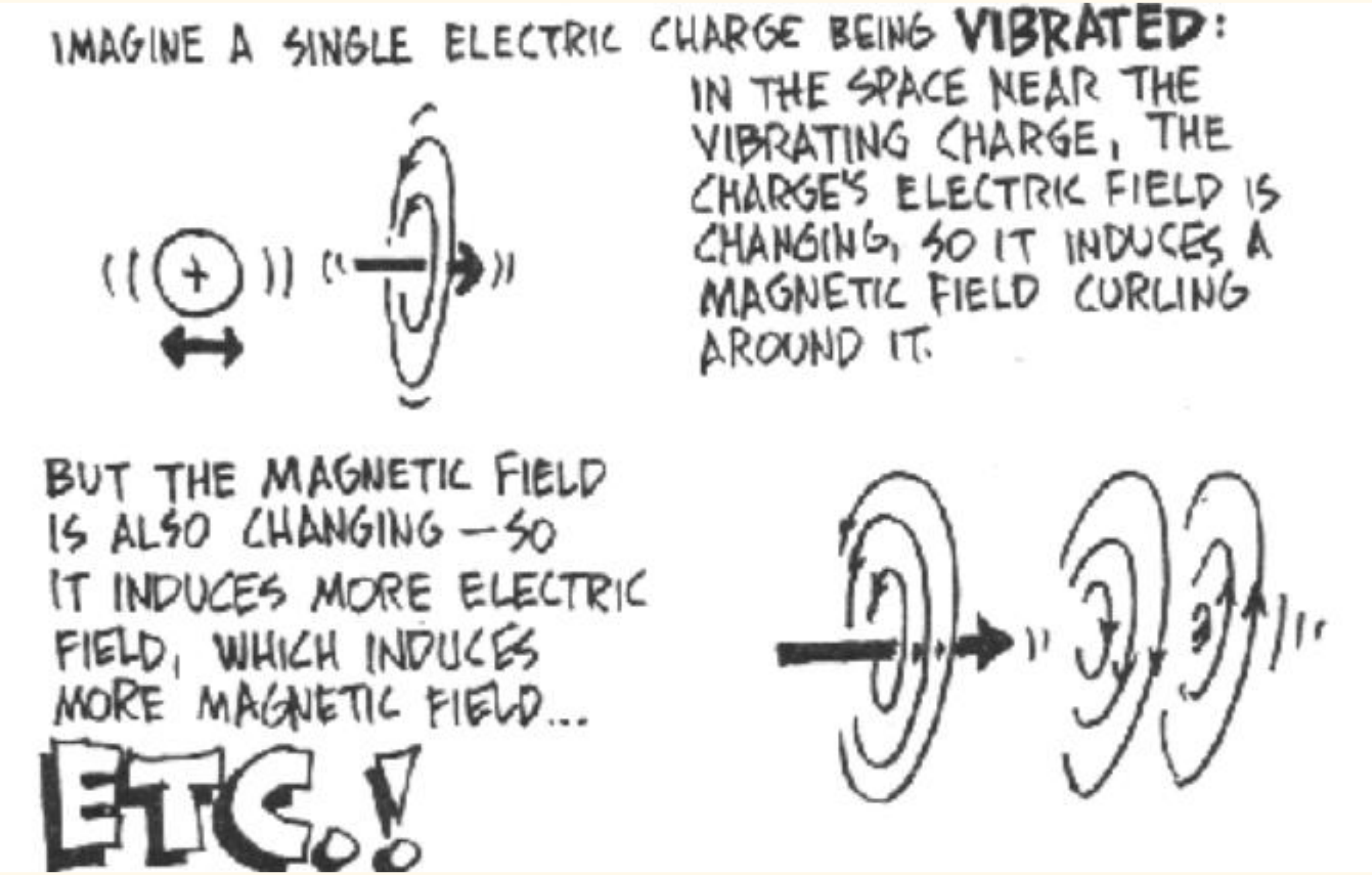
λ: ~ 780 – 390 nm

Only a small portion of the spectrum

THz domain: λ: ~ 3 mm – 30 μm

A lot of research in this domain

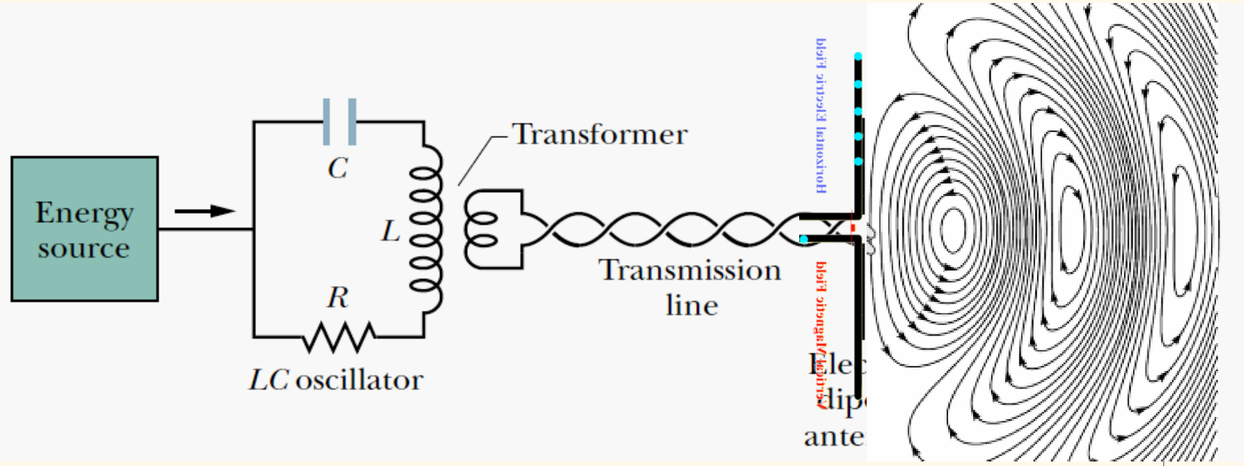



IMAGINE A SINGLE ELECTRIC CHARGE BEING VIBRATED:
IN THE SPACE NEAR THE VIBRATING CHARGE, THE CHARGE'S ELECTRIC FIELD IS CHANGING, SO IT INDUCES A MAGNETIC FIELD CURLING AROUND IT.
+
BUT THE MAGNETIC FIELD IS ALSO CHANGING — SO IT INDUCES MORE ELECTRIC FIELD, WHICH INDUCES MORE MAGNETIC FIELD...
ETC.!

Generation of EM
→ oscillating electric charges

Example for λ>1m :
LC oscillator coupled to a dipole antenna

$$\omega = \frac{1}{\sqrt{LC}}$$



Current oscillates at ω

Dipole oscillates at ω

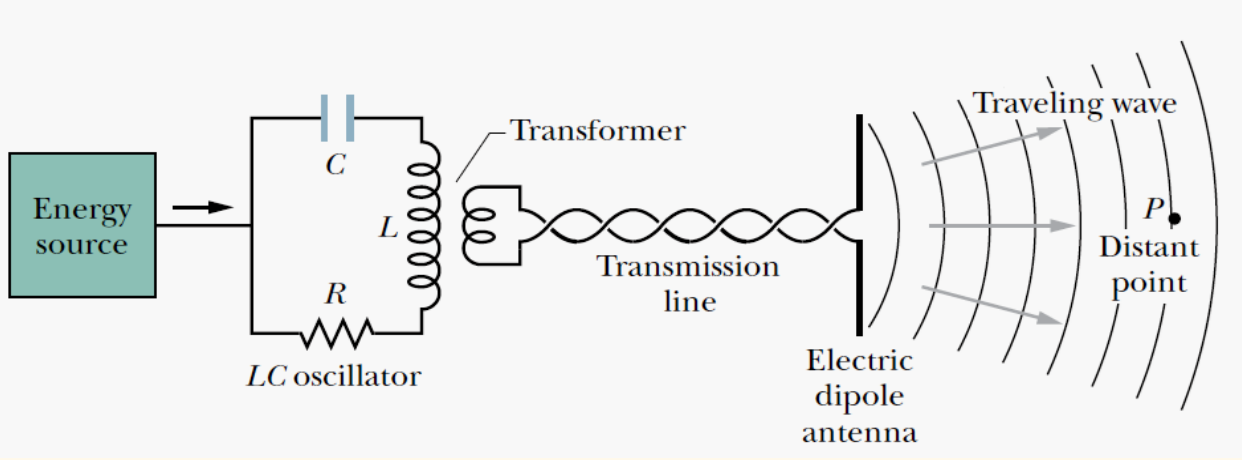
Electric field and Magnetic field oscillate at ω

Change over space ≠ instantaneous
EM wave of angular freq. ω traveling at speed c

Structure of the EM wave

→ **Far from the antenna** so the *curvature of the wave* can be be neglected: **Plane wave**

Out of the plane wave approximation physical description of EM waves is more complex



Current oscillates at ω

Dipole oscillates at ω

Electric field and **Magnetic field** oscillate at ω

Change over space ≠ instantaneous
EM wave of angular freq. ω traveling at speed c

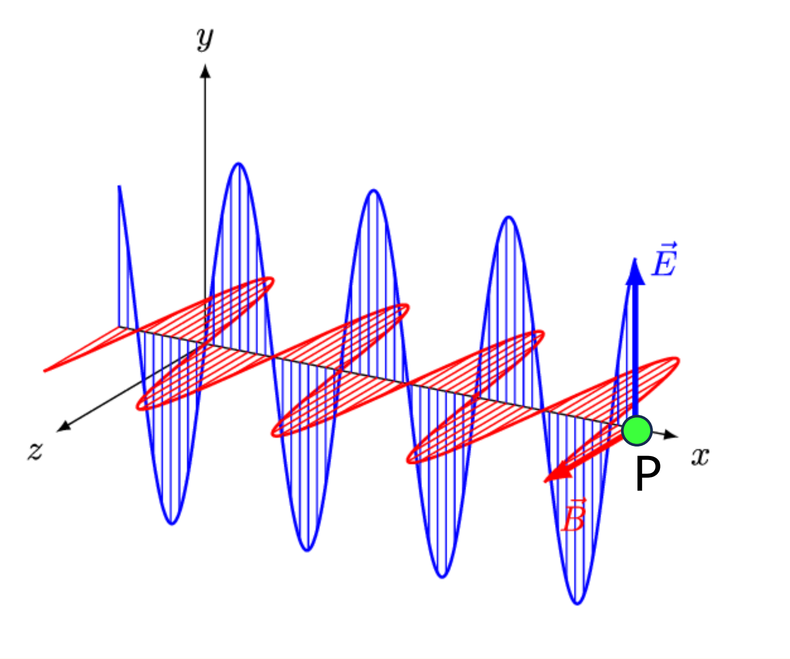
Structure of the EM traveling plane wave

$\vec{E}$ and $\vec{B}$ are **perpendicular to the direction of propagation → transverse wave**

$\vec{E}$ is **perpendicular** to $\vec{B}$

$\vec{E} \times \vec{B}$ gives the **direction of propagation**

$\vec{E}$ and $\vec{B}$ vary sinusoidally at the **same ω in phase**



P fixed point in space

Structure of the EM traveling plane wave

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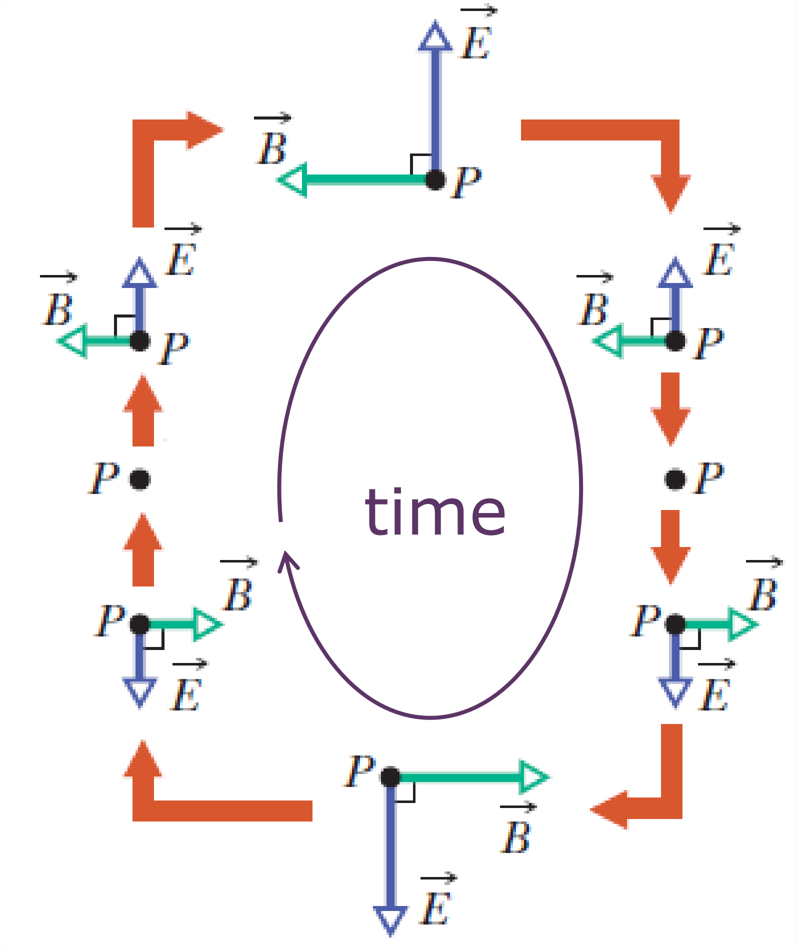


