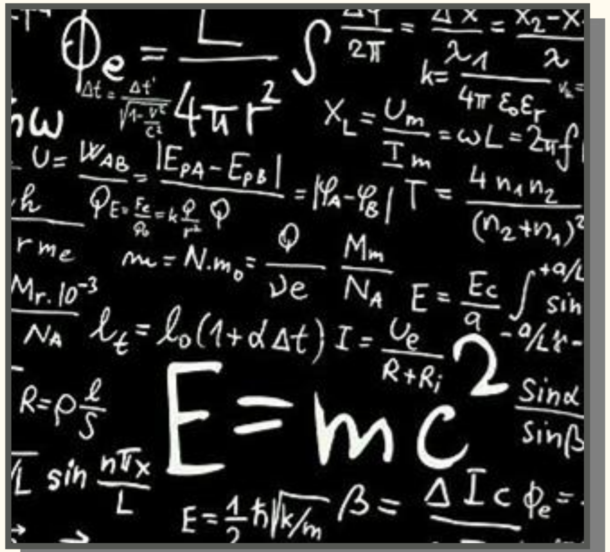
SUMMER PROGRAM – PHYSICS COURSES



UNIVERSITÉ CATHOLIQUE DE LILLE 1875

Lecturer for the 2nd part of the program

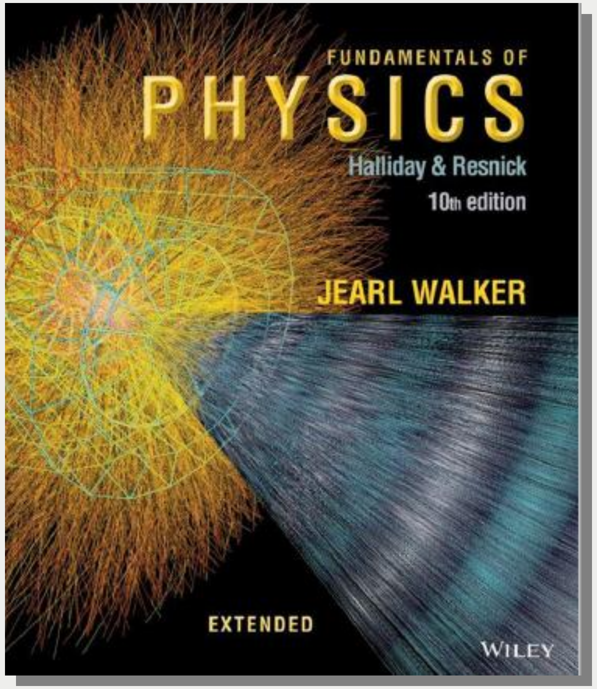
Dr. David MELE
david.mele@junia.com

Images: univ-catholique.fr wikipedia.org lilletourism.com

INTRODUCTION – 2nd PART OF THE COURSES

Content of this class

- Electromagnetic Oscillations & Alternating Current
- Maxwell's Equations & Magnetism of Matter
- Electromagnetic Waves
- Images
- Interference
- Diffraction



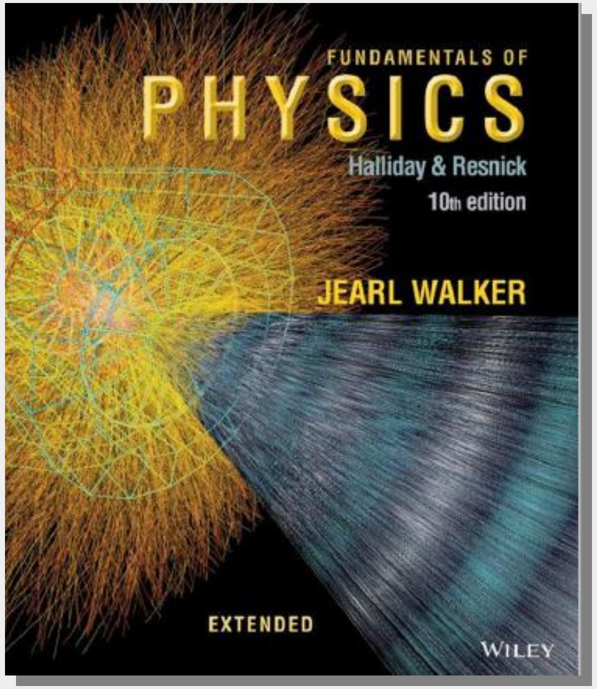
Textbook: *Fundamental of Physics, 10th edition*, Wiley, J. Walker, D. Halliday, R. Resnick,
Chapters 31 -36

INTRODUCTION – 2nd PART OF THE COURSES

This course will essentially follow the textbook

Reading assignments: Have a look at the next chapter before the lesson

Note: Expect specified otherwise, all figures in this presentations are adapted from the textbook



Textbook: *Fundamental of Physics, 10th edition*, Wiley, J. Walker, D. Halliday, R. Resnick,
Chapters 31 -36

INTRODUCTION – 2nd PART OF THE COURSES

Learn Int'l / UFL Program Draft Schedule - ESP2 2025 *subject to change

June	Friday 27	Saturday 28	Sunday 29	July	Monday 30	Tuesday 1	Wednesday 2	Thursday 3	Friday 4	Saturday 5	Sunday 6
9 am - 10 am	Chapter 31 Lecture			9 am - 10 am		Chapter 33 Lecture		Test (30min MCQ)	Chapter 36 Lecture		
10 am - 11am				10 am - 11am				Chapter 35 Lecture			
11 am - 12 pm	Chapter 31 Exercice			11 am - 12 pm					Chapter 36 Exercice		
12 pm - 1:30 pm				12 pm - 1:15 pm	Lunch time Group photo	Lunch time	Lunch time	Lunch time	Lunch time		
				1:15 - 2 pm	Chapter 31 Exercice	Chapter 33 Exercice					
2 pm - 3 pm				2 pm - 3 pm		Chapter 34 Lecture		Chapter 35 Exercice			
3 pm - 4 pm				3 pm - 4 pm	Chapter 32 Lecture			Chapter 36 Lecture			
4 pm - 4:15				4 pm - 4:15		Chapter 34 Exercice					

July	7	8	9	10	11	12	13
9 am - 12 pm 12 pm - 1:15 pm 1:15 pm - 4:15 pm	lab class			lab class			

July	14	15	16	17	18	19	20
9 am - 12 pm 12 pm - 1:15 pm 1:45 pm - 4 pm		IEMN Visit	Lab class	IEMN Visit	Lab class		

July	21	22	23	24	25	26	27
9 am - 12 pm 12 pm - 2 pm	EXAM (2h) chapters 31-33-34-35-36 (not 32)						
2 pm - 4 pm							

Intermediate evaluation (MCQ) + final test

Before we start

Physics requires to handle some mathematical tools
In this class, we will use accessible tools

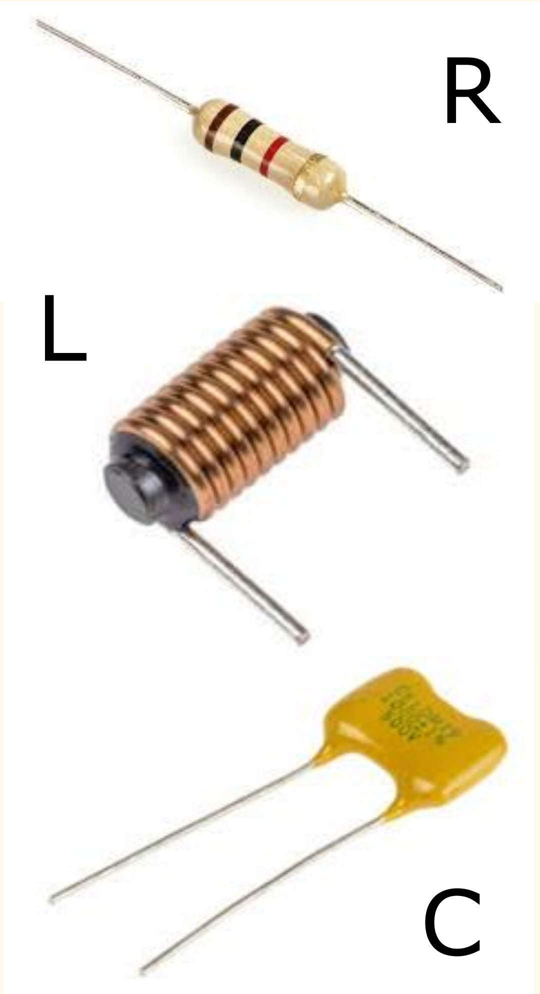
→ In the future of your studies, you will learn more advanced tools

→ We will have to admit some results

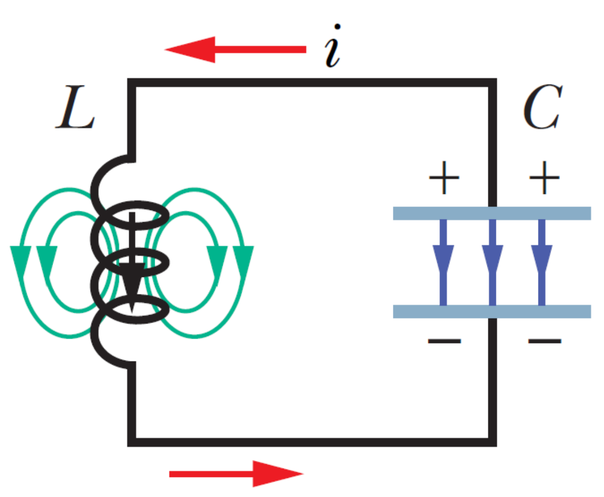
- **Electromagnetic Oscillations & Alternating Current**
- Maxwell’s Equations & Magnetism of Matter
- Electromagnetic Waves
- Images
- Interference
- Diffraction

Textbook: Chapter 31

- LC OSCILLATIONS
- DAMPED OSCILLATIONS IN AN RLC CIRCUIT
- FORCED OSCILLATIONS OF THREE SIMPLE CIRCUITS
- THE SERIES RLC CIRCUIT POWER IN ALTERNATING-CURRENT CIRCUITS
- TRANSFORMERS



Images: rs-online.com



Energy stored in the E field of the capacitor

$$U_E = \frac{q^2}{2C}$$

q: Charge stored (C)
C: Capacitance (F)

Energy stored in the B field of the inductor

$$U_B = \frac{Li^2}{2}$$

i: Current (A)
L: Inductance (H)

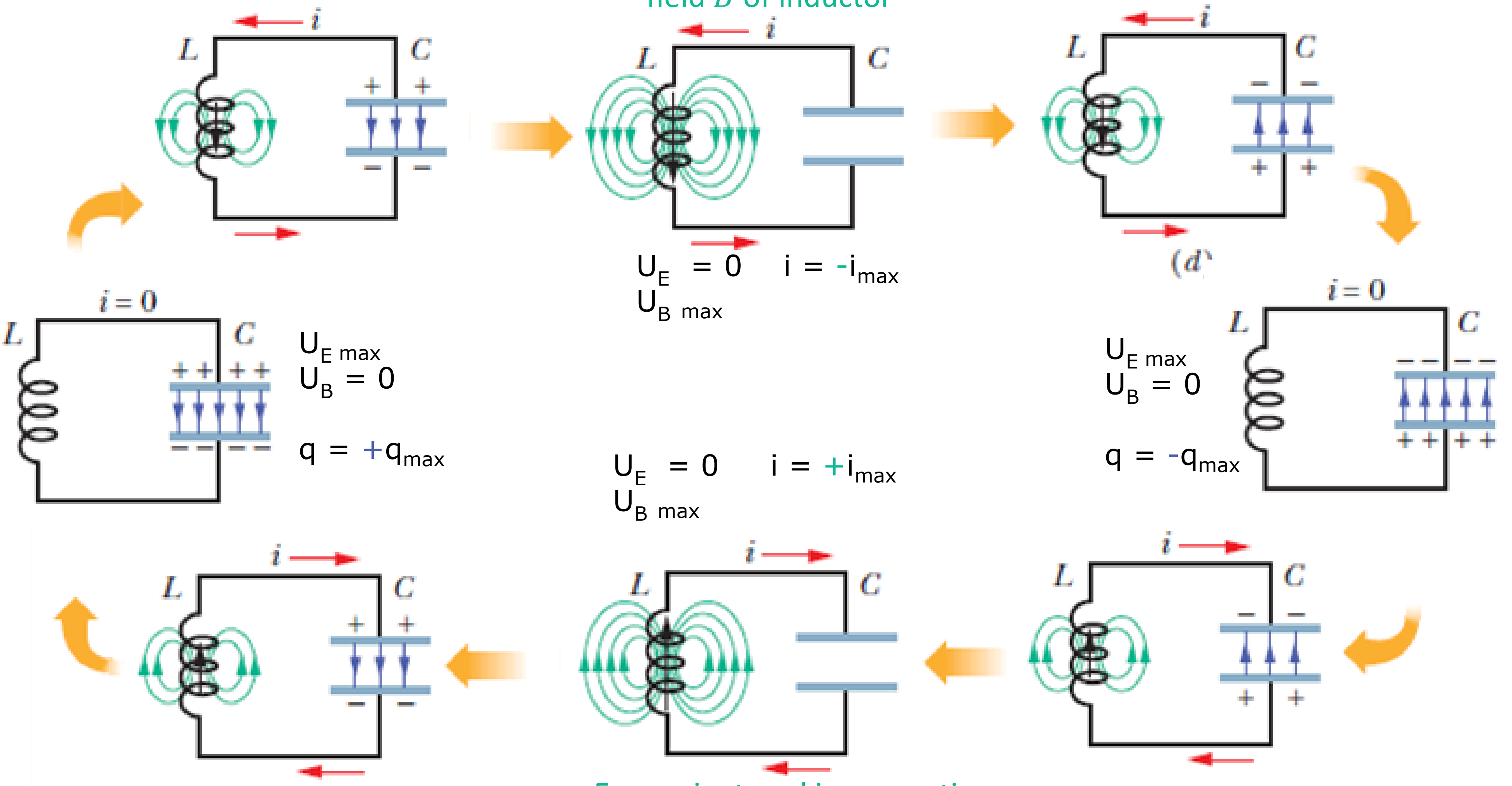
System: **LC** with an energy U distributed between **U_E** & **U_B**
Assuming **no losses**

What will happen ?
→ Discharge of C in L
→ C completely discharged
→ Charge of C by i
→ C completely charged
→ Discharge of C in L

And so on …

Note: q is the charge stored on a given plate

LC OSCILLATIONS



Energy is stored in magnetic field $\vec{B}$ of inductor

$U_E = 0$ $i = -i_{max}$
$U_{B\ max}$

Energy is stored in electric field $\vec{E}$ of capacitor

$U_{E\ max}$
$U_B = 0$

$q = +q_{max}$

$U_{E\ max}$
$U_B = 0$

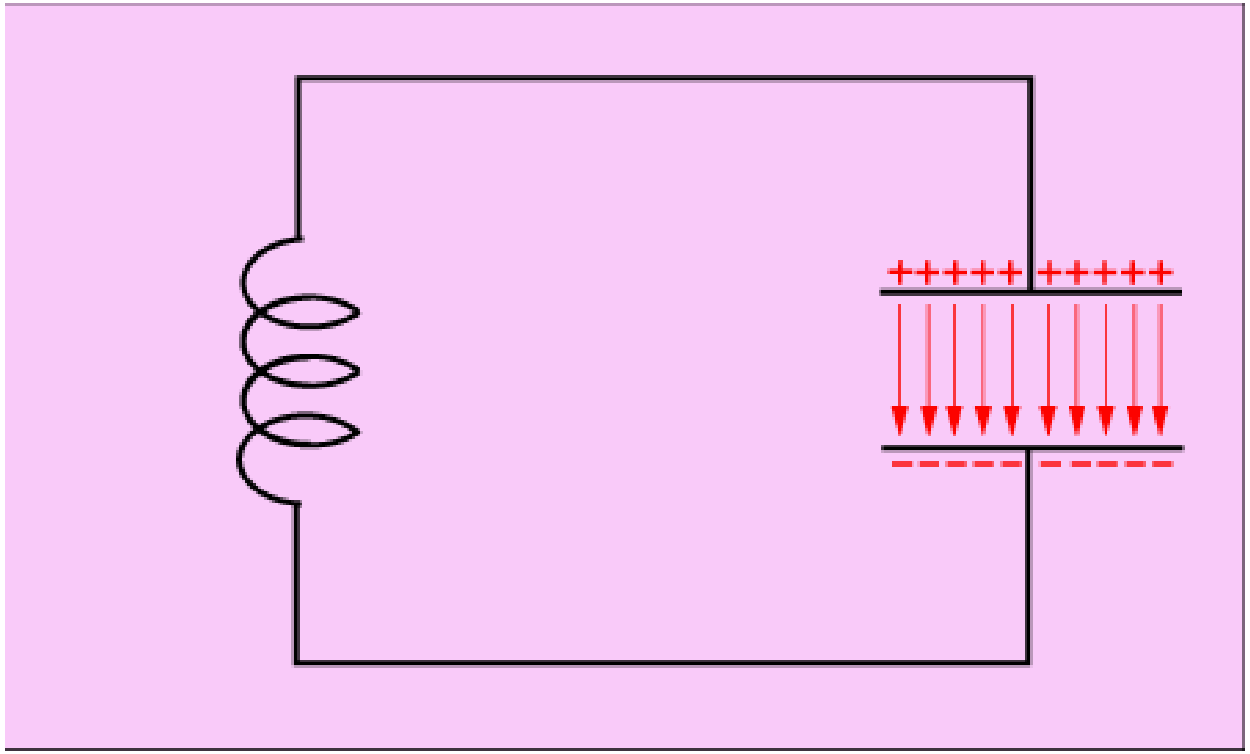
$q = -q_{max}$

Energy is stored in electric field $\vec{E}$ of capacitor

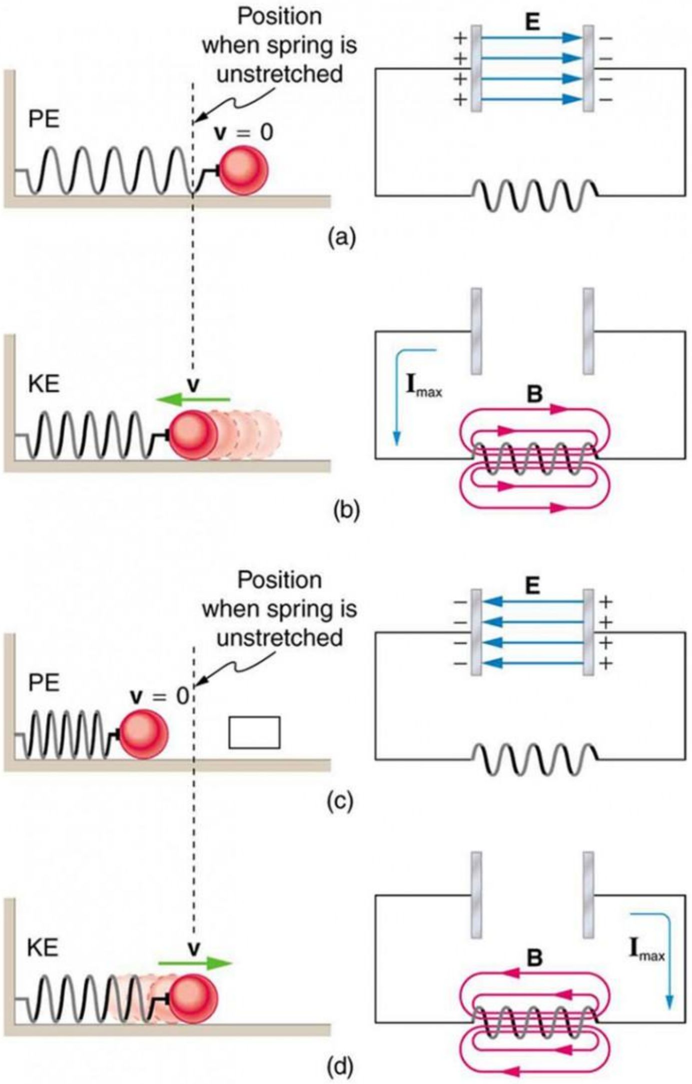
$U_E = 0$ $i = +i_{max}$
$U_{B\ max}$

Energy is stored in magnetic field $\vec{B}$ of inductor

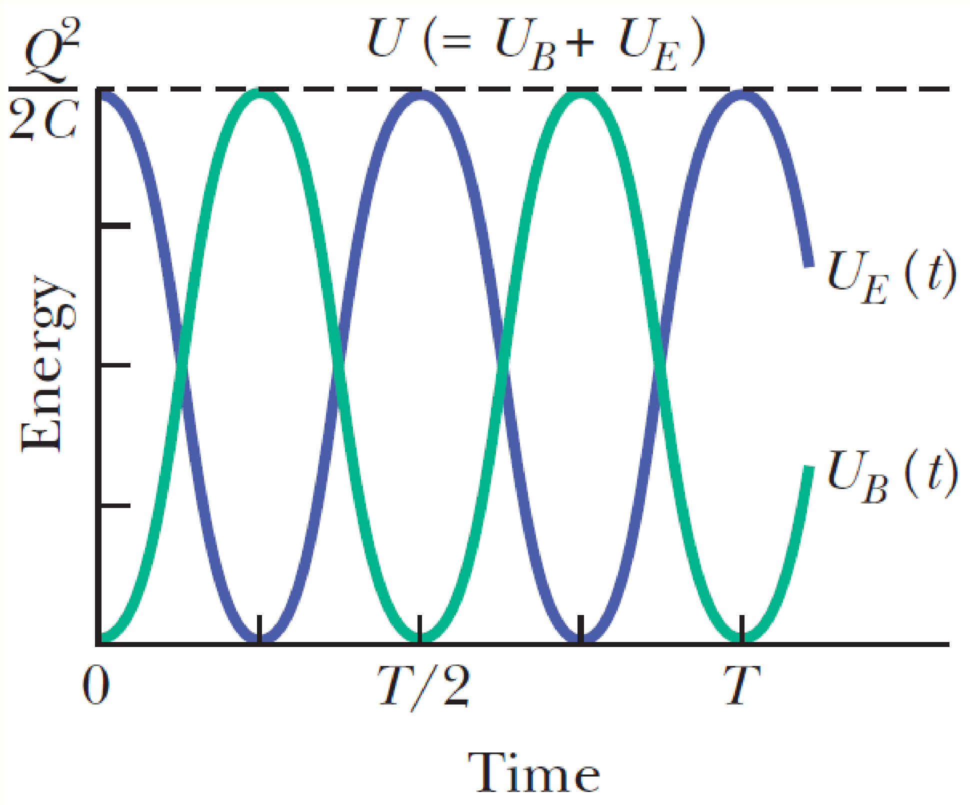
https://youtu.be/2_y_3_3V-so
http://tinyurl.com/ufbdsz3



Same physics as a mass-spring model !!!



LC OSCILLATIONS



Q: Max. charge stored (C)

U_E and U_B vary during time but the total amount of energy is constant

System: **LC** with an energy U distributed between **U_E** & **U_B**
Assuming **no losses**

What will happen ?
→ Discharge of C in L
→ C completely discharged
→ Charge of C by i
→ C completely charged
→ Discharge of C in L

And so on …

Analytical resolution:

$$U = U_B + U_E$$

$$U = \frac{Li^2}{2} + \frac{q^2}{2C}$$

Given that U is constant through time: $$\frac{dU}{dt} = 0$$

$$\frac{d}{dt}\left(\frac{Li^2}{2} + \frac{q^2}{2C}\right) = 0$$

$$\frac{L}{2}\frac{d}{dt}\left(i^2\right) + \frac{1}{2C}\frac{d}{dt}\left(q^2\right) = 0$$

$$\frac{d}{dx}\left(y_{(x)}^2\right) = 2y_{(x)}\frac{d}{dx}\left(y_{(x)}\right)$$

$$Li\frac{di}{dt} + \frac{1}{C}q\frac{dq}{dt} = 0$$

LC OSCILLATIONS

$$Li\frac{di}{dt} + \frac{1}{C}q\frac{dq}{dt} = 0$$

However, $i = \frac{dq}{dt}$, so, $\frac{di}{dt} = \frac{d^2q}{dt^2}$

So we have:

$$L\frac{dq}{dt}\frac{d^2q}{dt^2} + \frac{1}{C}q\frac{dq}{dt} = 0$$

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

The differential equation for the charge in an LC circuit without resistance

Solution: $q_{(t)} = Q\cos(\omega t + \phi)$ with $\omega = \frac{1}{\sqrt{LC}}$

Q: the maximum charge

ω: the natural angular frequency

ϕ: a phase constant

LC OSCILLATIONS

$$q_{(t)} = Q\cos(\omega t + \phi)$$

but, $$U_{E(t)} = \frac{q(t)^2}{2C}$$

and, $$U_{B(t)} = \frac{Li(t)^2}{2} = \frac{L\left(\frac{dq(t)}{dt}\right)^2}{2}$$

$$q_{(t)} = Q\cos(\omega t + \phi)$$

so, $U_{E(t)} = \dfrac{Q^2}{2C}\cos^2(\omega t + \phi)$

and, $U_{B(t)} = \dfrac{Q^2}{2C}\sin^2(\omega t + \phi)$

I = -ωQ is the max. current

from $\mathbf{i} = \dfrac{dq}{dt} = -\omega Q\sin(\omega t + \phi) = -\mathbf{I}\sin(\boldsymbol{\omega t - \phi})$

Periodic solutions that verify that $U = U_{B(t)} + U_{E(t)}$ is constant and equals $\dfrac{Q^2}{2C}$