P fixed point in space

Structure of the EM traveling plane wave

$\vec{E}$ and $\vec{B}$ vary sinusoidally at the **same ω in phase**

Assuming **forward propagation** along the **x** axis

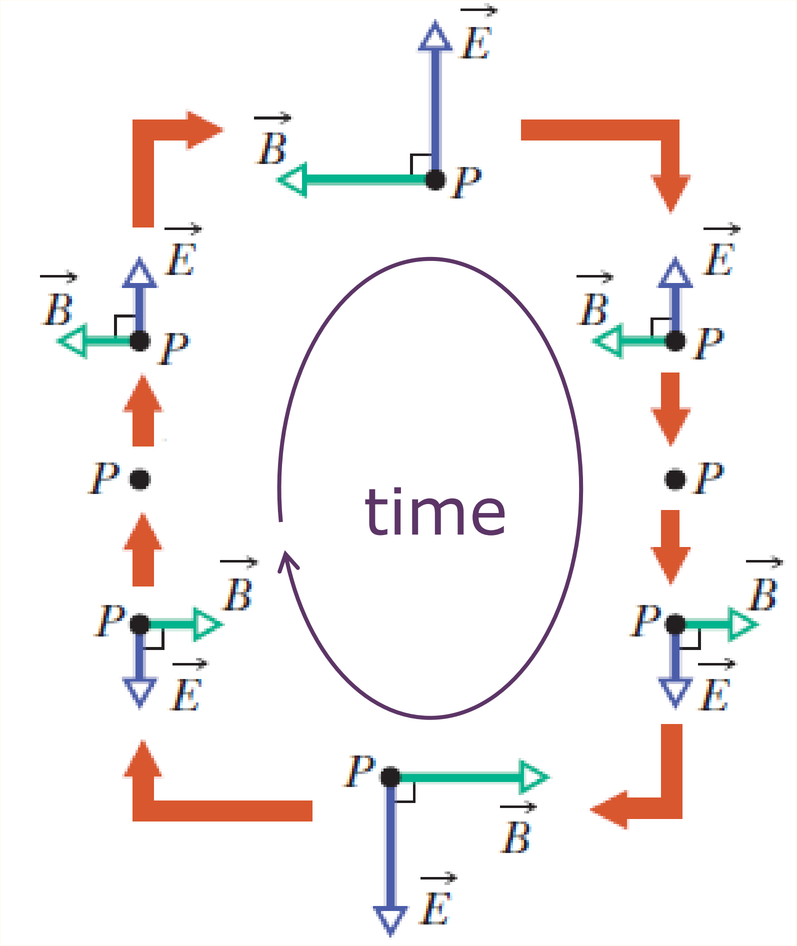
$$E = E_m \sin(kx - \omega t)$$
$$B = B_m \sin(kx - \omega t)$$

E_m, B_m: amplitudes of E and B
k: angular wave number
ω: angular frequency
c = ω / k speed of light in vacuum

Valid for linear or circular polarization

$$\frac{E_m}{B_m} = \frac{E}{B} = c$$

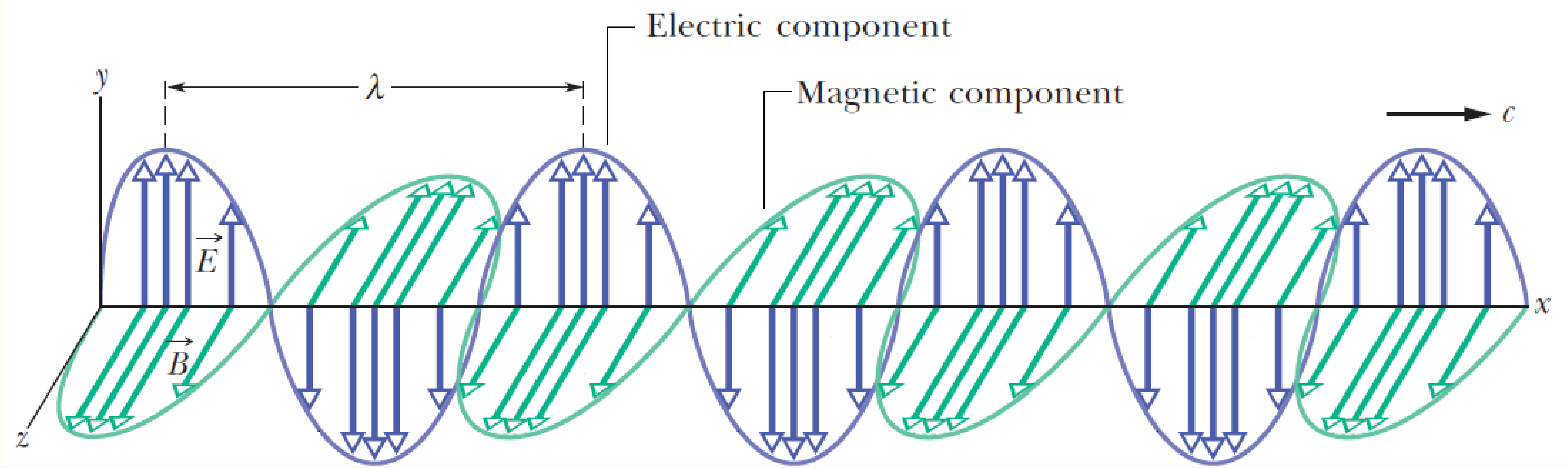
Amplitude & magnitude ratios



P fixed point in space

Representation of the wave:

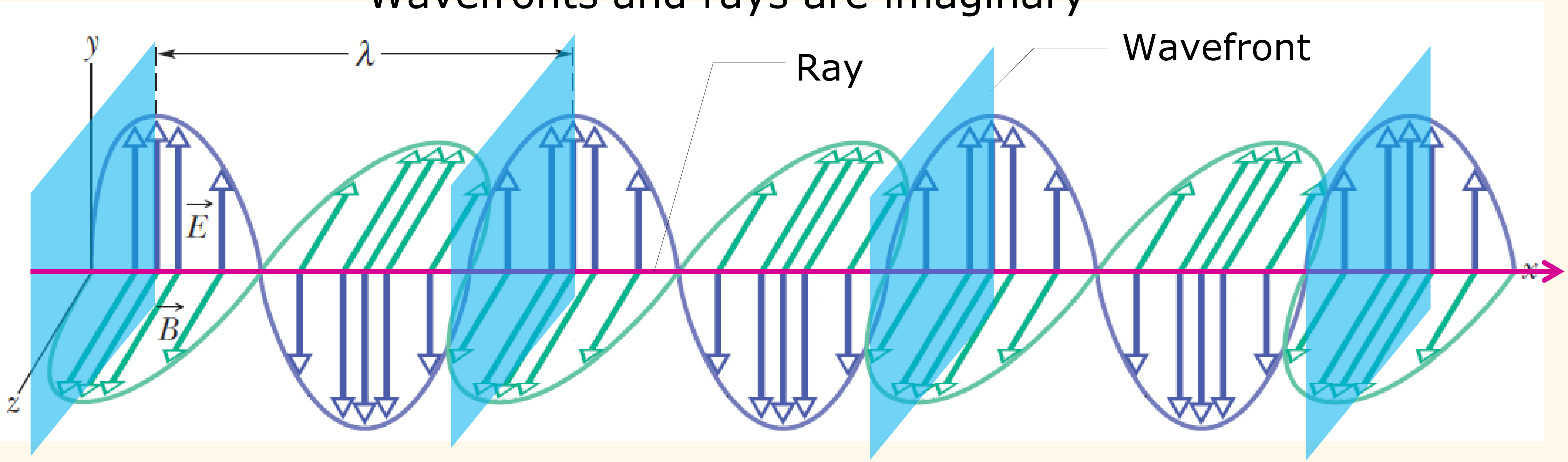
Representations of the **fields** through space at a **fixed instant**



Representation of the wave:

Wavefronts: Surfaces with E at **constant phase** separated by **λ**
Rays: Line in the **direction of propagation** perpendicular to the wavefront

Wavefronts and rays are imaginary



EM SPECTRUM AND TRAVELING EM WAVES

Specificity of EM waves

- Unlike other waves (e.g. sound) EM propagate **without medium**
- Special Relativity Theory:

The speed of light c is **the same no matter the motion** of the source or the observer

Propagation of EM waves

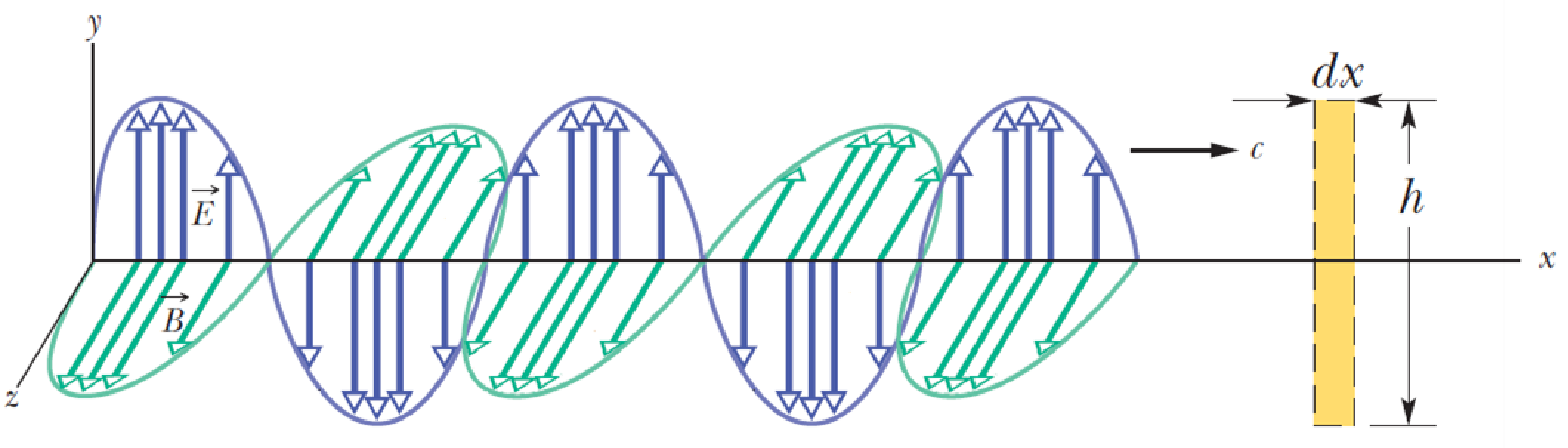
- Varying B field induces a varying E field (M-F)
- Varying E field induces a varying B field (M-A)
- And so on …

The two fields **continuously create each other** via induction

→ Next: Exploring this statement

Quantitative analysis

We consider a small rectangle (surface = h dx) perpendicular to Oz



Quantitative analysis

During dt $\vec{B}$ has varied → ϕ_B in the rectangle varies → induced $\vec{E}$ field

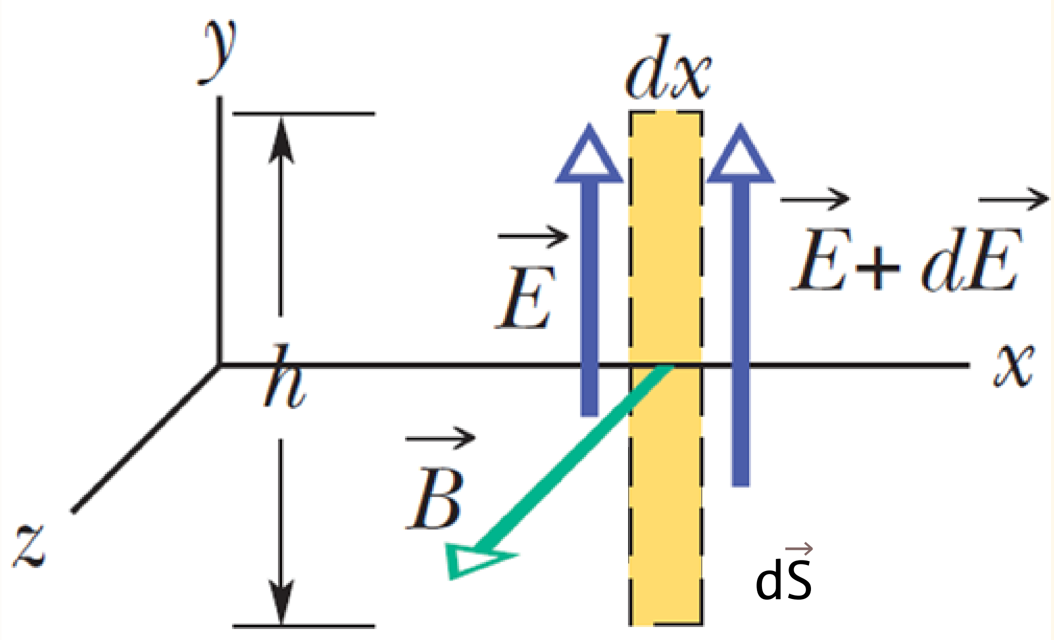
On one edge electric field is $\vec{E}$ and $\vec{E} + d\vec{E}$ on the other side

$$\oint \vec{E} . d\vec{S} = -\frac{d\phi_B}{dt} \quad (M - F)$$

$$0 + (E + dE)h + 0 - Eh = -\frac{d}{dt}(B\, h\, dx)$$

$$\frac{dE}{dx} = -\frac{dB}{dt} \qquad \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