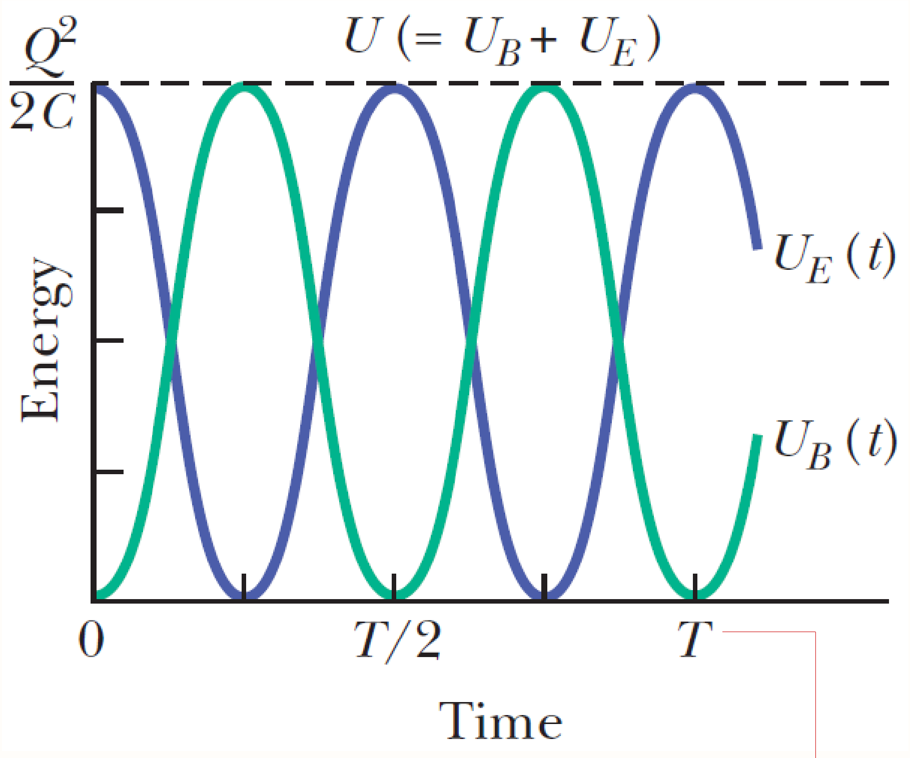
Remarks: When $U_E = Q^2/(2C)$ is max., U_B is zero, and conversely

The potential difference across C, $V_C = q/C$ is also periodic

When $q_{(t)}$ = + or - Q, $i_{(t)}$ is zero

When $I_{(t)}$ = + or – I, $q_{(t)}$ is zero

LC OSCILLATIONS



The period T is $\frac{1}{f} = \frac{1}{\omega/2\pi} = 2\pi\sqrt{LC}$

frequency (Hz or s^{-1})   angular frequency ($rad.s^{-1}$)

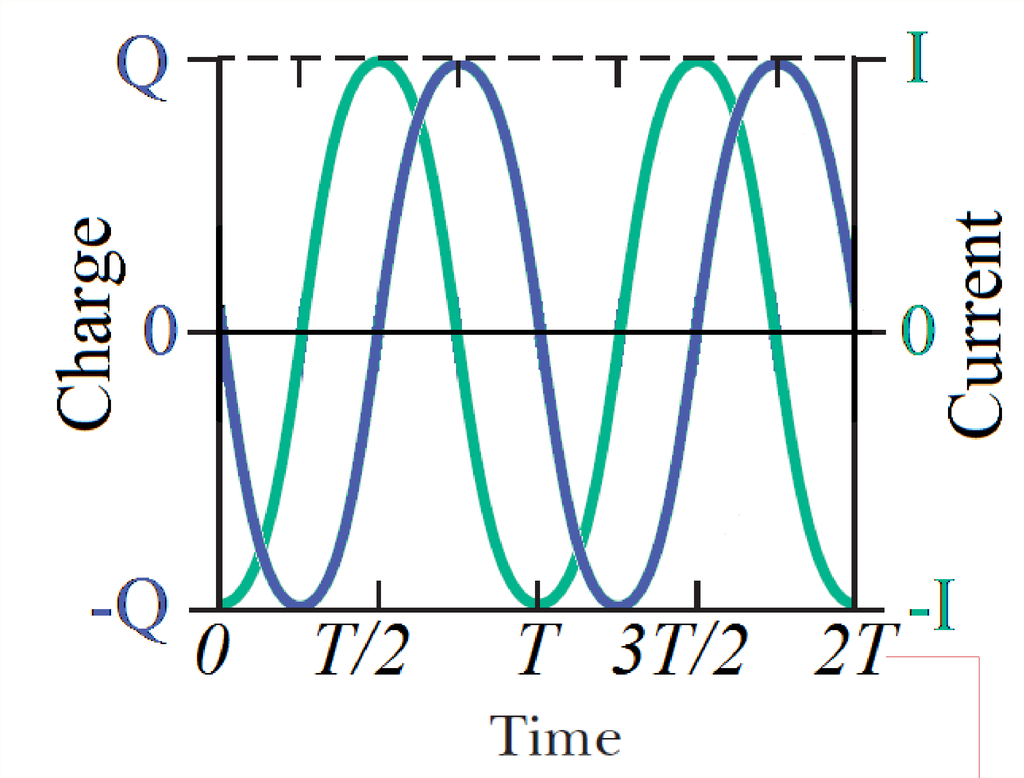
System: **LC** with an energy U distributed between $\mathbf{U_E}$ & $\mathbf{U_B}$
Assuming **no losses**

We demonstrated that U_E & U_B **oscillate periodically**

The **natural angular frequency**

ω equals $\frac{1}{\sqrt{LC}}$

LC OSCILLATIONS



note that T is relative to q and i

System: **LC** with an energy U distributed between $\mathbf{U_E}$ & $\mathbf{U_B}$
Assuming **no losses**

We demonstrated that U_E & U_B **oscillate periodically**

The **natural angular frequency**

ω equals $\dfrac{1}{\sqrt{LC}}$

So far we assumed all components had no resistance
→ no loss of energy by thermal dissipation

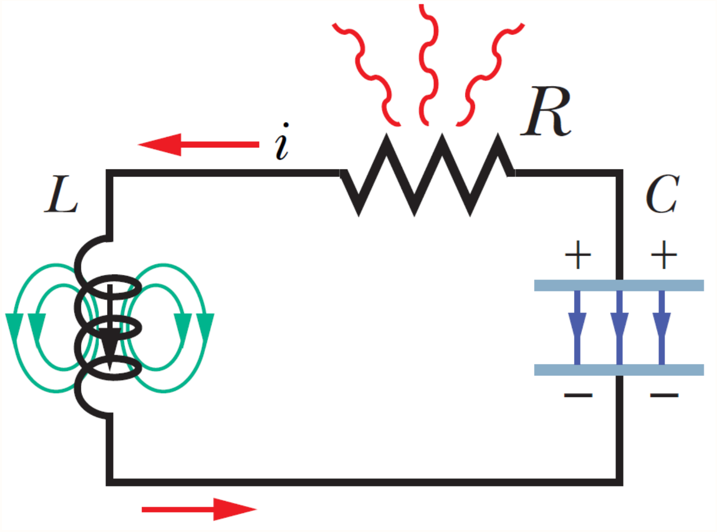
What will happen with losses ?

System: **LC** with an energy U distributed between U_E & U_B
Assuming **no losses**

We demonstrated that U_E & U_B **oscillate periodically**

The **natural angular frequency**

ω equals $\frac{1}{\sqrt{LC}}$



Energy stored in the E field of the capacitor

$$U_E = \frac{q^2}{2C}$$

q: Charge stored (C)
C: Capacitance (F)

Energy stored in the B field of the inductor

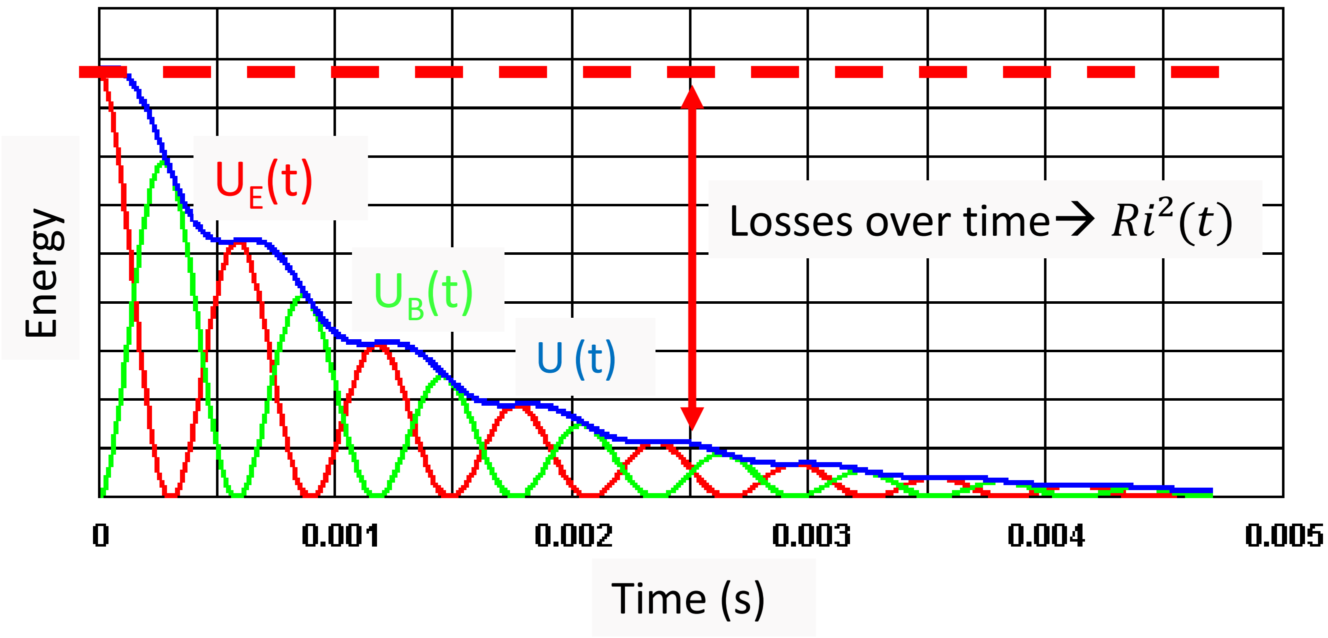
$$U_B = \frac{Li^2}{2}$$

i: Current (A)
L: Inductance (H)

System: R**LC** with an energy U distributed between $\mathbf{U_E}$ & $\mathbf{U_B}$
Loss of energy with R

Rate of energy dissipation
→ $\mathbf{i^2R}$

R: resistance of the resistor (Ω)



Measurement of V_C in an RLC circuit

The system oscillates but the amplitude decreases over time

System: R**LC** with an energy U distributed between **U_E** & **U_B**
Loss of energy with R
What will happen ?
→ Discharge of C in L
→ C completely discharged
→ Charge of C by i
→ C completely charged
→ Discharge of C in L

And so on … **until it stops**

Analytical resolution:

$$U = U_B + U_E$$

This is still true for RLC

$$U = \frac{Li^2}{2} + \frac{q^2}{2C}$$

But now U is no more constant through time:

$$\frac{dU}{dt} = -Ri^2$$

minus sign because we loose energy

Following the same route than for LC, we have:

$$Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = -Ri^2$$

or

$$Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} + Ri^2 = 0$$

Again, we substitute $i = \frac{dq}{dt}$ and $\frac{di}{dt} = \frac{d^2q}{dt^2}$

To obtain: $L\frac{dq}{dt}\frac{d^2q}{dt^2} + \frac{1}{C}q\frac{dq}{dt} + R\left(\frac{dq}{dt}\right)^2 = 0$

or

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

DAMPED OSCILLATIONS IN AN RLC CIRCUIT

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

The differential equation for the **charge decay** in an RLC circuit

That admits as solution:

$$q_{(t)} = Q\, e^{\frac{-Rt}{2L}} \cos(\omega' t + \phi)$$

With:

$$\omega' = \sqrt{\omega^2 - \left(\frac{R}{2L}\right)^2}$$

the angular frequency of the damped oscillations,

$$\omega = \frac{1}{\sqrt{LC}}$$

the natural angular frequency, ϕ a phase constant and Q the max. stored charge

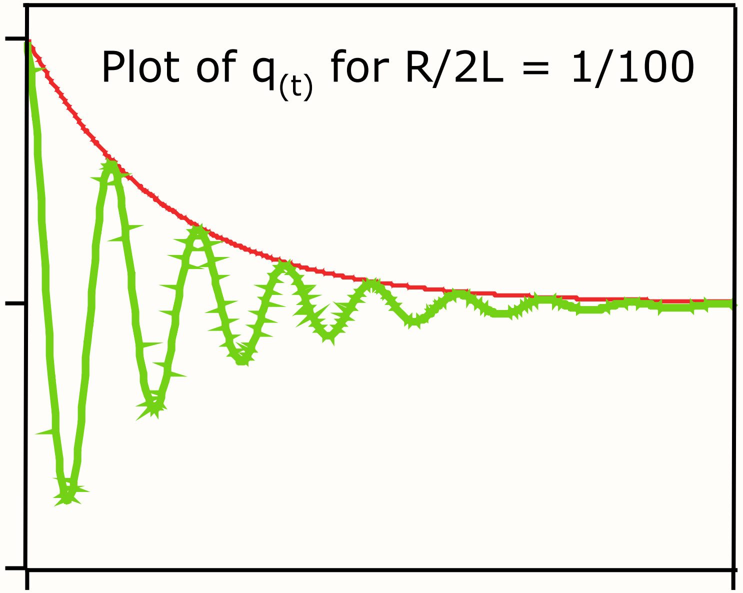
Remarks:

We have: $\omega' = \sqrt{\omega^2 - \left(\frac{R}{2L}\right)^2} < \omega$ but for R small, $\omega' \simeq \omega$

Expression of the charge $q_{(t)} = Q\, e^{\frac{-Rt}{2L}} \cos(\omega' t + \phi)$

$Q\, e^{\frac{-Rt}{2L}}$ → Damped Amplitude

$\cos(\omega' t + \phi)$ → Oscillations



DAMPED OSCILLATIONS IN AN RLC CIRCUIT

$$q_{(t)} = Q\, e^{\frac{-Rt}{2L}} \cos(\omega' t + \phi)$$

But, $U_E = \dfrac{q^2}{2C}$

So, $U_{E(t)} = \dfrac{Q^2}{2C} e^{\frac{-Rt}{L}} \cos^2(\omega' t + \phi)$

- $\dfrac{Q^2}{2C} e^{\frac{-Rt}{L}}$ → Damped Amplitude
- $\cos^2(\omega' t + \phi)$ → Oscillations

System: R**LC** with an energy U distributed between **U_E** & **U_B**
Loss of energy with R

Energy loss can be easily calculated for U_E

→ **Decay of the electric energy**

So far, we only considered circuits that have been given some energy and studied their behavior during time

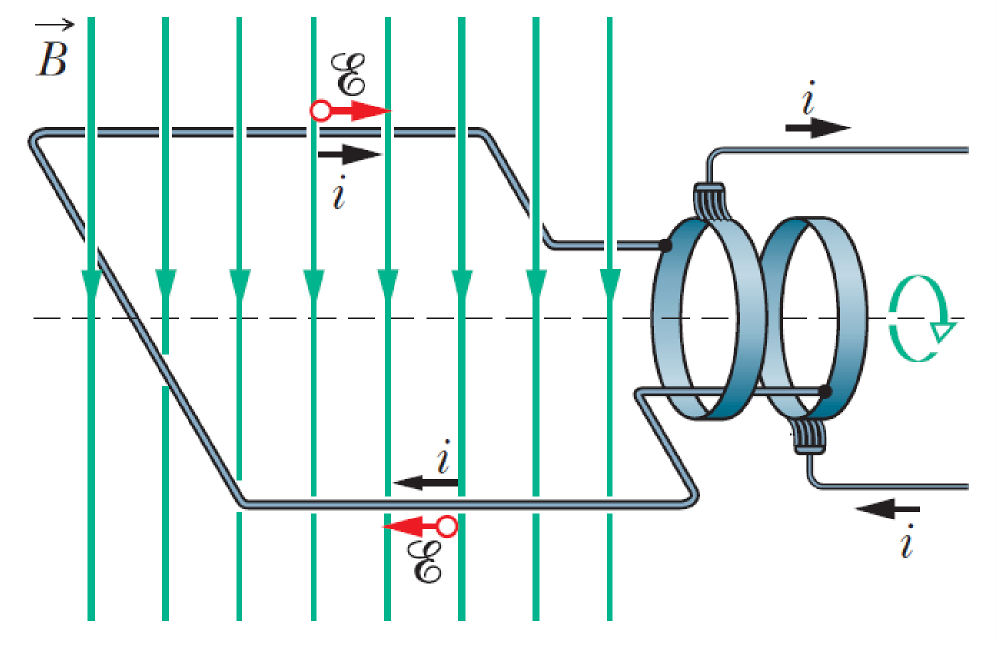
→ Now we will **continuously supply energy** to circuits at a **given frequency**

e.g. supply of an alternative current to an RLC circuit

Note on alternative currents:
ac: Alternative Current
→ **oscillating**

dc: Direct Current
→ **nonoscillating**

By extension, we also use the terms ac and dc for voltages



ω_d

Generates an **alternative voltage** that can drive an **alternative current**

d in ω_d → drive

Example: a conducting loop rotates in an uniform magnetic field with angular frequency ω_d

→ **Electromotive Force (emf)**

$$\xi = \xi_m \sin(\omega_d t)$$

ξ_m : amplitude

→ **Current if the loop is closed**

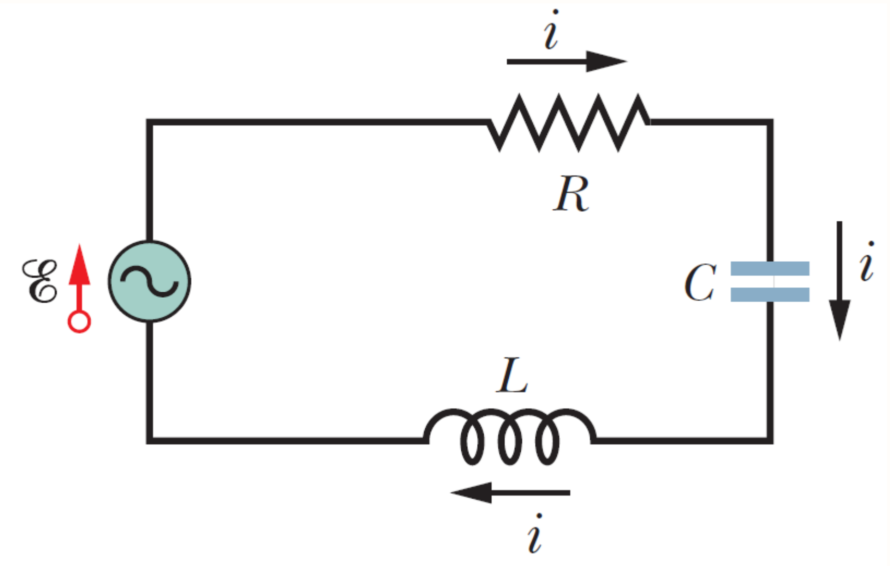
$$i = I \sin(\omega_d t - \phi)$$

I: amplitude, **ϕ**: phase constant

I and ξ are not always in phase

LC and RLC (with R <<) circuits oscillate at their natural angular frequency $\omega = 1/\sqrt{LC}$

What will happen if we continuously drive this circuits with an external supply operating at ω_d ?



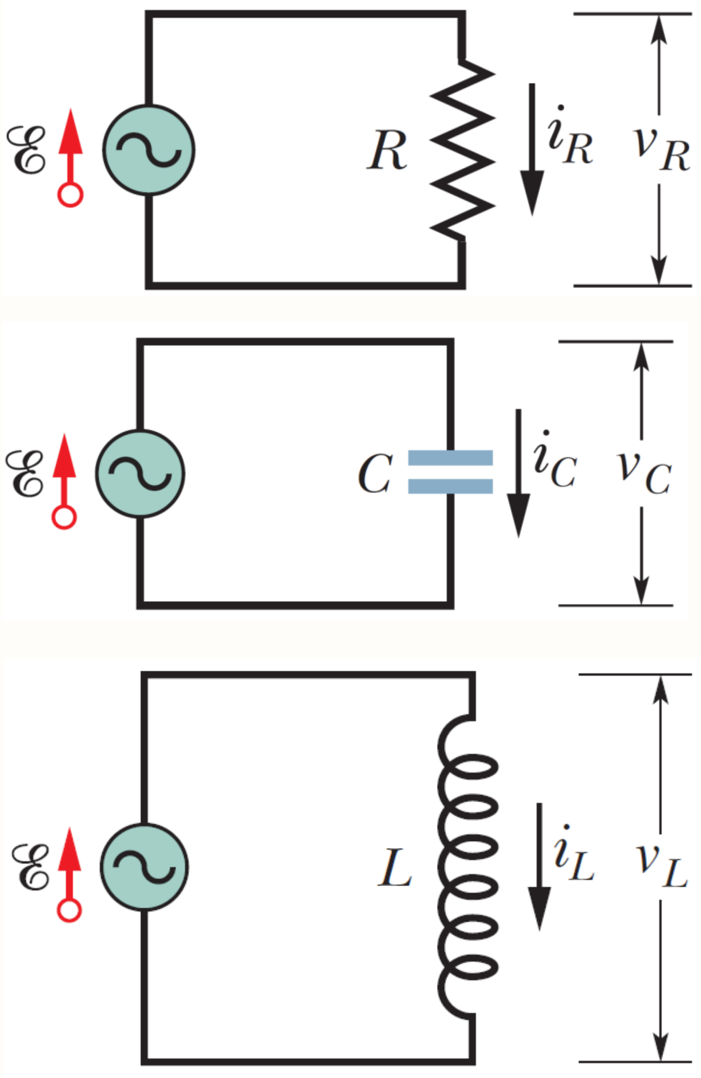
The system **will oscillate at ω_d** even if its natural frequency is ω.

→ **Forced Oscillations**

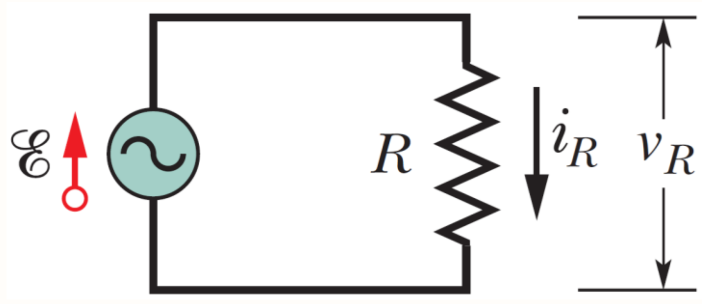
First, simpler circuits than RLC:

- **A Resistive Load**
- **A Capacitive Load**
- **An Inductive Load**

What are the relation between bias and current in these forced circuits ?



Resistive Load



$\xi = \xi_m \sin(\omega_d t)$

Loop rule: $\xi - v_R = 0 \rightarrow v_R = \xi$

$v_R = \xi_m \sin(\omega_d t)$

$v_R = V_R \sin(\omega_d t)$

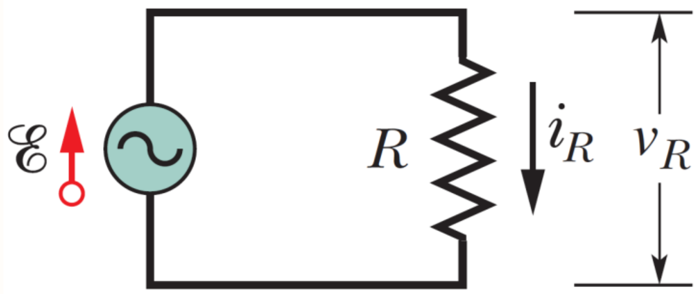
V_R is the amplitude of v_R equals to ξ_m

Ohm's law: $i_R = \frac{v_R}{R} = \frac{V_R}{R} \sin(\omega_d t + 0)$

General expression of I: $i_R = I_R \sin(\omega_d t - \phi)$ I_R is the amplitude of i_R

By identification: $I_R = V_R/R$ and $\phi = 0$ $\rightarrow$ $i_R = I_R \sin(\omega_d t)$ and $V_R = R\, I_R$

Resistive Load



$$v_R = V_R \sin(\omega_d t)$$

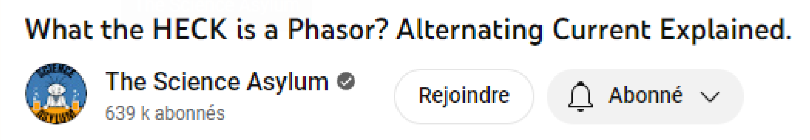
$$i_R = I_R \sin(\omega_d t)$$

v_R and i_R are in phase → $\phi = 0$

→ v_R and i_R *peak at the same time*

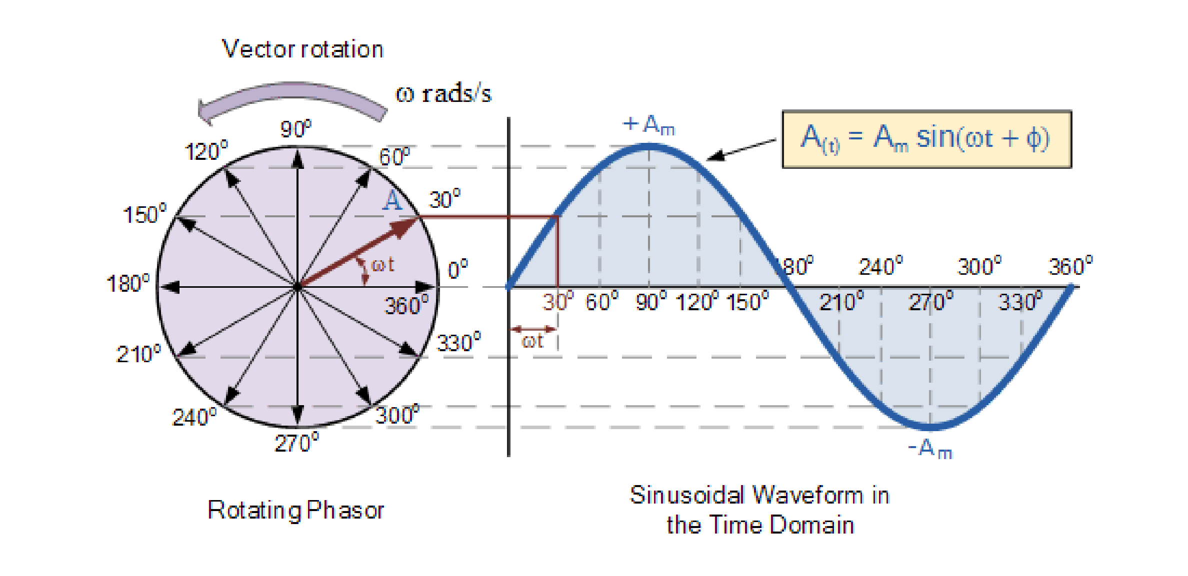
Representation with time traces and phasors :

Reminder



https://youtu.be/7weMCsff0xw

Representation with time traces and phasors :



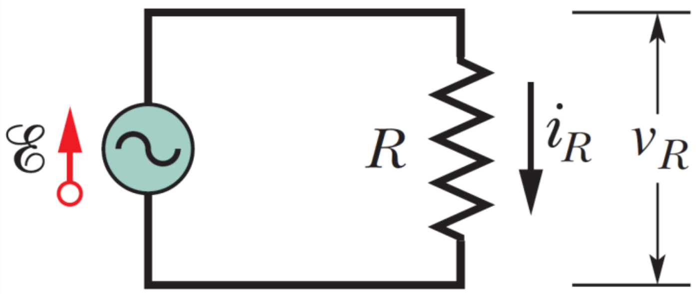
Vertical projection of the vectors gives the value