Note: 2 variables → partial derivative



Quantitative analysis

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$E = E_m \sin(kx - \omega t)$$
$$B = B_m \sin(kx - \omega t)$$

$$\frac{\partial E}{\partial x} = E_m\, k \cos(kx - \omega t)$$

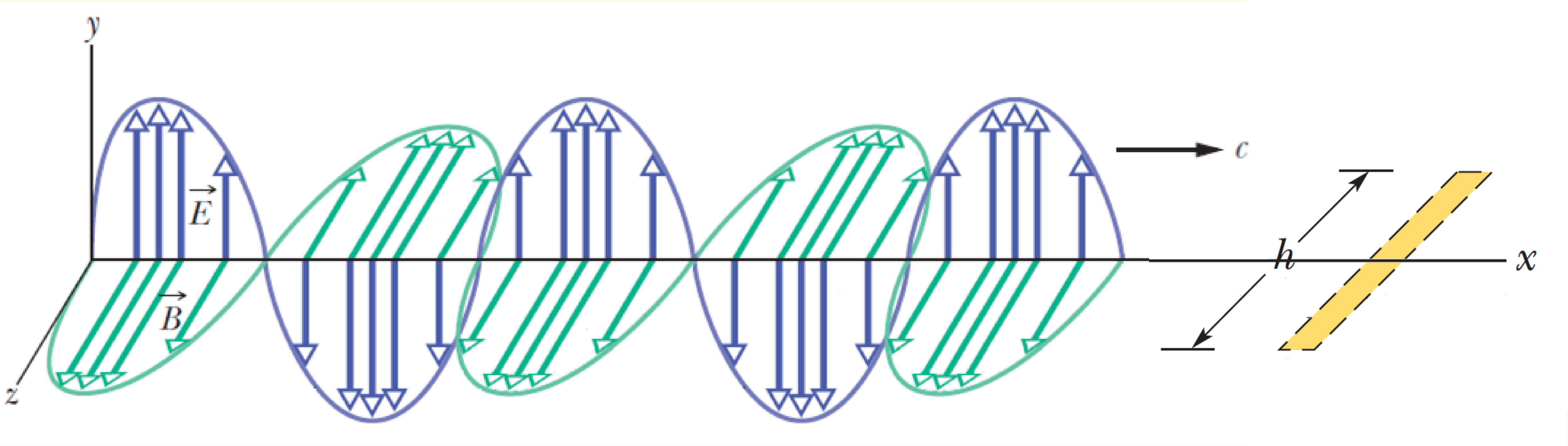
$$\frac{\partial B}{\partial t} = -B_m\, \omega \cos(kx - \omega t)$$

$$E_m\, k \cos(kx - \omega t) = B_m\, \omega \cos(kx - \omega t)$$

That demonstrates $\dfrac{E_m}{B_m} = \dfrac{\omega}{k} = c$

Quantitative analysis

We consider a small rectangle (surface = h dx) perpendicular to Oy



Quantitative analysis

During dt $\vec{E}$ has varied → ϕ_E in the rectangle varies → induced B field

On one edge electric field is $\vec{B}$ and $\vec{B} + d\vec{B}$ on the other side

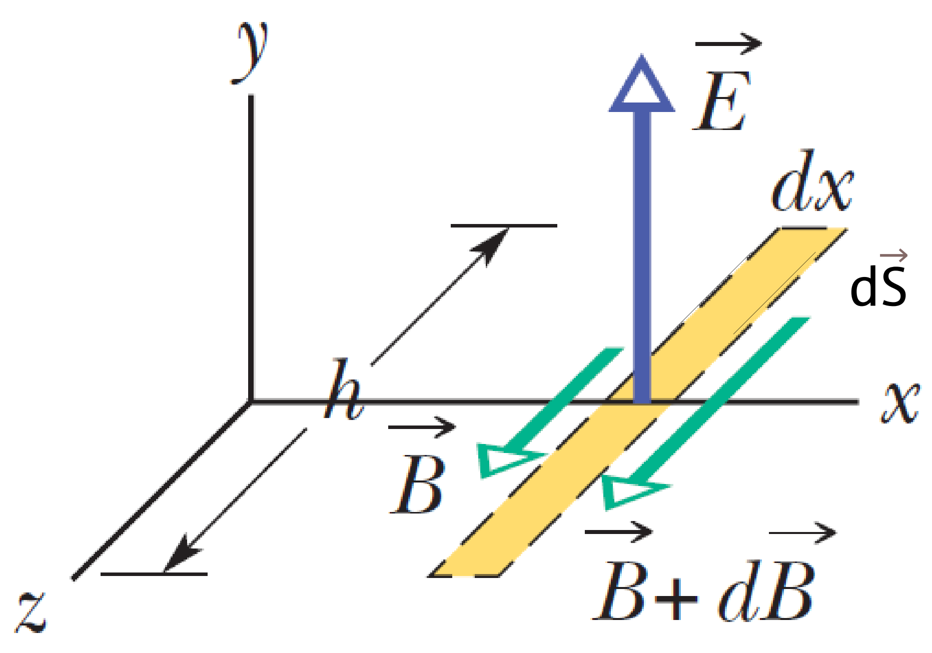
$$\oint \vec{B} \cdot d\vec{S} = \mu_0 \epsilon_0 \frac{d\phi_E}{dt} + \mu_0 i_{enc} \quad (M - A)$$

Note: we assume free space propagation: $i_{enc} = 0$

$$0 - (B + dB)h + 0 + Bh = \mu_0 \epsilon_0 \frac{d}{dt}(E\, h\, dx)$$

$$-\frac{dB}{dx} = \mu_0 \epsilon_0 \frac{dE}{dt}$$

$$-\frac{\partial B}{\partial x} = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$



Note: 2 variables → partial derivative

Quantitative analysis

$$-\frac{\partial B}{\partial x} = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$E = E_m \sin(kx - \omega t)$$

$$B = B_m \sin(kx - \omega t)$$

$$\frac{\partial E}{\partial t} = -E_m \,\omega \cos(kx - \omega t)$$

$$\frac{\partial B}{\partial x} = B_m \,k \cos(kx - \omega t)$$

$$-B_m \,k \cos(kx - \omega t) = -\mu_0 \epsilon_0 E_m \,\omega \cos(kx - \omega t)$$

$$\frac{E_m}{B_m}\frac{\omega}{k} = \frac{1}{\mu_0 \epsilon_0}$$

But also $\frac{E_m}{B_m} = \frac{\omega}{k} = c$ $\quad c^2 = \frac{1}{\mu_0 \epsilon_0}$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

ENERGY TRANSPORT AND POYNTING VECTOR

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

Since $\vec{E}$ and $\vec{B}$ are perpendicular:

- $\vec{S}$ is along the direction of propagation
- $S = \frac{EB}{\mu_0} = \frac{1}{c\mu_0} E^2$

EM waves carry energy

Characterized by the
Poynting vector $\vec{S}$

S relates to the instantaneous energy transfer rate per area

$$S = \left(\frac{energy/time}{surface}\right)_{inst} \quad (W\ /\ m\text{^}2\ )$$

Note: Poynting vector ≠ spin

$$I = S_{avg} = \left(\frac{1}{c\mu_0}E^2\right)_{avg} = \frac{1}{c\mu_0}(E^2)_{avg}$$

$$E = E_m \sin(kx - \omega t)$$

$$E^2 = E_m^2 \sin^2(kx - \omega t)$$

$$E^2{}_{avg} = E_m^2 \left(\sin^2(kx - \omega t)\right)_{avg}$$

$$E^2{}_{avg} = E_m^2/2$$

$$I = \frac{E_m^2}{2c\mu_0}$$

EM waves carry energy

Characterized by the **Poynting vector $\vec{S}$**

Power transfer characterized by the **intensity I**

$$I = S_{avg} = \left(\frac{energy\ /\ time}{surface}\right)_{avg}$$

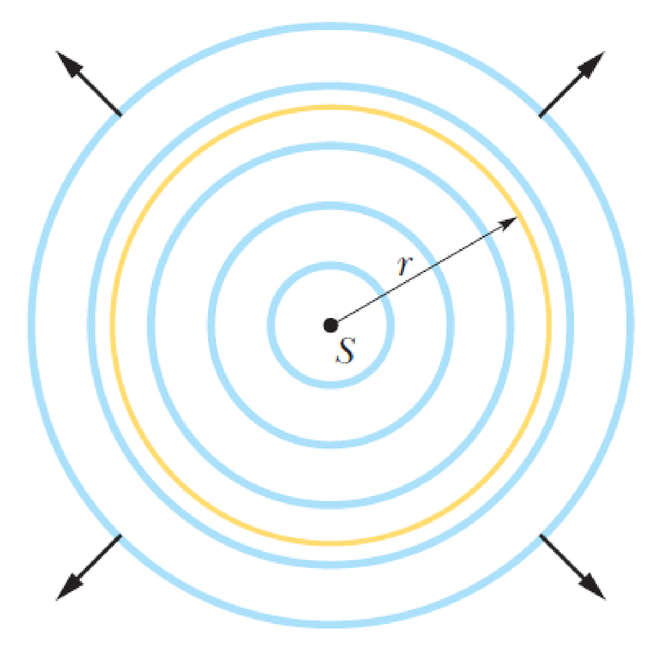
$$= \left(\frac{power}{surface}\right)_{avg}$$

Note: we usualy find the notation : $\langle S \rangle = (S)_{avg}$

Variation of intensity with distance

We assume the EM comes from a point source of power P_S
→ spherical wavefronts close to the source

$$I = \frac{power}{area} = \frac{P_S}{4\pi r^2}$$



Intensity decreases with the squared distance from the source

RADIATION PRESSURE

Light does not have mass but **carries momentum**

according to quantum mechanics $\boldsymbol{p} = \frac{\boldsymbol{U}}{\boldsymbol{c}}$

U: *Energy carry by the beam of light*

c: *speed of light*

EM waves carry energy

Energy & momentum transfer to an irradiated object

RADIATION PRESSURE

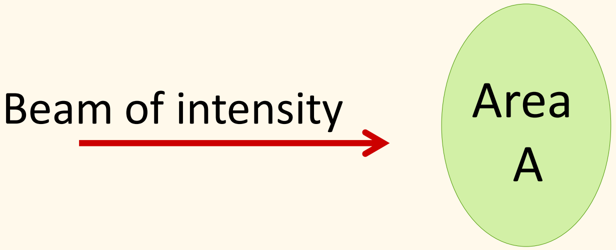
Light does not have mass but **carries momentum**

according to quantum mechanics $\boldsymbol{p} = \frac{\boldsymbol{U}}{\boldsymbol{c}}$ ← *Energy carry by the beam of light*; c ← *speed of light*

$$\Delta p = \frac{\Delta U}{c} \quad \text{(total absorption of light)}$$

$$\Delta p = 2\frac{\Delta U}{c} \quad \text{(total reflection of light)}$$

$\Delta\vec{p}$ in direction of propagation of the incoming light



$$\Delta U = IA\Delta t$$

$$F = \frac{\Delta p}{\Delta t}$$

EM waves carry energy

Energy & momentum transfer to an irradiated object

→ Force on the object

$$F = \frac{\Delta p}{\Delta t} = \frac{IA}{c} \quad \text{(total absorption)}$$

$$F = 2\frac{IA}{c} \quad \text{(total reflection)}$$

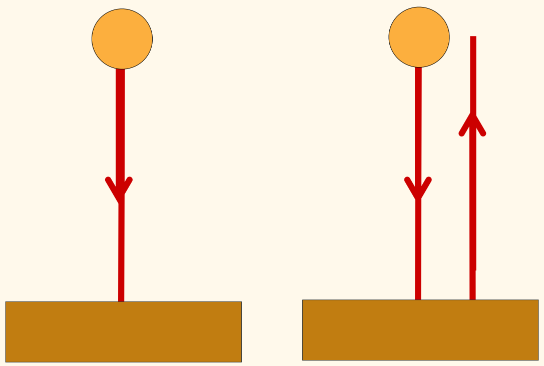
RADIATION PRESSURE

$$\Delta p = \frac{\Delta U}{c} \qquad F = \frac{IA}{c} \qquad \text{(total absorption of light)}$$

$$\Delta p = 2\frac{\Delta U}{c} \qquad F = 2\frac{IA}{c} \qquad \text{(total reflection of light)}$$

Factor 2 between total absorption and total reflection

→ More momentum is transferred during an elastic collision rather than an inelastic collision



Note: if not total absorption or total reflection
The factor is between 1 and 2

RADIATION PRESSURE

$$I = \frac{power}{area} = \frac{rate\ of\ doing\ work}{area}$$

$$W = \int F.dl = F.\Delta x \quad in\ 1D\ and\ conservative\ force$$

$$= \frac{(F.\Delta x)/_{\Delta t}}{area}$$

$$\Delta x/_{\Delta t} = speed = c$$

$$in\ case\ of\ light$$

$$I = P_r c$$

$$\frac{F}{area} = pressure = p_r$$

$$p_r = \frac{I}{c}$$

also called "radiation pressure"

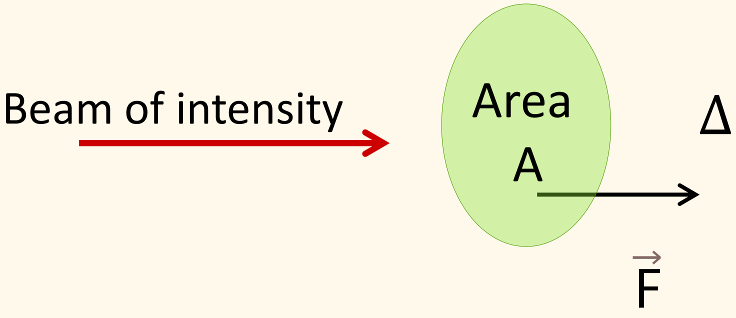
Note: Be careful not to confuse the symbol p_r for radiation pressure with the symbol p for momentum.