Length is the **amplitude**

Angle is the **phase** at time t

Angular speed is **angular frequency**

Resistive Load



$$v_R = V_R \sin(\omega_d t)$$

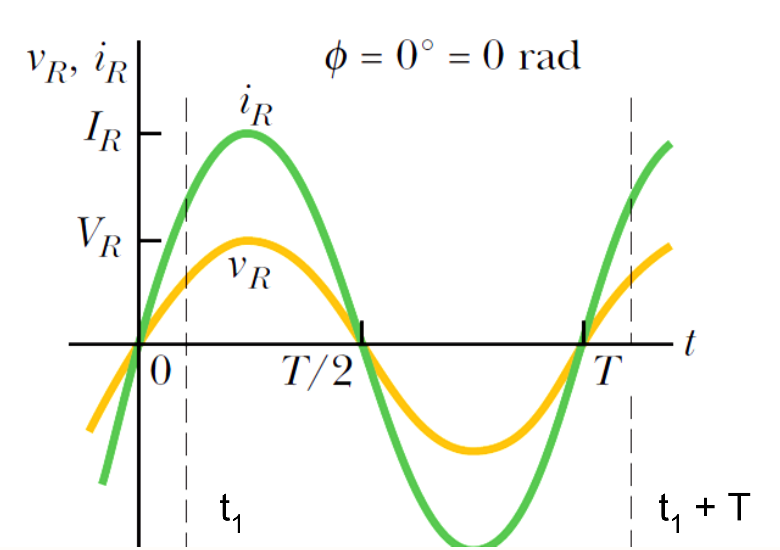
$$i_R = I_R \sin(\omega_d t)$$

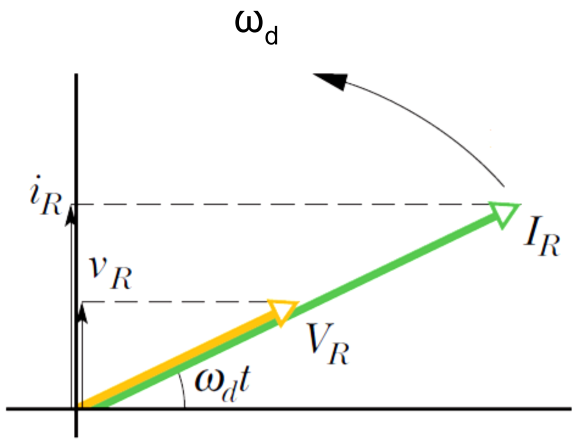
v_R and i_R are in phase → $\phi = 0$

→ v_R and i_R *peak at the same time in the time trace*

→ $\phi = (\vec{V_R}, \vec{I_R}) = 0$ in phasor

Representation with time traces and phasors :





Phasors at t_1

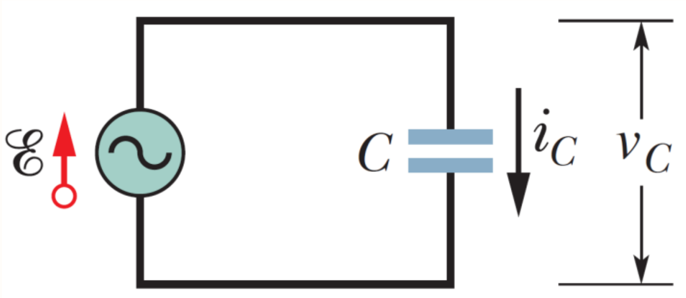
Vertical projection of the vectors gives the value

Length is the amplitude $V_R = R\, I_R$

Angle is the phase ω_d t

Angular speed is ω_d

Capacitive Load



$$\xi = \xi_m \sin(\omega_d t)$$

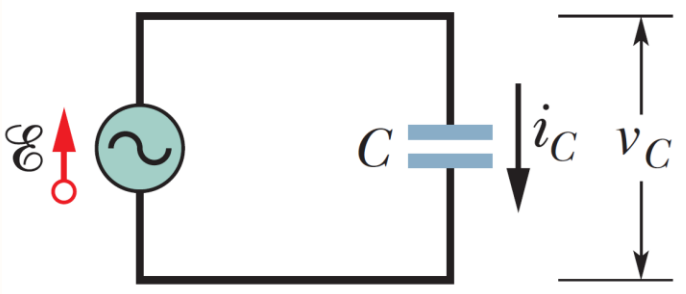
Loop rule: $\xi - v_C = 0 \rightarrow v_C = \xi$

$$v_C = \xi_m \sin(\omega_d t)$$

$$v_C = V_C \sin(\omega_d t)$$

V_C is the amplitude of v_C equals to ξ_m

Capacitive Load



We have: $q = C\, v_C$ and $i = \frac{dq}{dt}$ so $i_C = C \frac{dv_C}{dt}$

$v_C = V_C \sin(\omega_d t)$

$= CV_C \frac{d}{dt}(\sin(\omega_d t))$

$= CV_C \omega_d \cos(\omega_d t)$

$= CV_C \boldsymbol{\omega_d} \sin(\omega_d t + 90°)$

$i_C = \frac{V_C}{X_C} \sin(\omega_d t + 90°)$

We define the **capacitive reactance X_C (Ω)**

$$X_C = \frac{1}{\omega_d C}$$

By identification with : $i_C = I_C \sin(\omega_d t - \phi)$

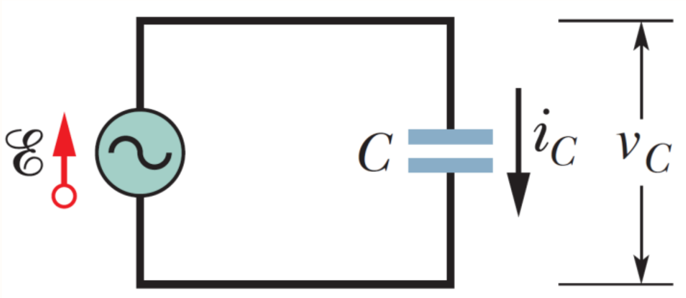
I_C is the amplitude of i_C

$I_C = V_C / X_C$ and $\phi = -90°$ → $i_C = I_C \sin(\omega_d t + 90°)$

and $V_C = X_C I_C$

Note: the second relation resemble to $V_R = R\, I_R$, but here X_C depends of ω_d

Capacitive Load



$$v_C = V_C \sin(\omega_d t)$$

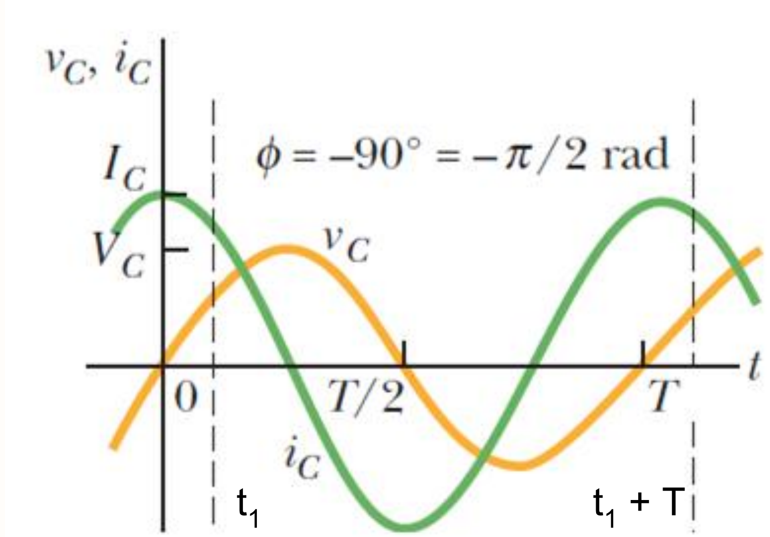
$$i_C = I_C \sin(\omega_d t + 90°)$$

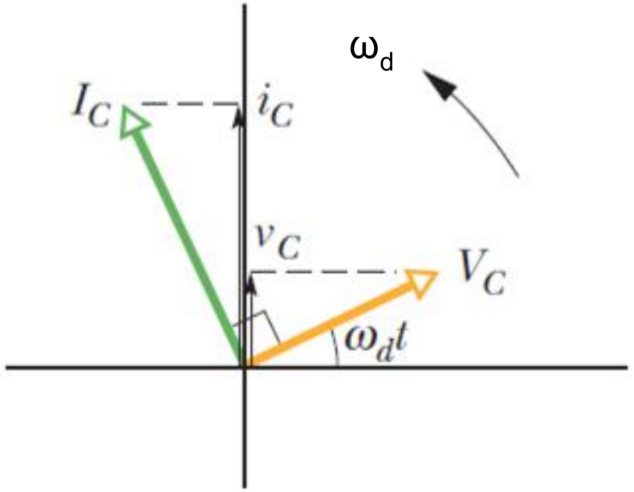
v_c and i_c are one quarter of cycle out of phase → ϕ = -90°

→ i_c leads v_c by 90° *(it peaks first in time trace)*

→ $\phi = (\vec{V_C}, \vec{I_C}) = -90° = -\frac{\pi}{2}$ rad in phasor

Representation with time traces and phasors :



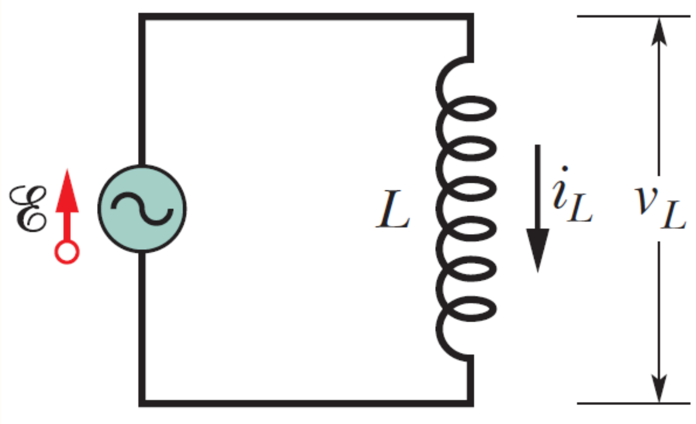


Phasors at t_1

$$V_C = X_c I_C$$

Note: the relation resemble to $V_R = R\,IR$, but here X_C depends of ω_d

Inductive Load



$\xi = \xi_m \sin(\omega_d t)$

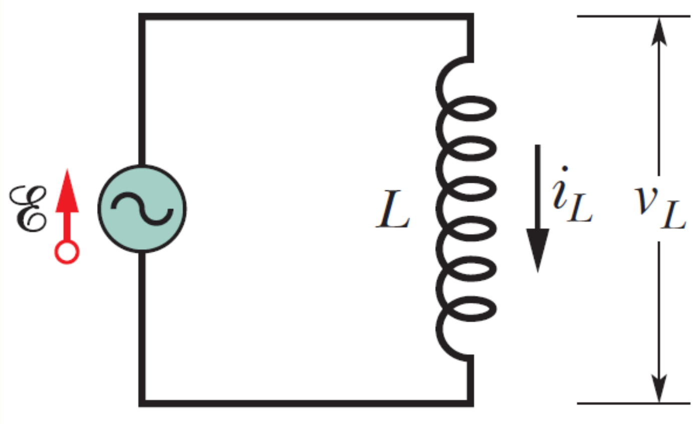
Loop rule: $\xi - v_L = 0 \rightarrow v_L = \xi$

$v_L = \xi_m \sin(\omega_d t)$

$v_L = V_L \sin(\omega_d t)$

V_L is the amplitude of v_L equals to ξ_m

Inductive Load



We have: $v_L = L\frac{di_L}{dt}$ so $\frac{di_L}{dt} = \frac{v_L}{L} = \frac{V_L}{L}sin(\omega_d\, t)$

Integrating the expression: $i_L = \int \frac{V_L}{L}sin(\omega_d\, t)dt$

$$= -\frac{V_L}{\omega_d\, L}\cos(\omega_d\, t)$$

$$= \frac{V_L}{\omega_d\, L}\sin(\omega_d\, t - 90°)$$

We define the
inductive reactance X_L (Ω)
$X_L = \omega_d L$

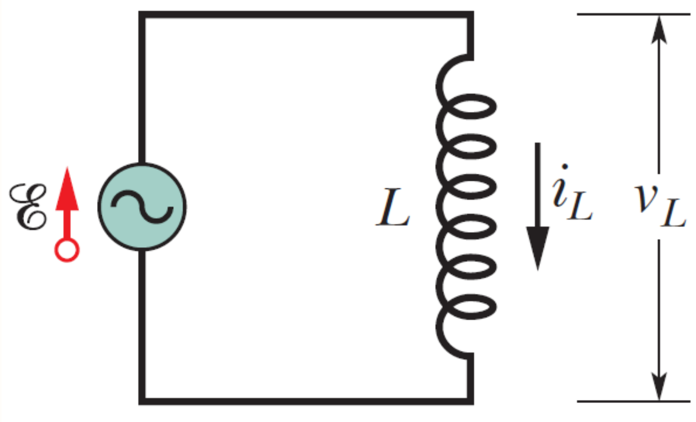
$$i_L = \frac{V_L}{X_L}\sin(\omega_d\, t - 90°)$$

By identification with: $i_L = I_L \sin(\omega_d\, t - \phi)$

I_L is the amplitude of i_L

$I_L = V_L/X_L$ and $\phi = +90°$ → $i_L = I_L \sin(\omega_d\, t - 90°)$ and $V_L = X_L\, I_L$