RADIATION PRESSURE

$$P_r = \frac{I}{c} \quad \text{(total absorption of light)}$$

$$P_r = 2\frac{I}{c} \quad \text{(total reflection of light)}$$

If I >>
and A << → (focused laser) P_r >>



$$\Delta U = IA\Delta t$$

EM waves carry energy

Energy & momentum
transfer to an irradiated object

→ Force on the object
→ Pressure (P_r) on the object

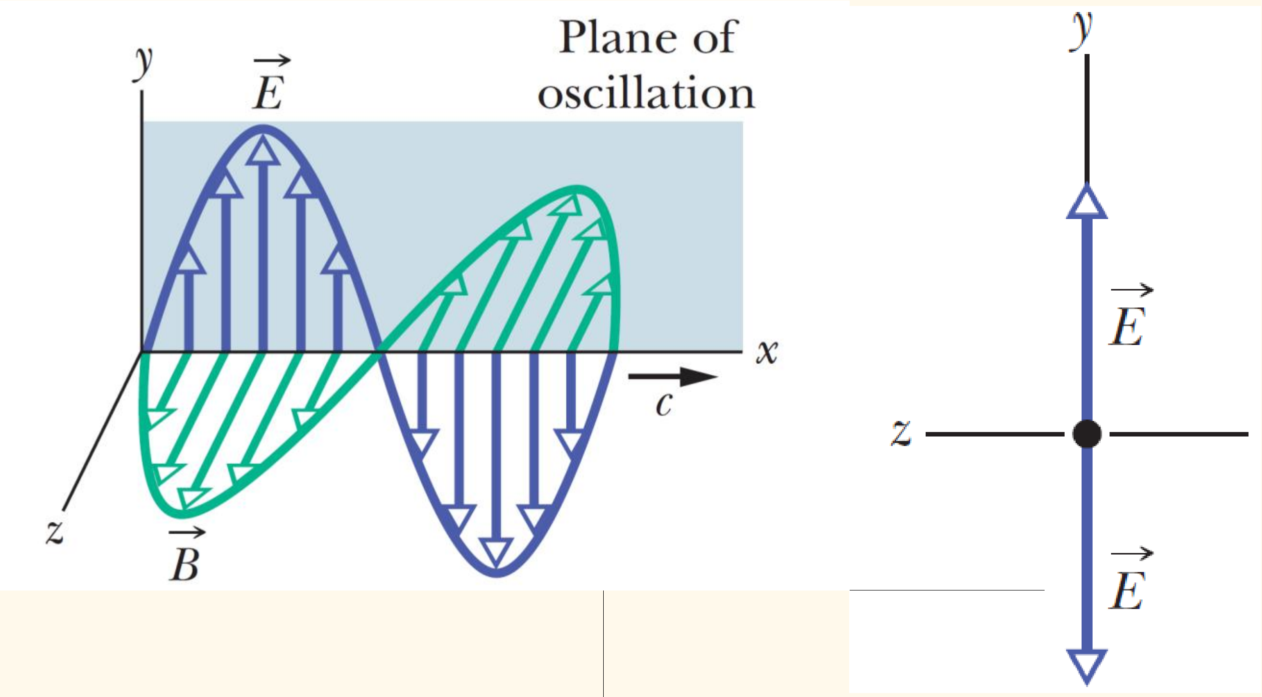
$$P_r = \frac{F}{A}$$

Radiation Pressure

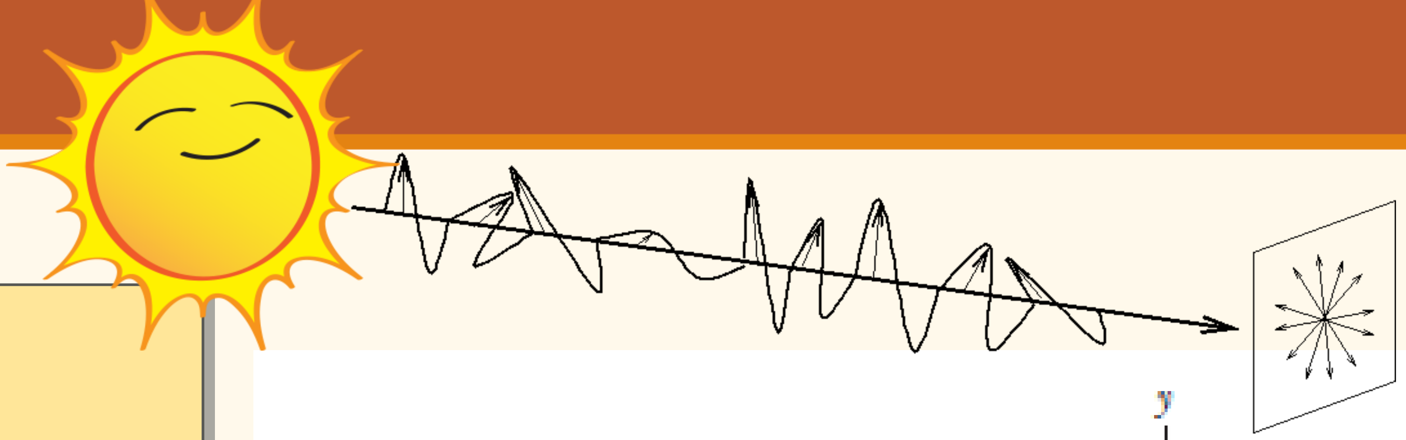
So far we represented the EM wave like this:

$\vec{E}$ is always in the (Oxy) plane
→ **Linear polarization** along Oy
Vertical polarization
→ (Oxy): **plane of oscillation**

But light is not always linearly polarized



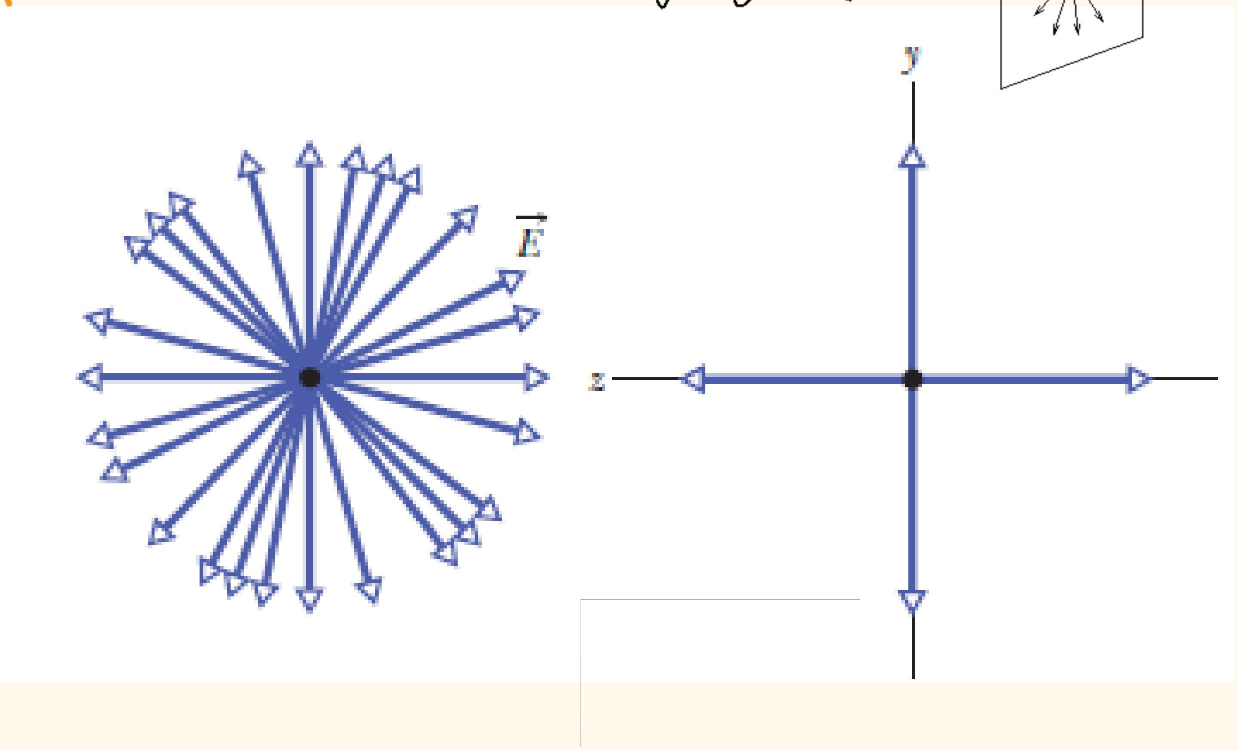
Schematic representation of an EM wave polarized along Oy



Unpolarized light
(e.g. sun, lightbulb, …)

$\vec{E}$ **randomly oriented** in a plane perpendicular to the direction of propagation

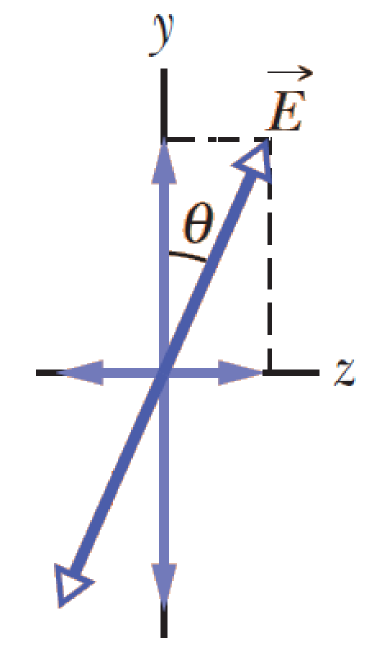
Separation of $\vec{E}$ in **2 components**
→ along Oz & Oy: E_z and E_y
Both oscillate as linearly polarized electric fields

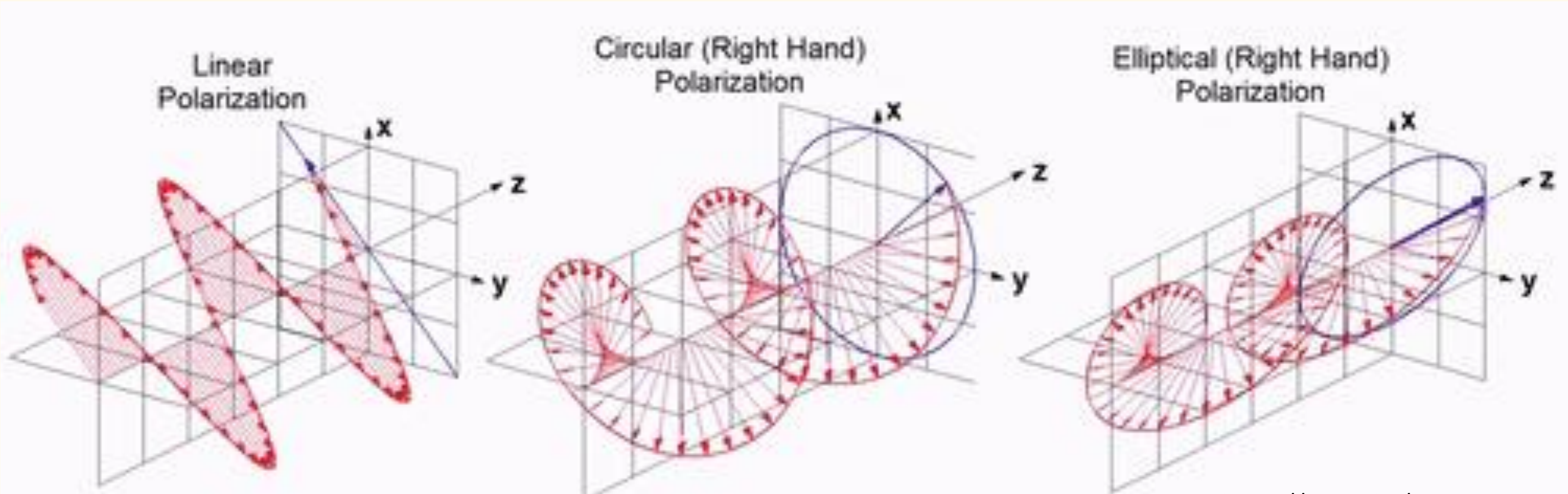


Schematic representation of an unpolarized EM wave

Note on polarization:

- It is possible to have a **partially polarized** light





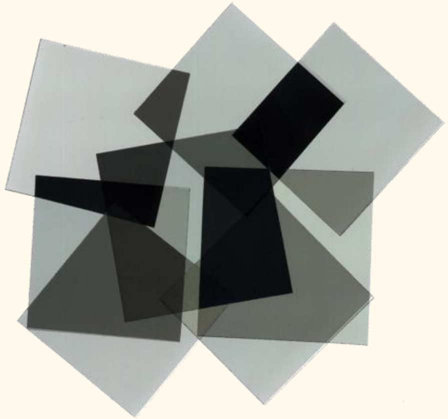
https://youtu.be/Q0qrU4nprB0

- If both components are in phase → back to **linear polarization** oriented with angle θ with respect to Oy

- If E_z and E_y have a $\pm\pi/2$ phase shift and the same amplitude → **circular polarization**

- If E_z and E_y have a $\pm\pi/2$ phase shift → **elliptic polarization**

POLARIZATION

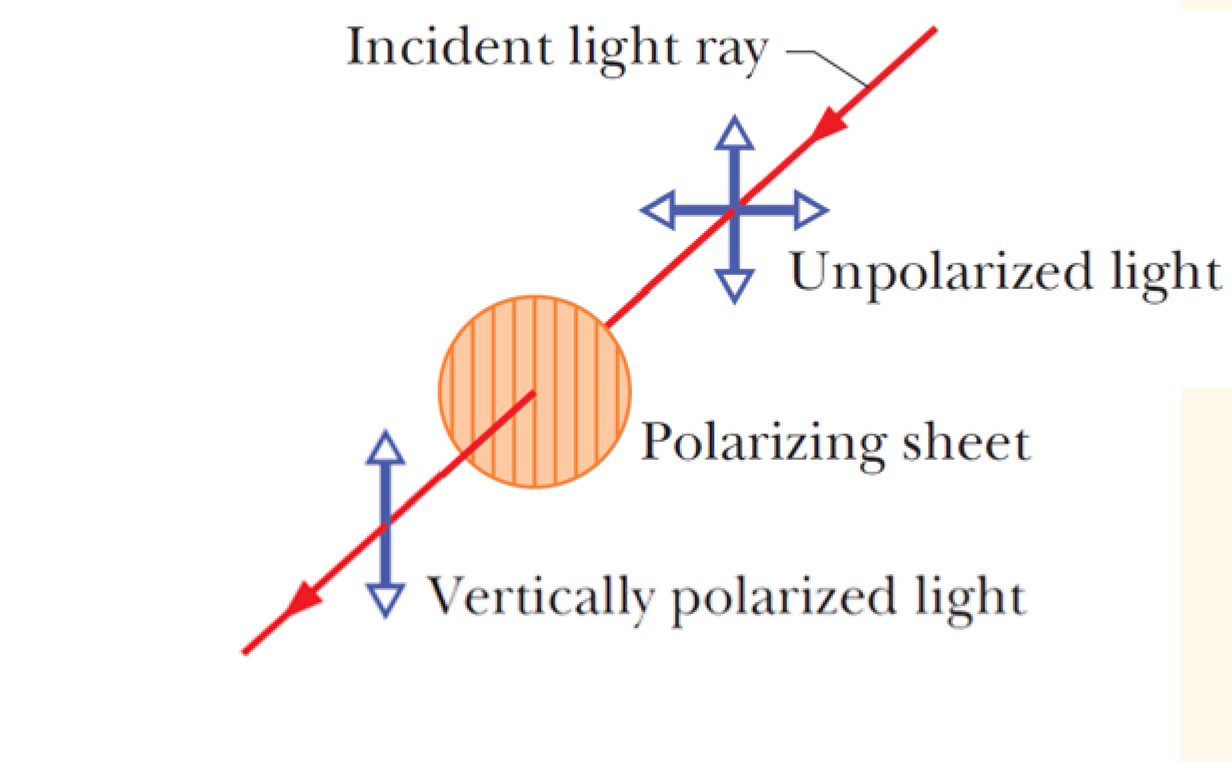


Unpolarized → Polarized light

Use of **polarizing sheet**

→ material that **transmit light** polarized along **one direction**
= Polarizing direction

POLARIZATION



Emerging light is linearly polarized parallel to the polarizing direction

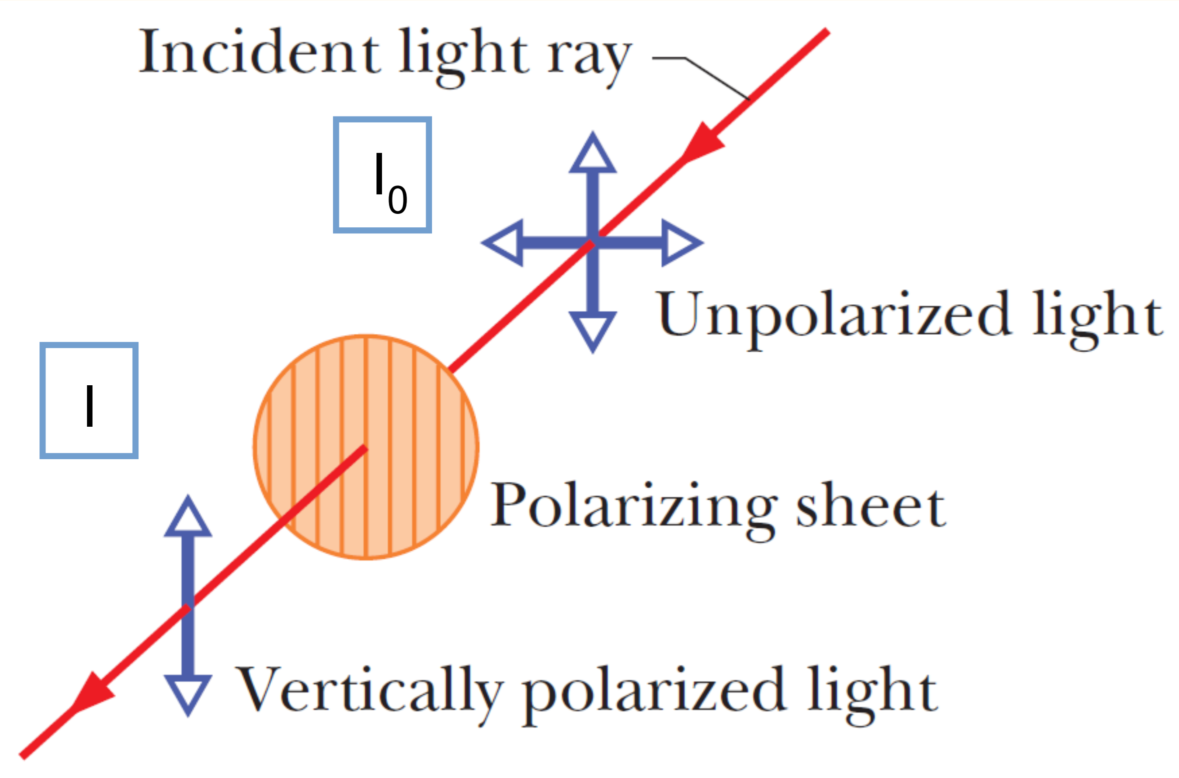
Unpolarized → Polarized light

Use of **polarizing sheet**

→ material that **transmit light** polarized along **one direction**
= Polarizing direction

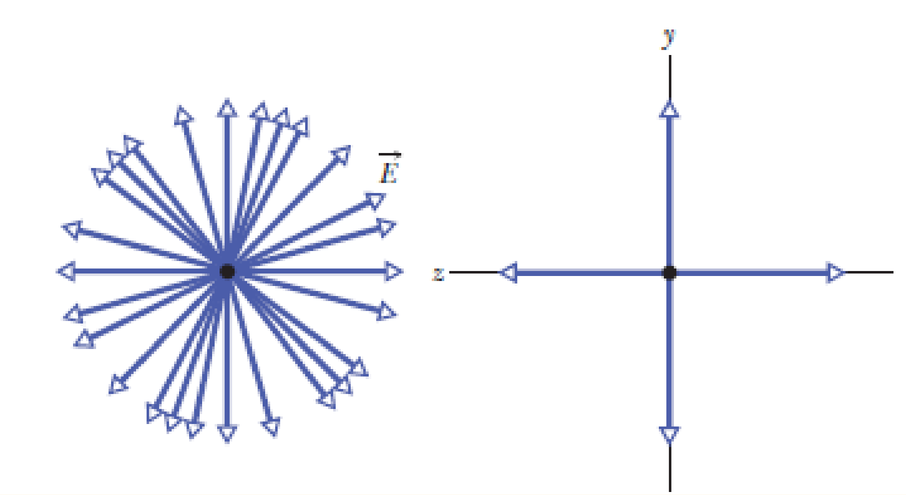
→ **absorbs light** polarized along the **perpendicular direction**

Note: like always in physics, it's more complicated than that. Polarizers don't really act like filters by « absorbing » some light. We have to deal with quantum mechanics for a full understanding

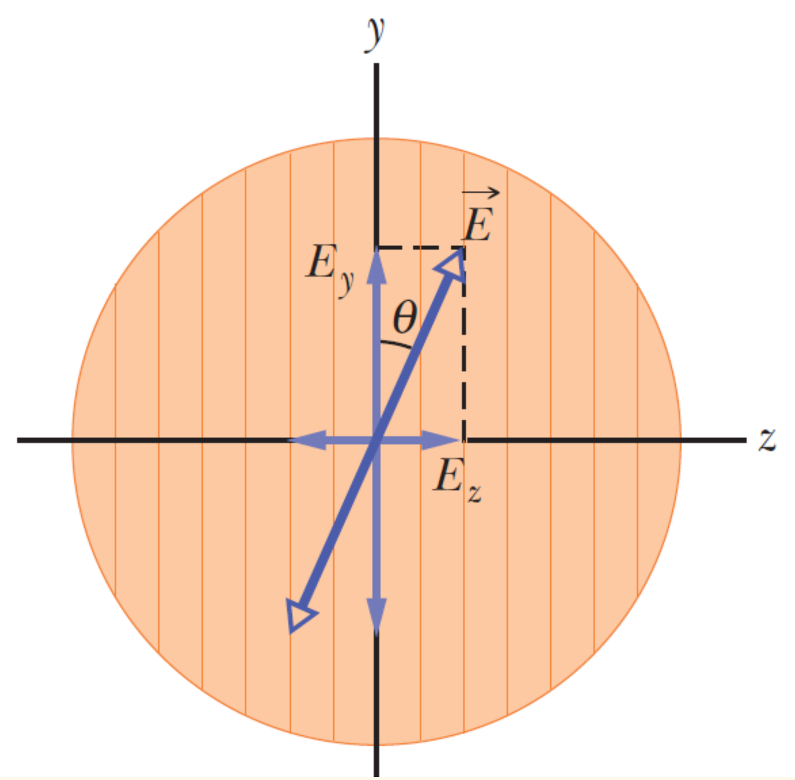


Intensity of the emerging light:

- For unpolarized light



$$I = I_0 / 2$$ (one-half rule)



Intensity of the emerging light:

→ For linearly incident polarized light

$$\vec{E} = E_y\overrightarrow{u_y} + E_z\overrightarrow{u_z} = E\cos(\theta)\overrightarrow{u_y} + E\sin(\theta)\overrightarrow{u_z}$$

$$I_0 = \frac{E^2}{2c\mu_0}$$

→ For Emerging light (sheet polarized along Oy)

$$\vec{E} = E_y\overrightarrow{u_y} = E\cos(\theta)\overrightarrow{u_y}$$

$$I = \frac{E^2}{2c\mu_0}\cos^2(\theta)$$

$$I = I_0\cos^2(\theta)$$

(Malus law)

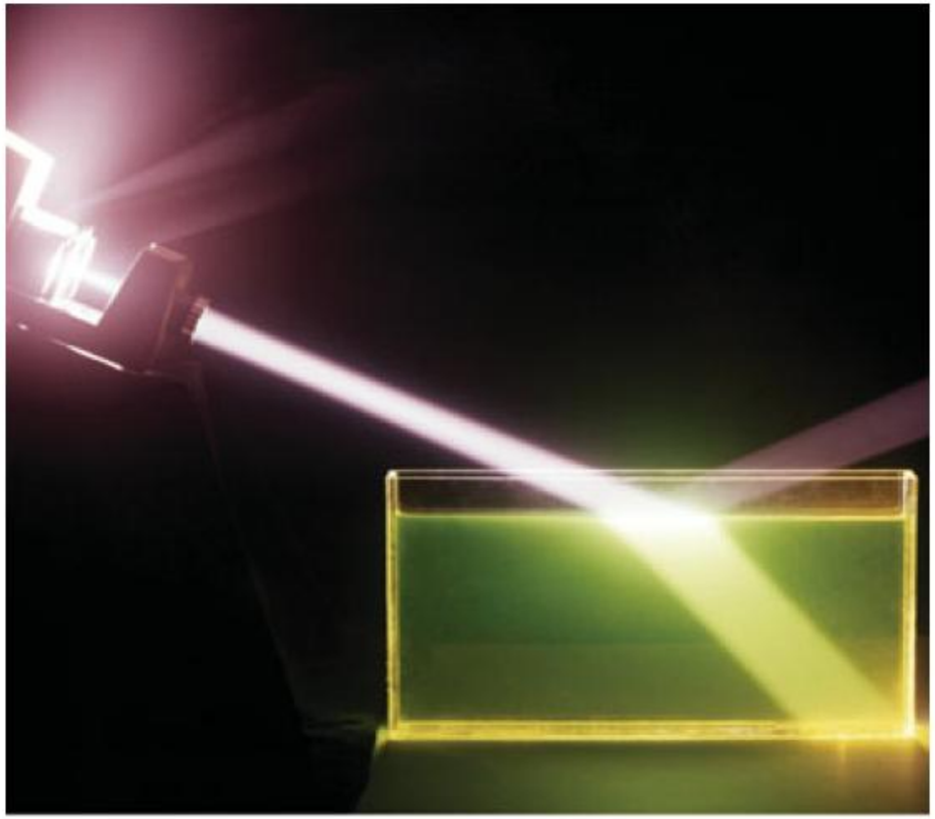
Note: 2 polarizers with perpendicular polarizing directions transmit no light

Geometrical optics

We only consider **rays of light** and do not treat light as a wave (but rays have a wavelength)
→ **approximation**

Easier to treat propagation of light:

- Interfaces between mediums
- Design of optical instruments



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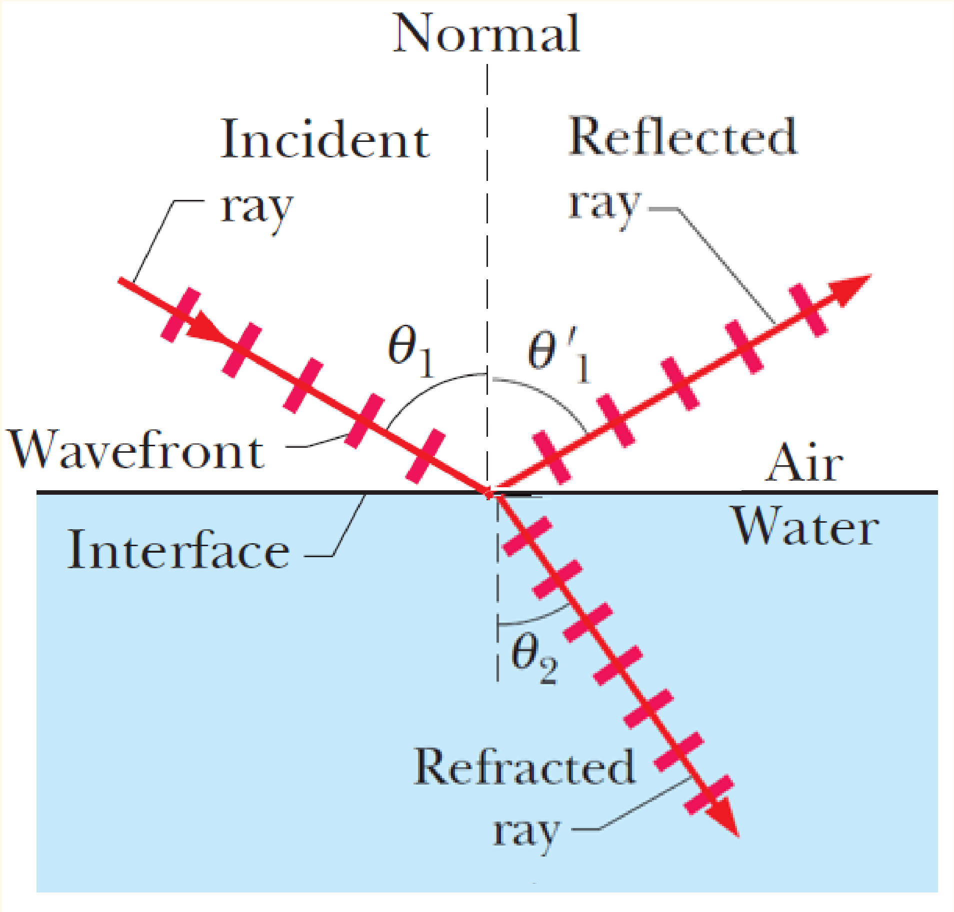
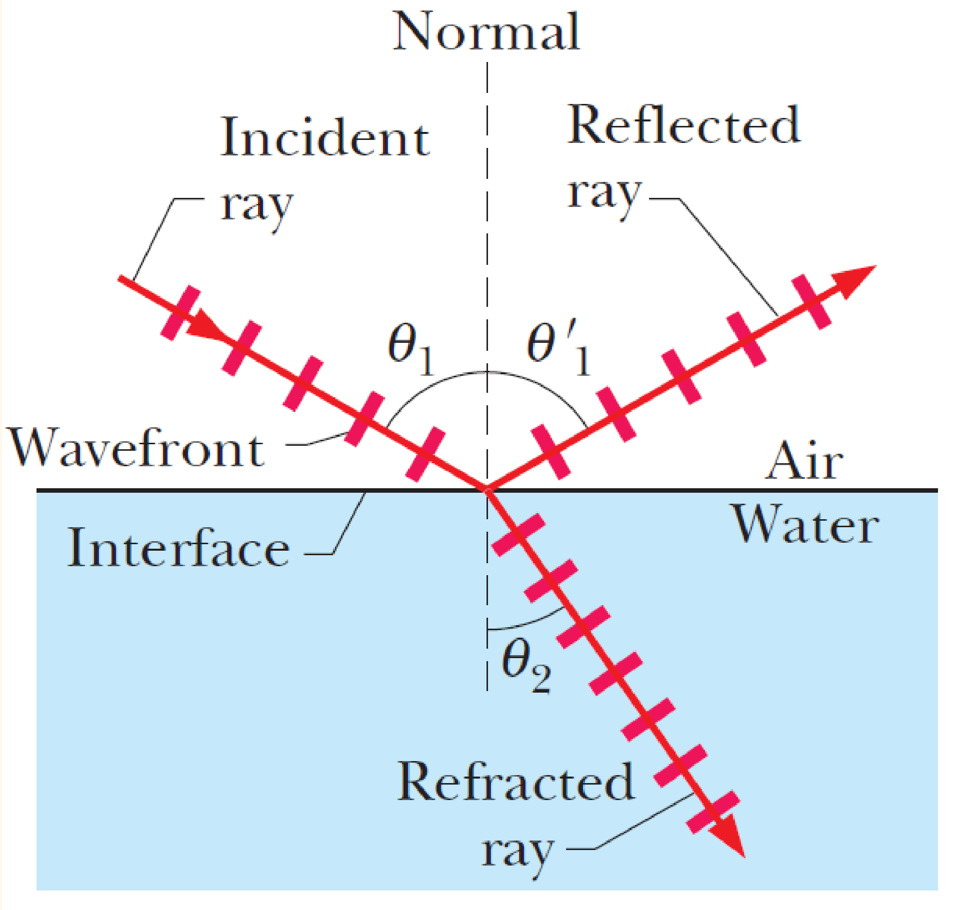


An **Incident ray** of light arrives at an interface (here air / water) With an angle θ with respect to the **normal to the interface**

- Some light is reflected
→ **Reflected ray**

- Some light travels in the other medium
→ **Refracted ray**

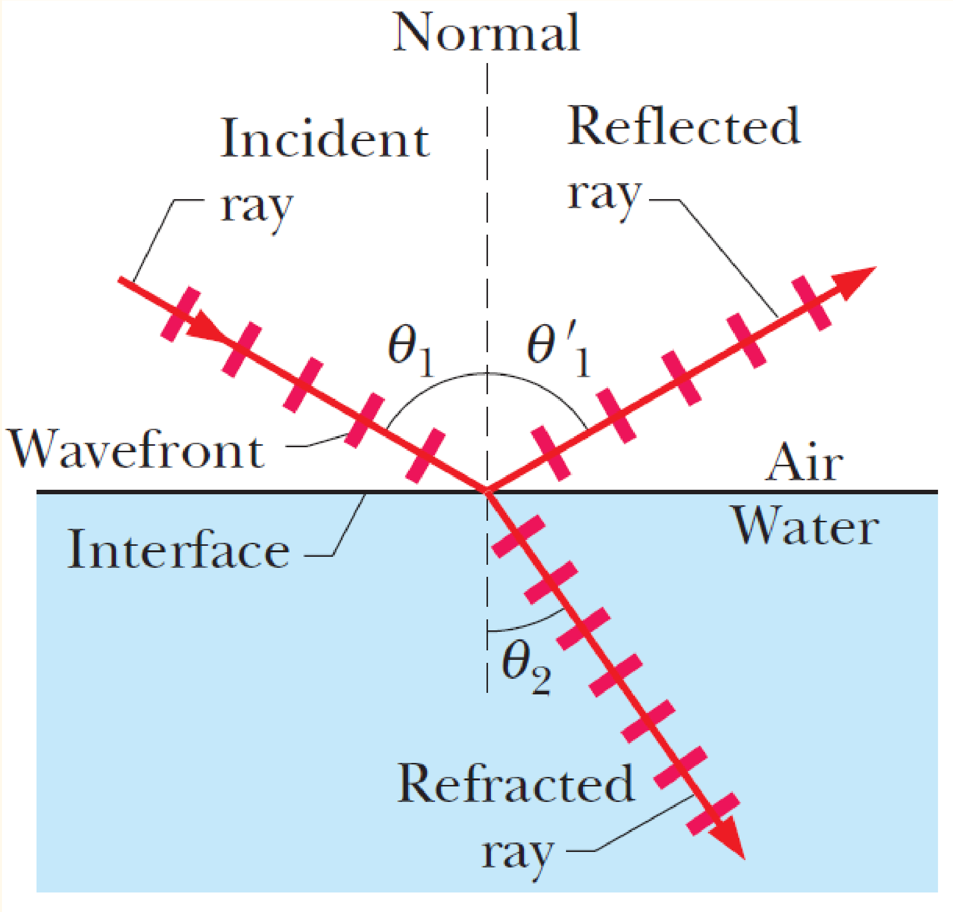


Vocabulary

θ_1 is the **angle of incidence**
θ'_1 is the **angle of reflection**
θ_2 is the **angle of refraction**

Angles between the rays and the normal

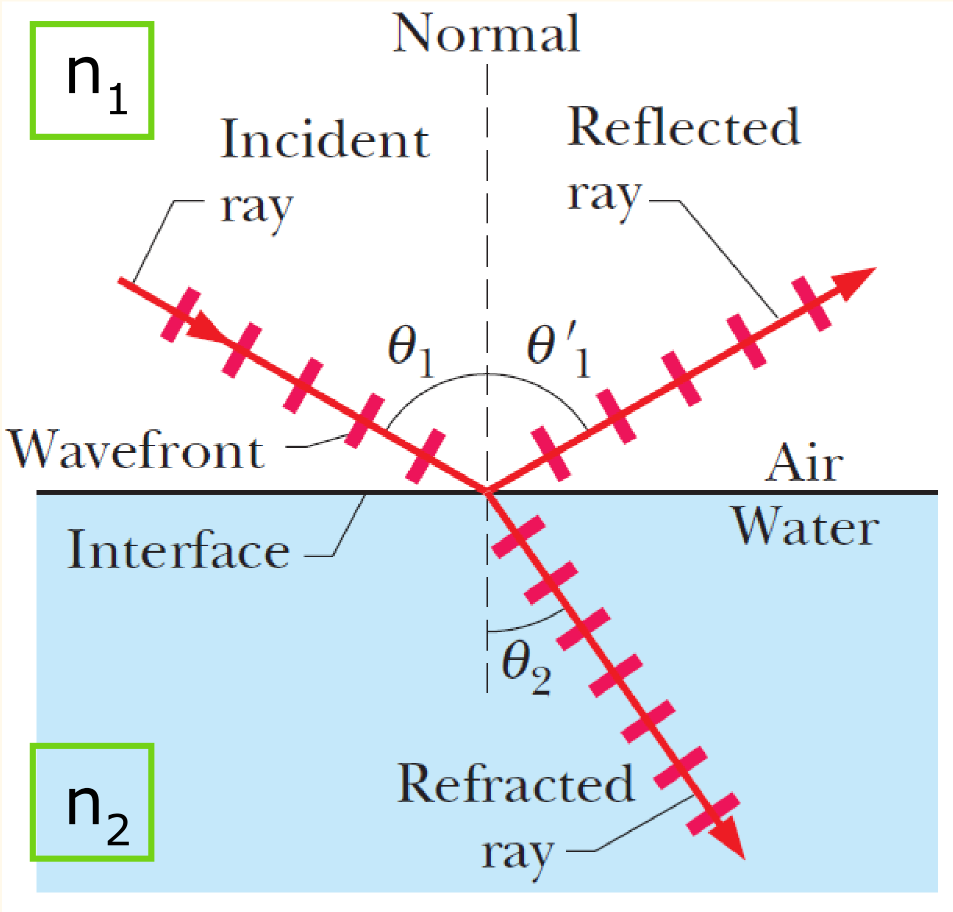
The normal and the incident ray define the **plane of incidence**



An **Incident ray** of light arrives at an interface (here air / water) With an angle θ with respect to the **normal to the interface**

Directions of the reflected ray and the refracted ray are determined by Snell's laws

Note: In France we call them *The Laws of Snell – Descartes* (S-D)



1st S-D law:
The **reflected ray** and the **refracted ray** are in the **plane of incidence**

2nd S-D law:
The **angle of reflection** is equal to the **angle of incidence**

$$\theta'_1 = \theta_1$$

3rd S-D law:
The **angle of refraction** is given by

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

n_1, n_2: **indexes of refraction** of the mediums

$$n = \frac{speed\ of\ light\ in\ the\ vacum}{\mathbf{speed\ of\ light\ in\ the\ medium}} = \frac{c}{v}$$