Inductive Load



$$v_L = V_L \sin(\omega_d\, t)$$

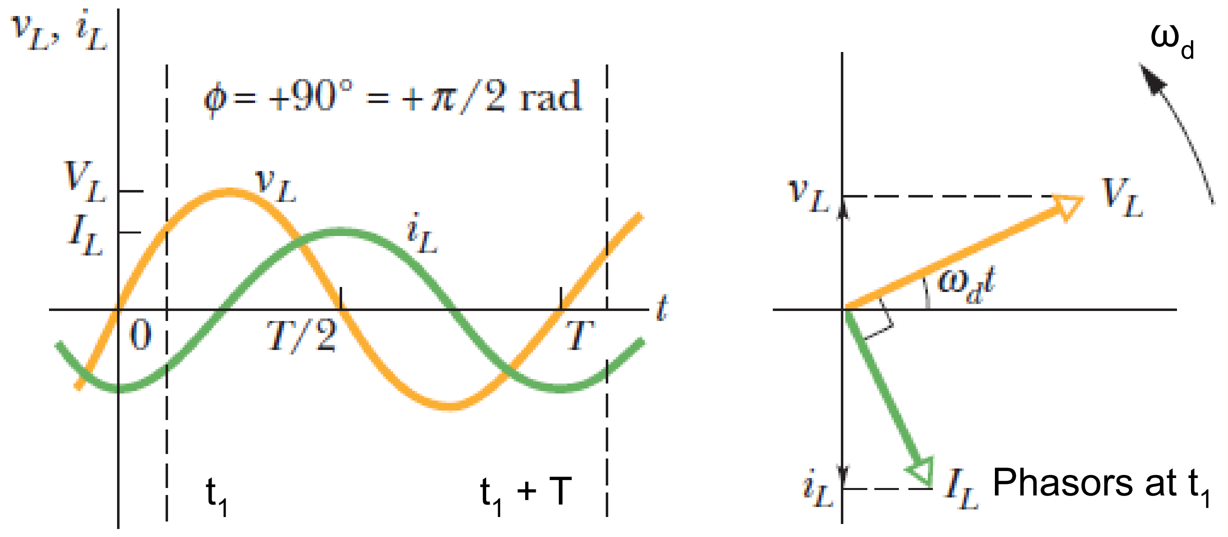
$$i_L = I_L \sin(\omega_d\, t - 90°)$$

v_L and i_L are one quarter of cycle out of phase → $\phi = +90°$

→ i_L lags v_L by 90° (*it peaks after*)

→ $\phi = (\vec{V_L}, \vec{I_L}) = 90° = \frac{\pi}{2} rad$ in phasor

Representation with time traces and phasors :



$$V_L = X_L\, I_L$$

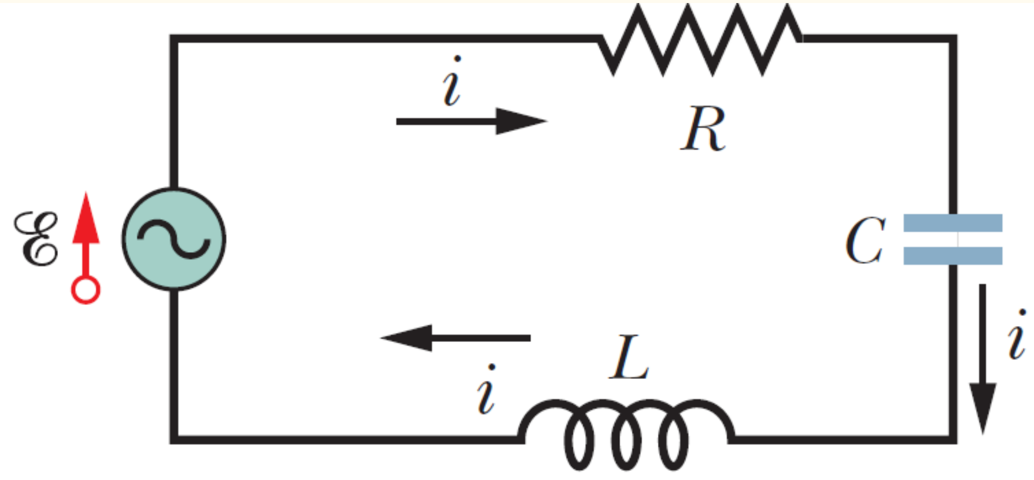
RLC (with R <<) circuit forced at ω_d by an external supply of energy

$$\xi = \xi_m \sin(\omega_d t)$$

Components in series

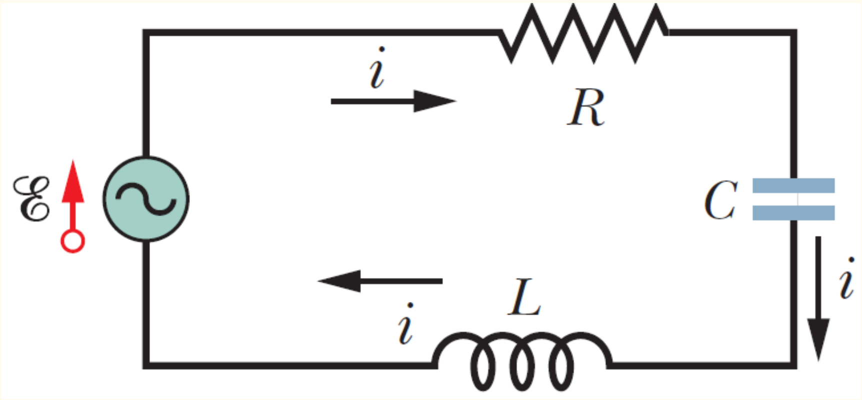
$$I_R = IC = i_L = I = I \sin(\omega_d t - \phi)$$

Dependence of I and ϕ with ω_d ?



The system **will oscillate at ω_d** even if its natural frequency is ω.

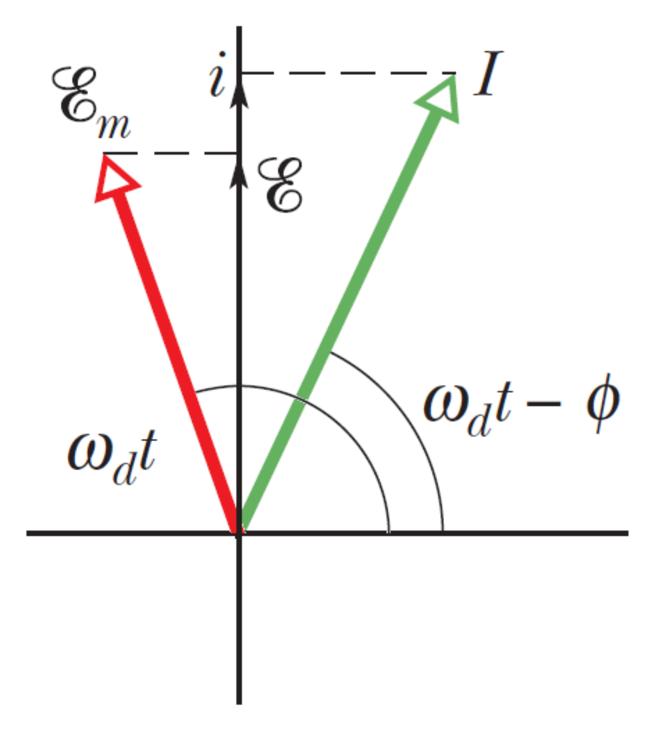
→ **Forced Oscillations**

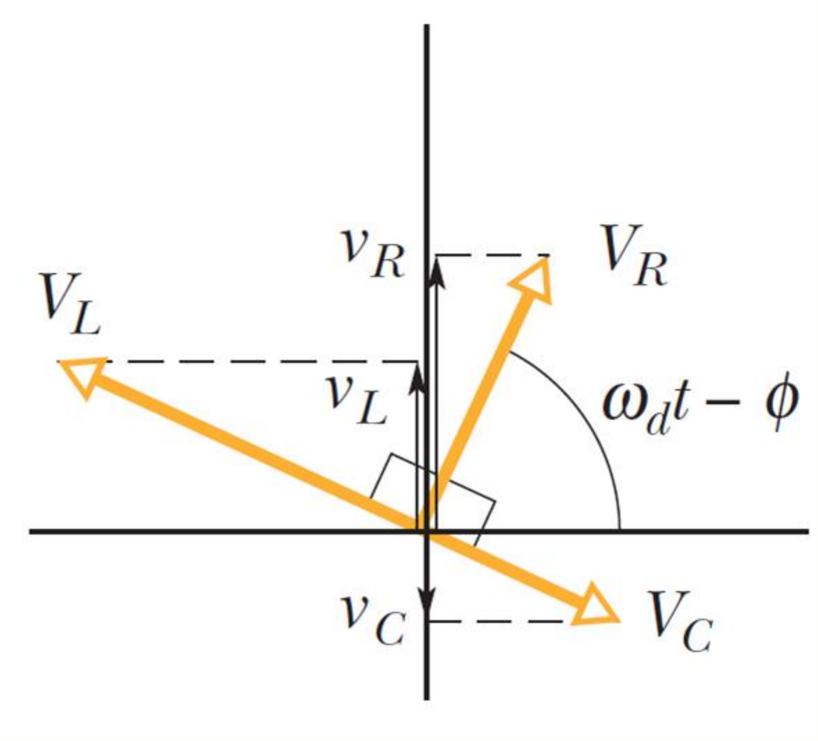


Loop rule: $\xi = v_R + v_C + v_L$

General expressions of ξ and i :

$\xi = \xi_m \sin(\omega_d t)$ $i = I_m \sin(\omega_d t - \phi)$





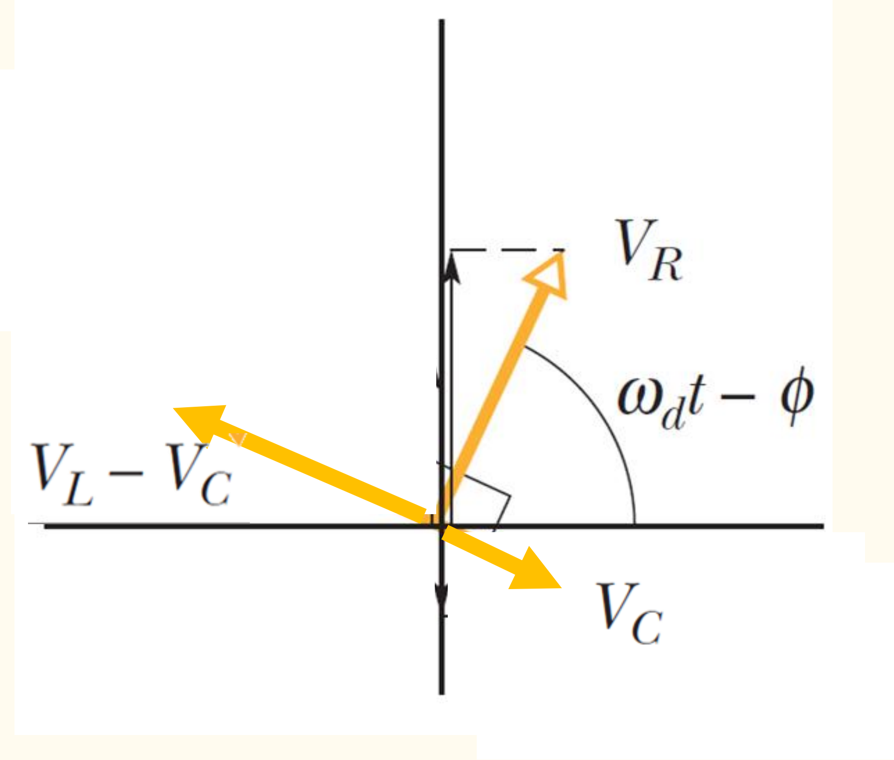
From the previous part, we know that:

- i is in phase with v_R
- i leads v_C by 90°
- i lags v_L by 90°

→ Representation with phasors

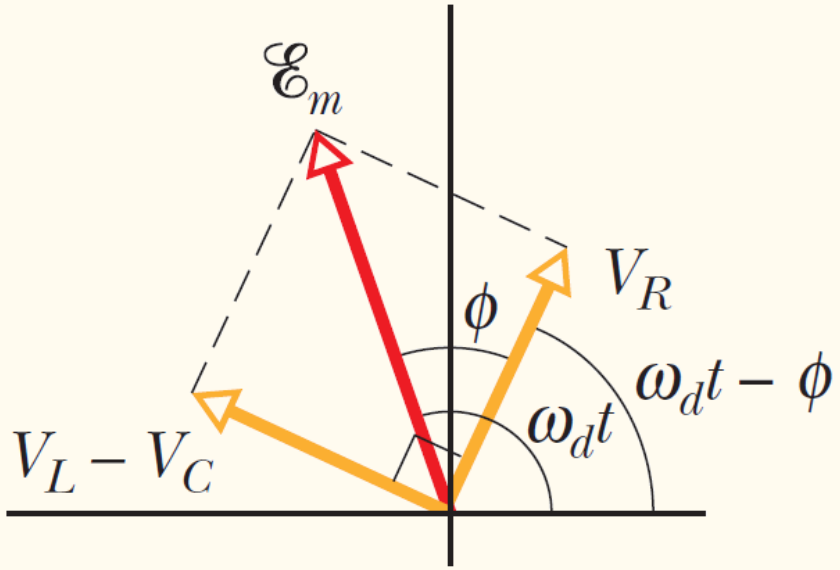
Geometrical resolution:

We redraw the phasors as follows and sum the phasors of v_L & v_C to simplify:



Geometrical resolution:

We redraw the phasors as follows and sum the phasors of v_L & v_C to simplify:



To satisfy the loop rule, the sum of the vertical projections of the phasors v_R, v_L and v_C must **always equals** the projection of the phasor of ξ

→ To verify this, the **vector sum** of the phasors must equals the phasor ξ

Applying the Pythagorean theorem, we have:

$$\xi_m{}^2 = V_R{}^2 + (V_L - V_C)^2$$

$$\xi_m{}^2 = V_R{}^2 + (V_L - V_C)^2$$ can be written as $$\xi_m{}^2 = (IR)^2 + (IX_L - IX_C)^2$$

(We still have $V_R = IR$, $V_C = IX_C$ and $V_L = IX_L$)

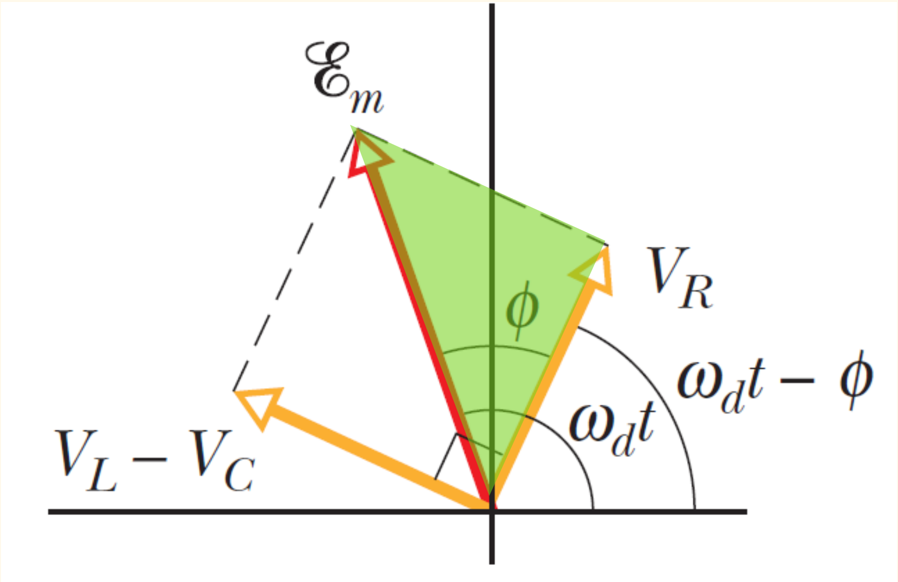
That leads to; $$I = \frac{\xi_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\xi_m}{Z}$$

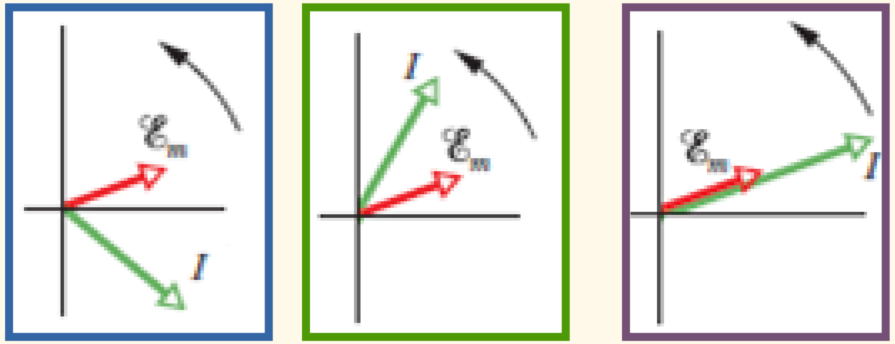
Z is the **impedance** of the circuit

Detailing all the terms, we have: $$I = \frac{\xi_m}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}}$$

Note: this expression holds for steady-state current after some time already passed

Now, we only need an expression for the phase constant





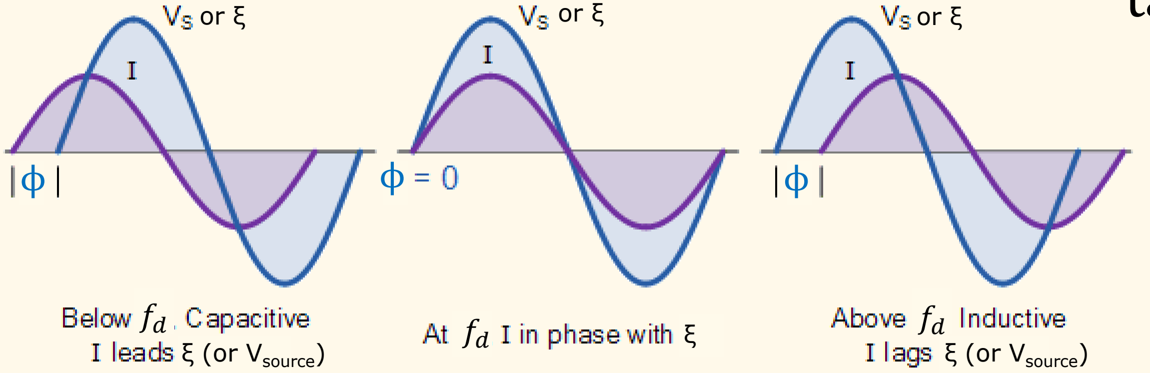
$$\tan(\phi) = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}$$

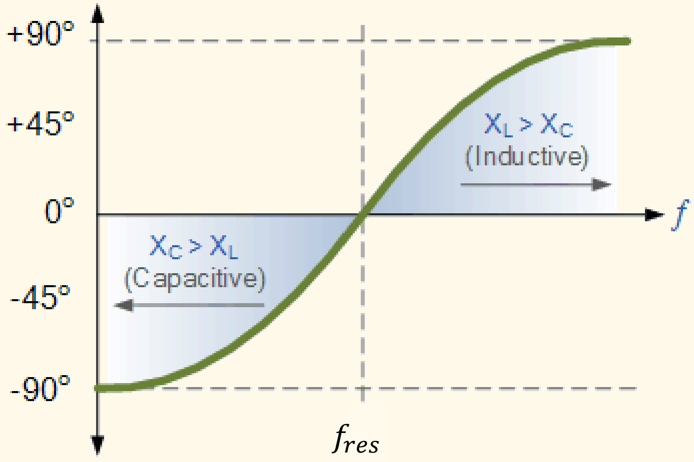
- If $X_L - X_C > 0$, then, $\tan(\phi) > 0$, so, $0<\phi<90°$
System more inductive than capacitive at ω_d
→ i rotates behind of ξ

- If $X_L - X_C < 0$, then, $\tan(\phi) < 0$, so, $-90<\phi<0°$
System more capacitive than inductive at ω_d
→ i rotates ahead of ξ

- If $X_L - X_C = 0$, then, $\tan(\phi) = 0$, so, $\phi = 0°$
System in resonance at ω_d
→ Phasors rotate together

Now, we only need an expression for the phase constant





$$\tan(\phi) = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}$$

- If $X_L - X_C > 0$, then, $\tan(\phi) > 0$, so, $0<\phi<90°$
System more inductive than capacitive at ω_d
→ i rotates behind of ξ

- If $X_L - X_C < 0$, then, $\tan(\phi) < 0$, so, $-90<\phi<0°$
System more capacitive than inductive at ω_d
→ i rotates ahead of ξ

- If $X_L - X_C = 0$, then, $\tan(\phi) = 0$, so, $\phi = 0°$
System in resonance at ω_d
→ Phasors rotate together