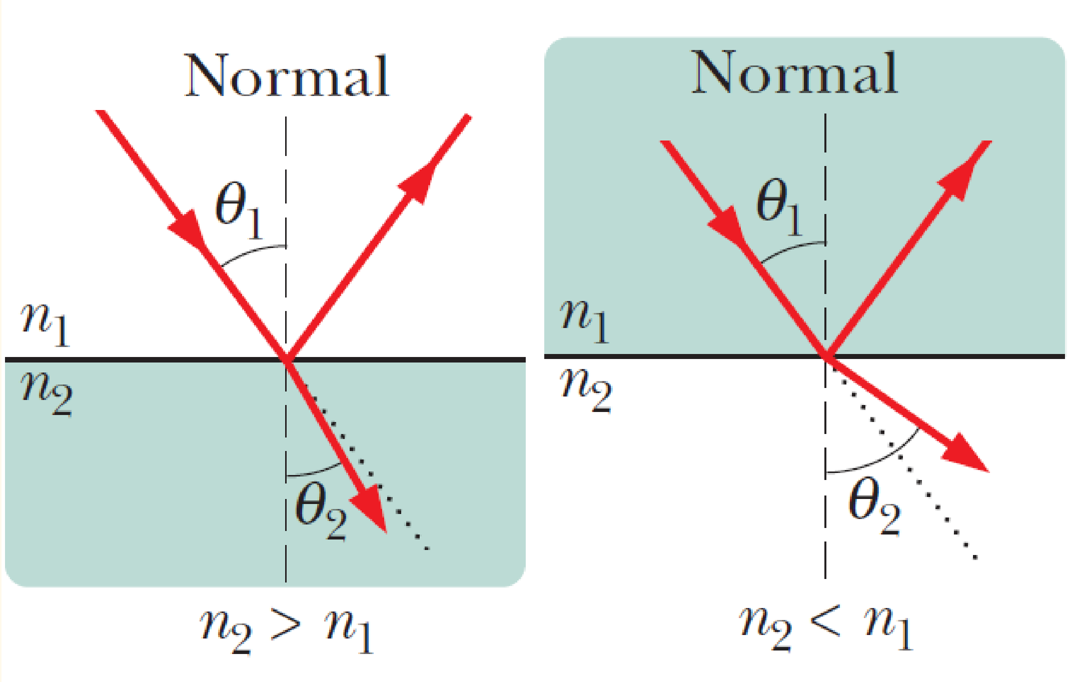
That implies: n ≥ 1

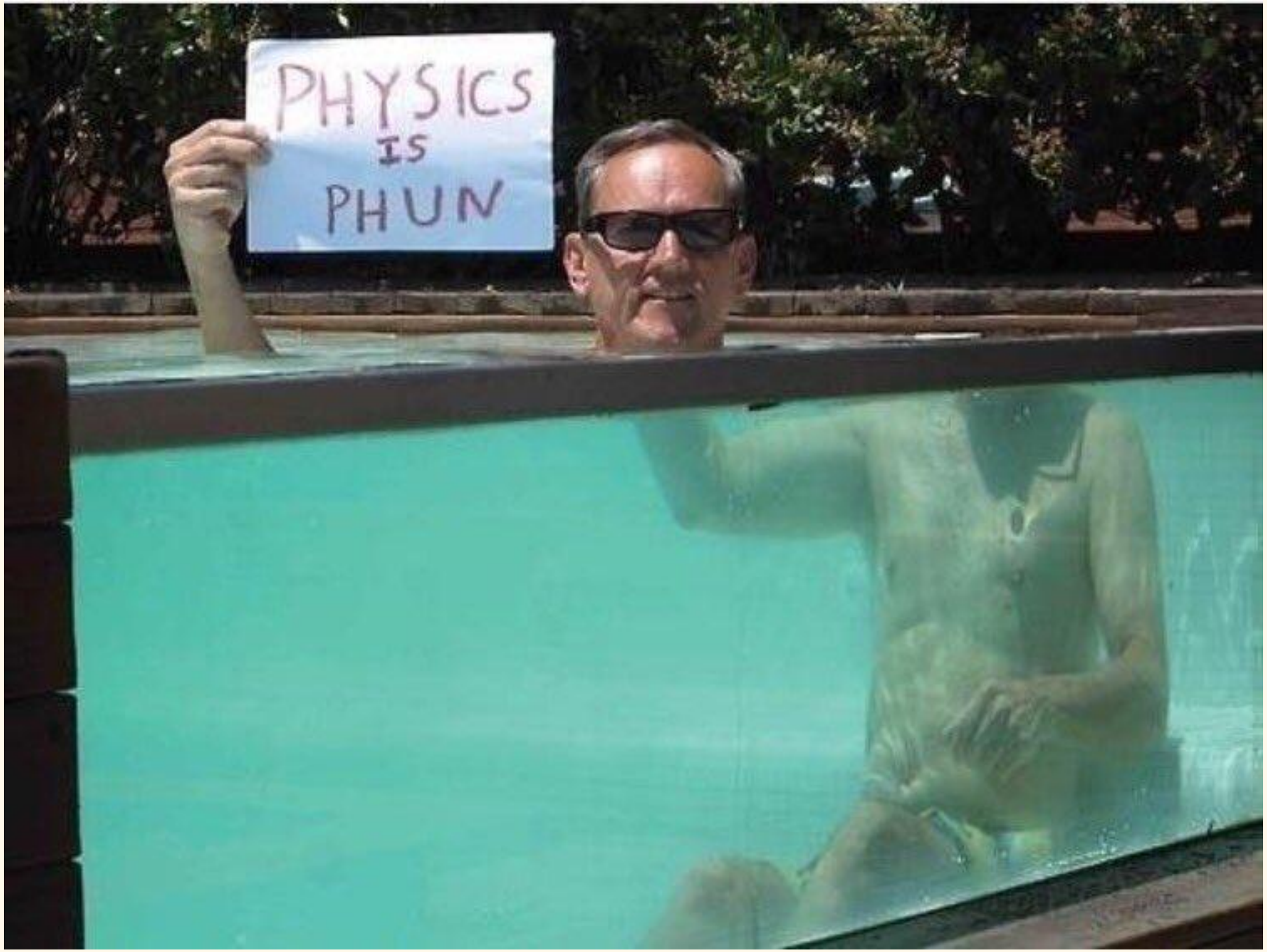
Typical values:
- $n_{air} \simeq 1$
- $n_{eau} \simeq 1.33$
- $n_{glass} \simeq 1.4 - 1.7$

Rewriting the 3rd S-D law

$$\sin(\theta_2) = \frac{n_1}{n_2}\sin(\theta_1)$$

If $n_2 > n_1$ (resp. $< n_1$)
the refracted ray is bend towards
(resp. away) from the normal

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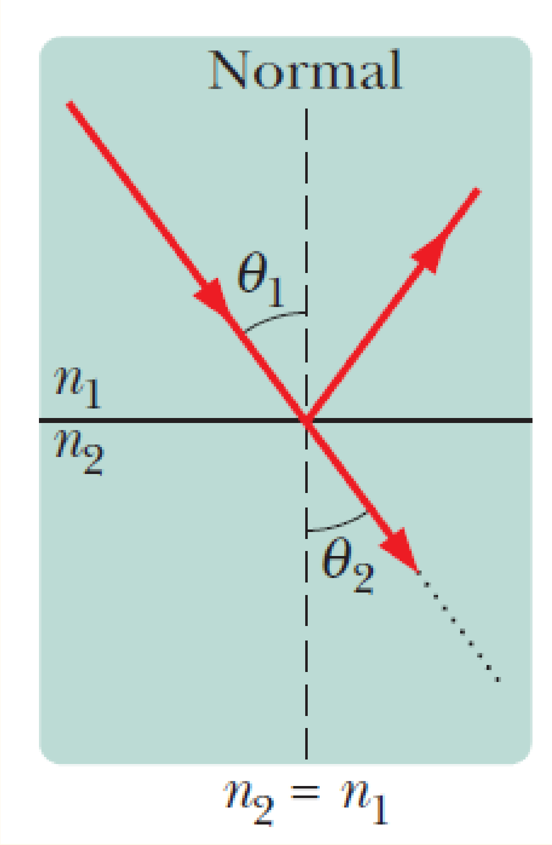
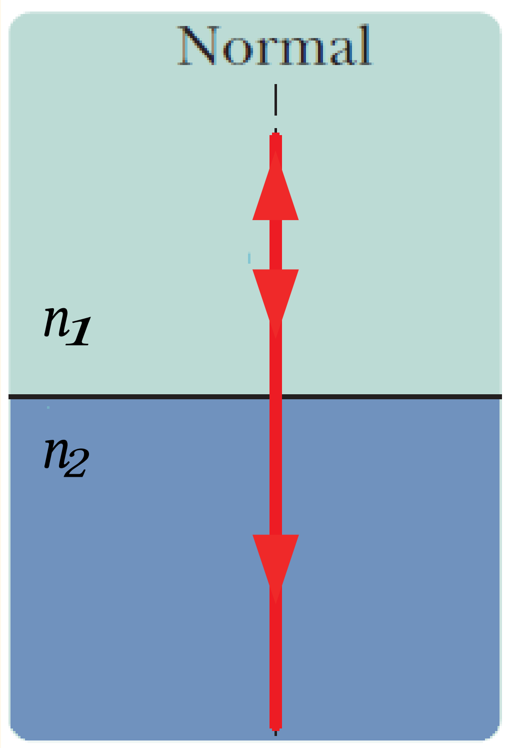
Note:



If $n_1 = n_2$ (index matching) the incident ray is not deviated



If $\theta_1 = 0°$ (normal incidence)

→ $\sin(\theta_1) = 0$

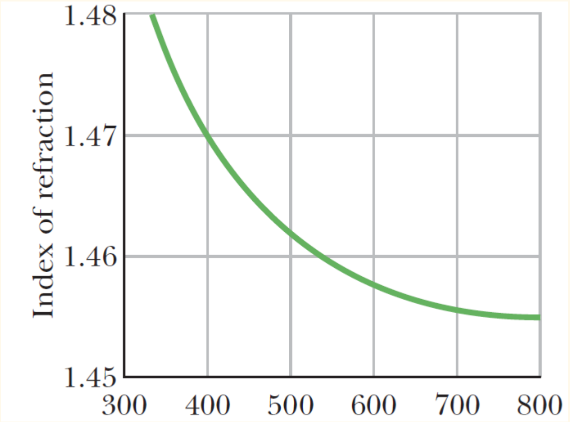
Thus, $\theta'_1 = 0$

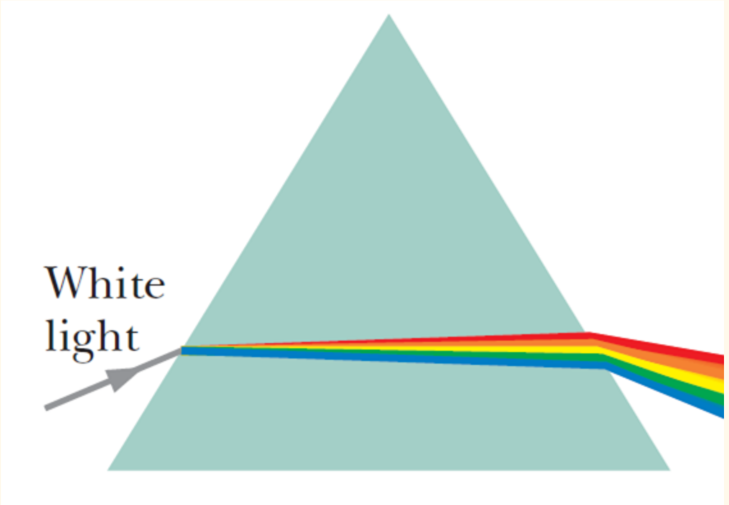
→ The reflected beam is superposed to the incident ray

And $\sin(\theta_2) = 0$, so $\theta_2 = 0$

→ The refracted ray is not deviated

Note that **n is a function of the wavelength** (e.g. $n_{(\lambda)}$ for fused quartz)





Beam of white light = superposition of **Monochromatic** rays (each with a different λ) with the same θ_1
→ different θ_2
→ **Chromatic dispersion**

Prisms separate wavelengths with two refractions

The same phenomena occurs in raindrops to form rainbows

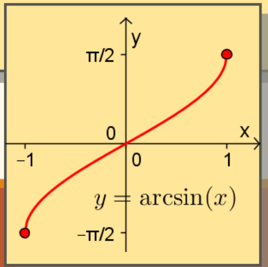
S-D law of refraction:

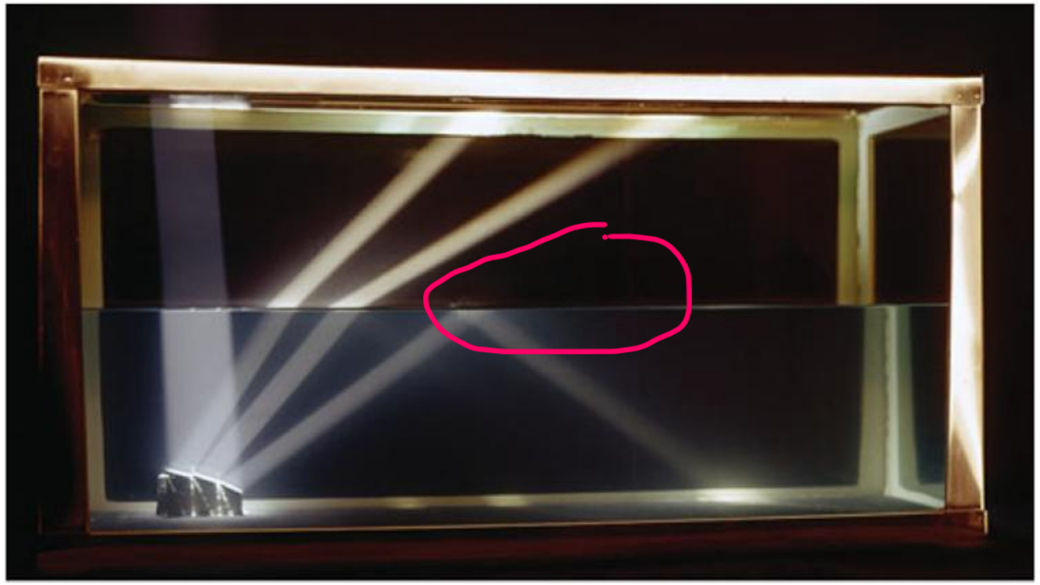
$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\rightarrow \boldsymbol{\theta_2 = asin\left(\frac{n_1}{n_2} sin(\theta_1)\right)}$$

We assume $\mathbf{n_1 > n_2}$
(e.g. medium 1 is water, 2 is air)

→ Above a certain value of θ_1, we cannot calculate θ_2
(**asin(x) with x > 1 is not defined**)





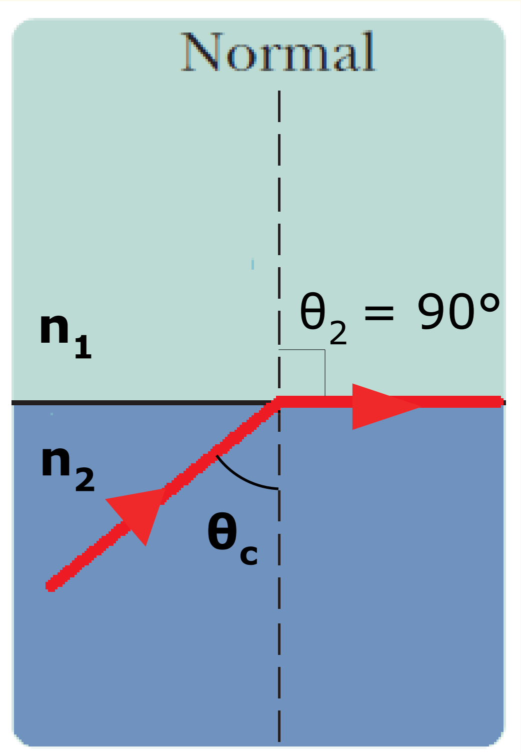
Ken Kay/Fundamental Photographs

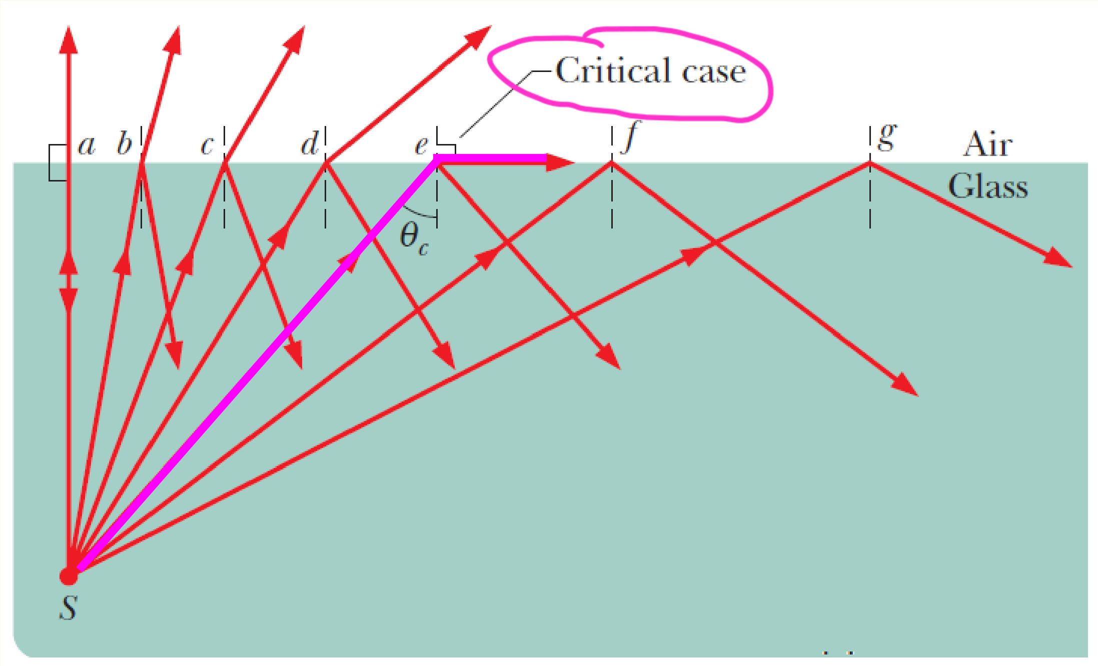
In such cases, the refracted ray **does not exist** → only reflection in medium 1

→ **Total Internal Reflection (TIR)**

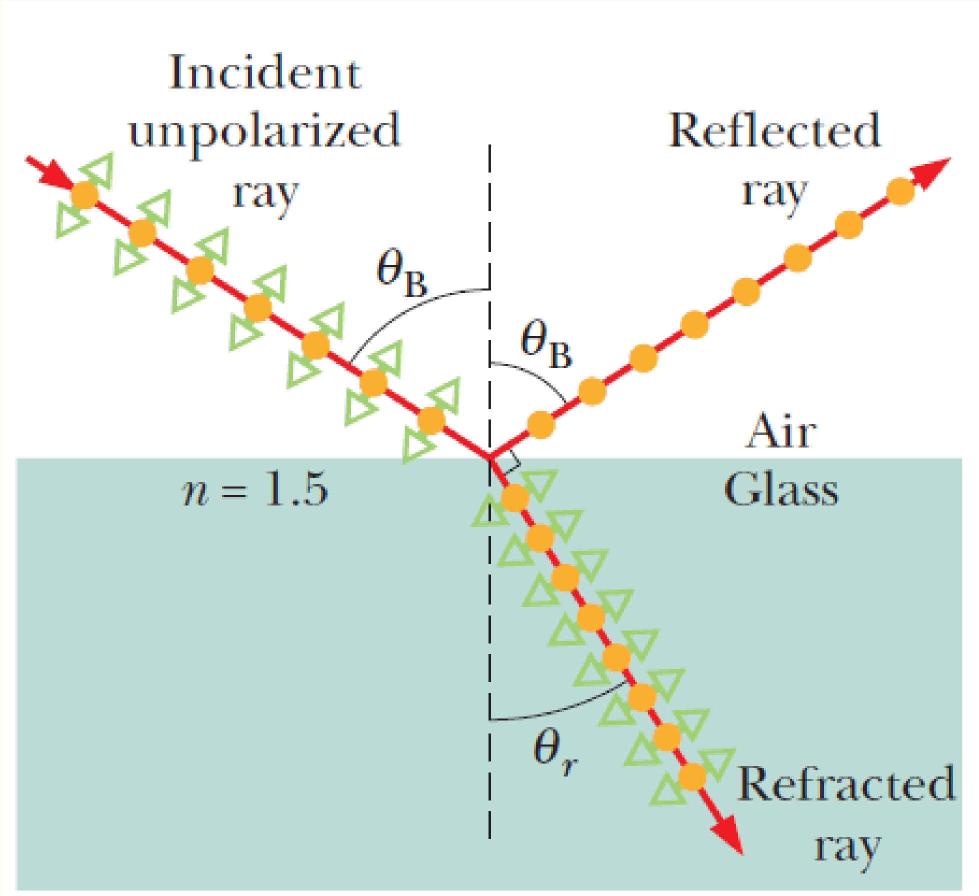
For TIR, we define the **critical angle θ_c** such that $\theta_2 = 90°$:
$n_1 \sin(\theta_c) = n_2 \sin(90) = n_2$

$$\theta_c = \operatorname{asin}\left(\frac{n_2}{n_1}\right)$$





TIR if $n_1 > n_2$ and $\theta > \theta_c$

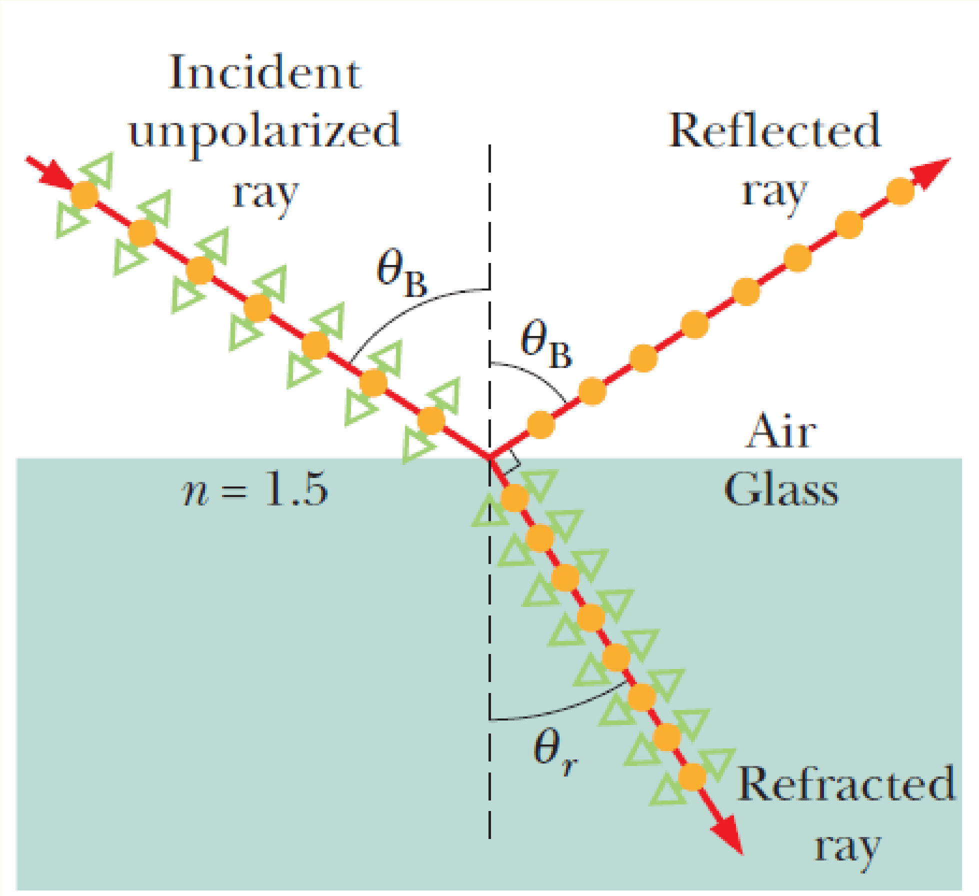


Previously, we saw that light passing through a polarizing sheet is polarized

Reflection Brewster angle θ_B
For $\boldsymbol{\theta_1 = \theta_B}$ (2^{nd} S-D)

→ **Reflected and refracted rays are perpendicular**

→ **Reflected ray is polarized perpendicularly to the plane of incidence**



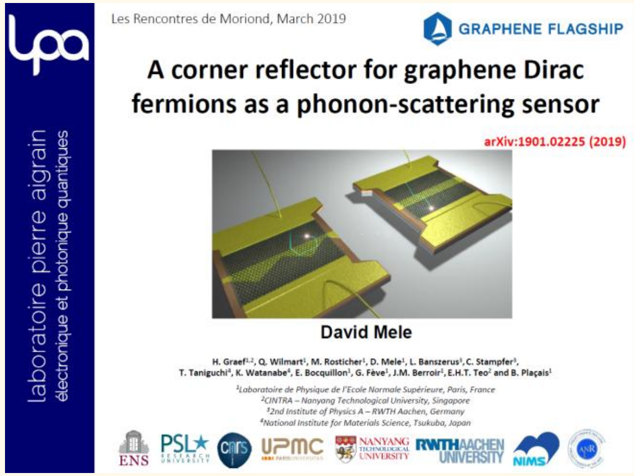
We have: $\theta_B + \theta_2 = 90$

with $n_1 \sin(\theta_B) = n_2 \sin(\theta_2)$ (3rd S-D)

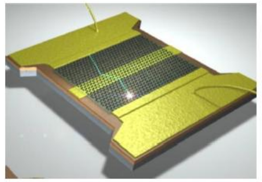
So: $n_1 \sin(\theta_B) = n_2 \sin(90 - \theta_B) = n_2 \cos(\theta_B)$

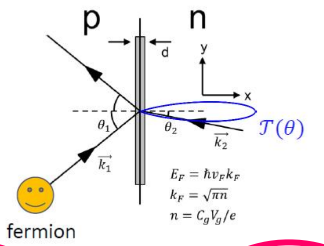
Then: $\tan(\theta_B) = \frac{n_2}{n_1}$ —— $\boldsymbol{\theta_B = arctan\left(\frac{n_2}{n_1}\right)}$

One of my research interest was to retrieve the Snell laws and total Internal Reflexion, not with light but with high speed electrons, called ballistic electrons that « behave like light » in electronics 2D based material



Dirac Fermion Optics (DFO) – Diopter



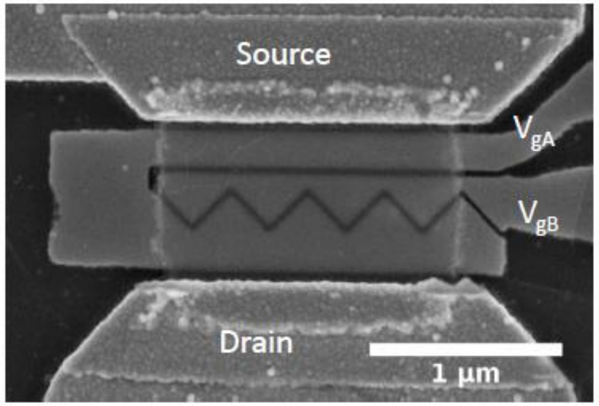


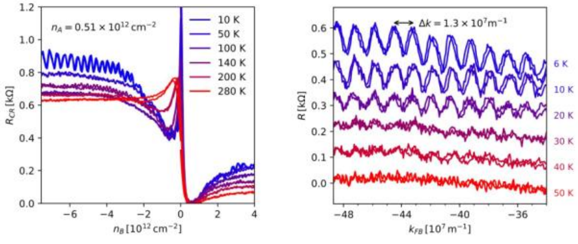
	Photon optics (3D)	Dirac fermion optics (2D)
Medium	transparent	ballistic
Phase velocity	$3 \cdot 10^8\ ms^{-1}$	$10^6\ ms^{-1}$
Snell-Descartes	$n_1 sin\theta_1 = n_2 sin\theta_2$	$E_{F1} sin\theta_1 = E_{F2} sin\theta_2$
Critical angle	$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$	$\theta_c = \arcsin\left(\frac{E_{F2}}{E_{F1}}\right)$
Fresnel relation	$R_s = \left\lvert\frac{n_1 cos\theta_i - n_2 cos\theta_t}{n_1 cos\theta_i + n_2 cos\theta_t}\right\rvert^2$	$\mathcal{T}(\theta) = e^{-\pi\frac{2d}{\lvert k_1 - k_2\rvert}k_2^2 \sin^2\theta}$

Cheianov et al., PRB 74 (2006) 041403(R) J. Cayssol et al., PRB 79 (2009) 075428 2

Light in two different optical medium

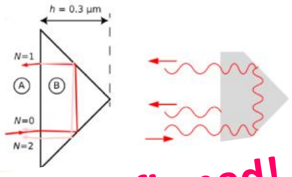
electron in two different electrostatic medium





Gate coupling calibration:

$$L = \frac{2\pi}{\Delta k_{FB}} \sim 0{,}5\ \mu m \sim 2h$$

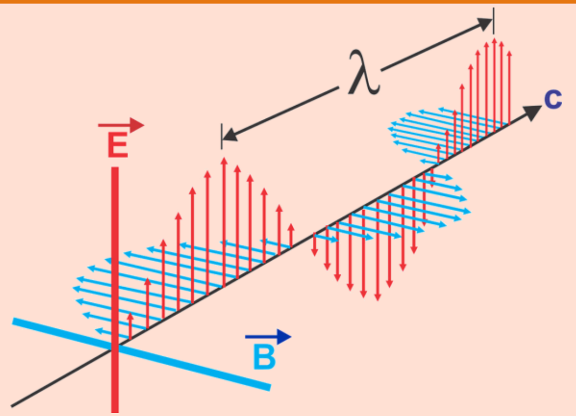


TIR with electron confirmed!

KEY POINTS

Light as an EM wave

Structure of the traveling plane EM wave



$$\frac{E}{B} = c$$

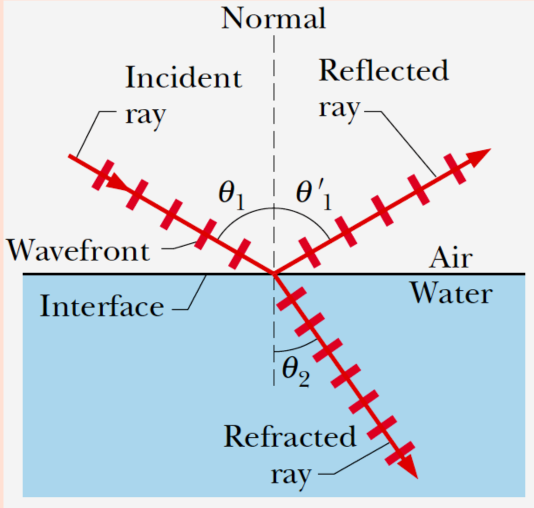
Poynting vector, intensity and radiation pressure

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} \qquad I = \frac{E_m^2}{2c\mu_0}$$

Polarized and unpolarized light

Snell-Descartes laws

TIR and Polarization by reflection



$$n_1 sin(\theta_1) = n_2 sin(\theta_2)$$

READING ASSIGNMENT

Chapter 34 of the textbook