At resonance: $X_L - X_C = 0$

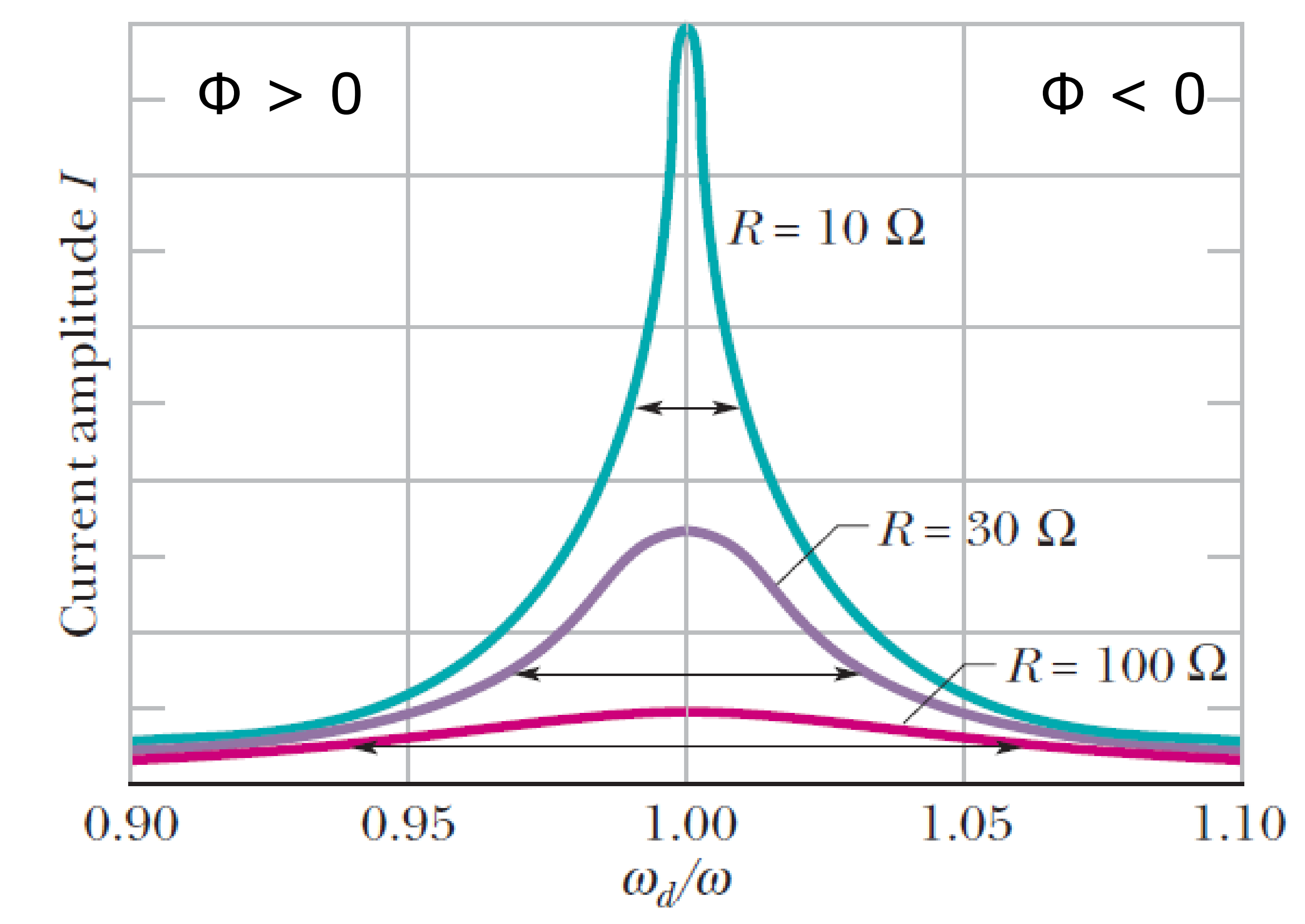
$$L\omega_d = \frac{1}{C\omega_d}$$

$$\omega_d^2 = \frac{1}{LC}$$

$$\omega_d = \frac{1}{\sqrt{LC}}$$

$$\boxed{\omega_d = \omega}$$

Resonance occurs when the driving frequency equals the natural frequency



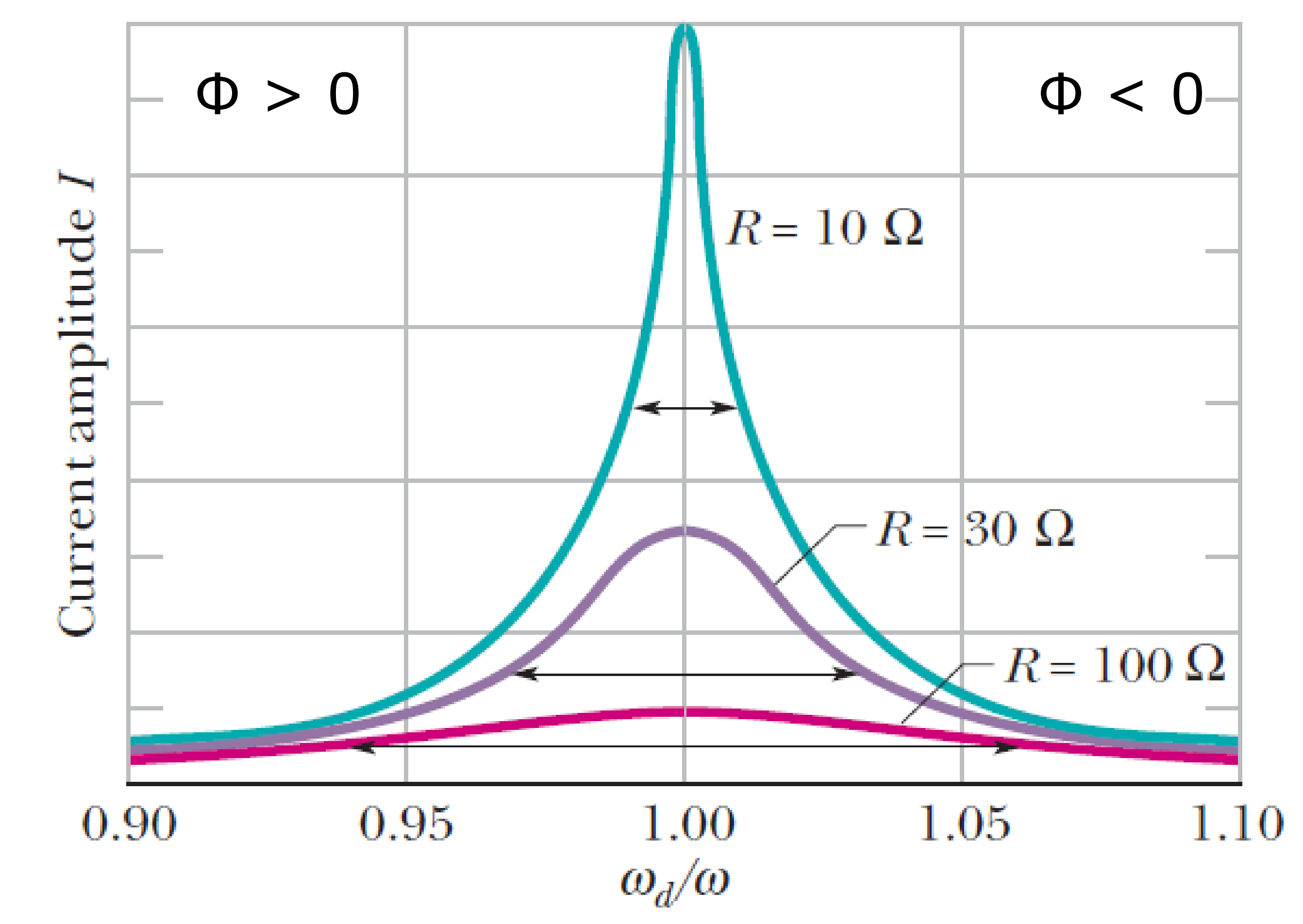
L = 100 μH, C = 100 pF

THE SERIES RLC CIRCUIT

$$I = \frac{\xi_m}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}}$$

The lower is R, the sharpest is the resonance

$$\omega = \frac{1}{\sqrt{LC}}$$



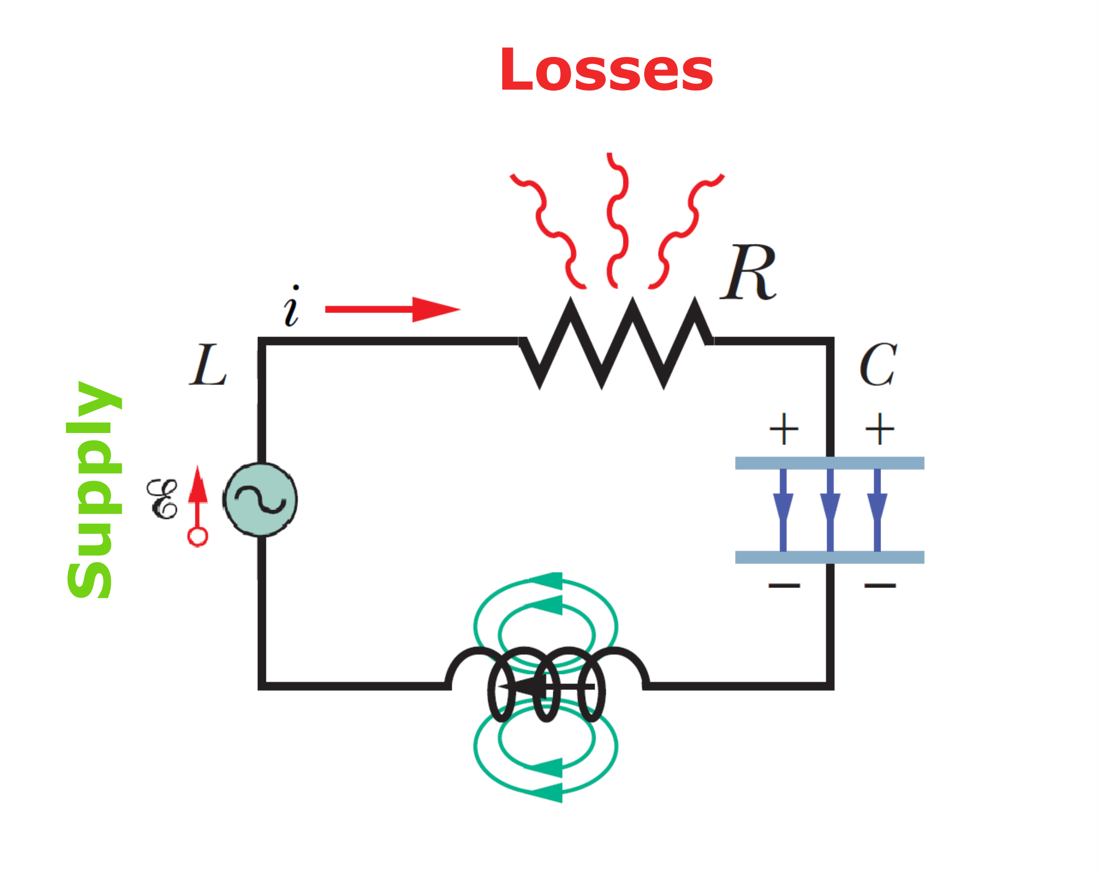
L = 100 μH, C = 100 pF

POWER IN ALTERNATIVE CURRENT CIRCUITS

We consider an RLC circuit forced by an external emf at ω_d

Some energy is stored in B, some in E, some is dissipated by R, some is provided by emf

In the steady-state the average amount of energy in the system is constant



POWER IN ALTERNATIVE CURRENT CIRCUITS

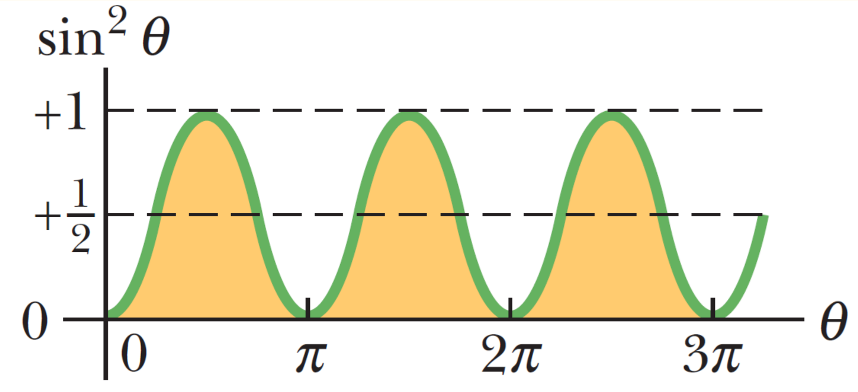
Instantaneous dissipated power in the resistor

$$P = i^2 R = I^2 R \sin^2(\omega_d t - \phi)$$

Average dissipated power

$$P_{avg} = \left(\frac{I}{\sqrt{2}}\right)^2 R = I_{rms}^2 R$$

I_{rms} is the **root-mean-square** value of I



Note: the average value of $\sin^2(\theta)$ is 1/2

For a **pure ac current**

$$I_{rms} = \frac{I}{\sqrt{2}}$$

For **pure ac emf and voltage**, we also have

$$\xi_{rms} = \frac{\xi_m}{\sqrt{2}} \qquad V_{rms} = \frac{V}{\sqrt{2}}$$

Note: a pure ac signal is a sinus (or cosinus) without offset (dc component)

Therefore, we can write: $I_{rms} = \dfrac{\xi_{rms}}{Z}$

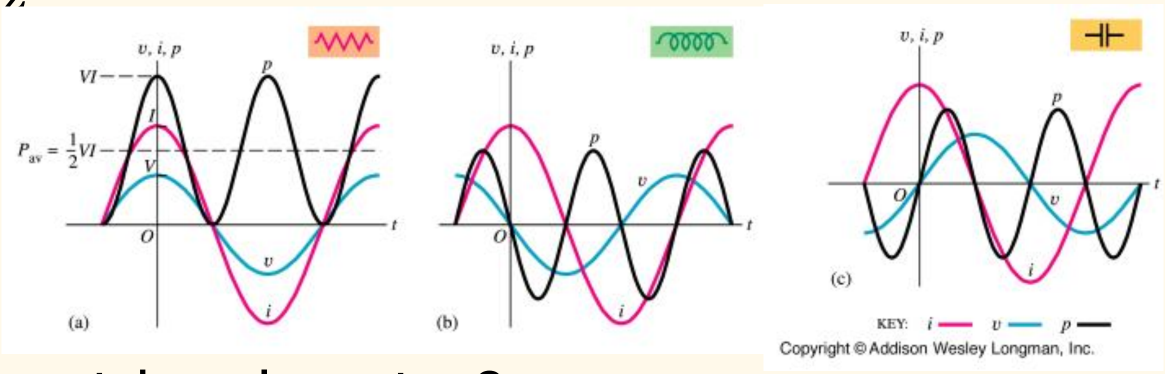
$$P_{avg} = \left(\frac{I}{\sqrt{2}}\right)^2 R = I_{rms}^2 R$$

And P_{avg} becomes, when replacing one I_{rms} by the last formula:

$P_{avg} = \left(\dfrac{\xi_{rms}}{Z}\right) I_{rms} R = \left(\dfrac{R}{Z}\right) I_{rms}\xi_{rms}$ with $\dfrac{R}{Z} = \cos\phi$ (demonstrated using $\tan\phi$ in Z)

$$P_{avg} = \cos\phi \, I_{rms}\xi_{rms}$$

$\cos\phi$ → Power factor



To maximize the energy transfer rate, ϕ must be close to 0

→ **At the resonance energy transfer is maximized**

Note on the rms values:

When we measure a voltage or a current with a multimeter there is **2 modes** e.g. **= V** and **~ V** for voltages

The **~ V** mode indicates the **rms value of the ac component** of the signal
→ Correspond to the the rms value for pure ac signals

$$V_{rms} = \frac{V}{\sqrt{2}}$$

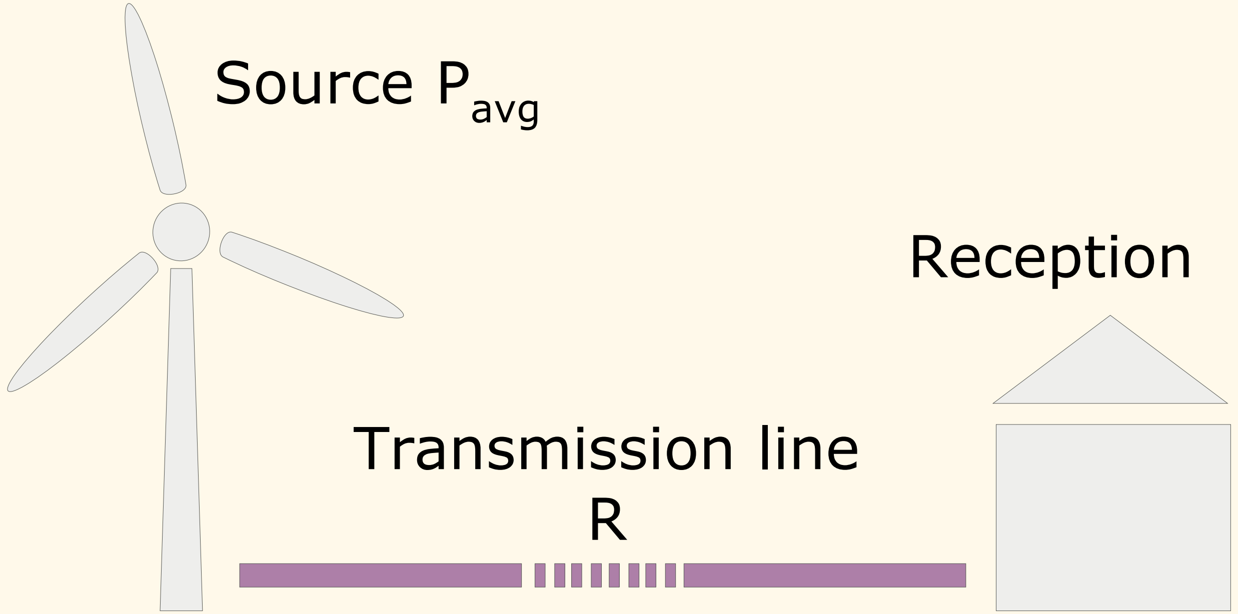
Amplitude / √2

The **= V** mode is the **average voltage**
→ 0 for a pure ac signal

Usually, we directly report rms values for alternating currents / voltages

Images: rs-online.com

Not these transformers



Note: in the following slides we do not write rms and assume that all the values are rms of pure ac signals

→ This is what is measured

Considering the situation where we need to **transmit energy in a power line**

→ R of the line ≠ 0

$$P_{avg} = \xi_{rms} I_{rms} = V_{rms} I_{rms}$$

Power losses: $R\, Irms^2$

→ Must be reduced

→ To transmit energy, for a given P_{avg}

$$V_{rms} >> \text{ and } I_{rms} <<$$

V >> and I <<

Problem: High bias are dangerous in domestic installations → Mismatch

Need to have low voltage at the end of the transmission

High V transmission
Low V consumption
Keep P=VI constant

Need to transform I and V ?
Transformer

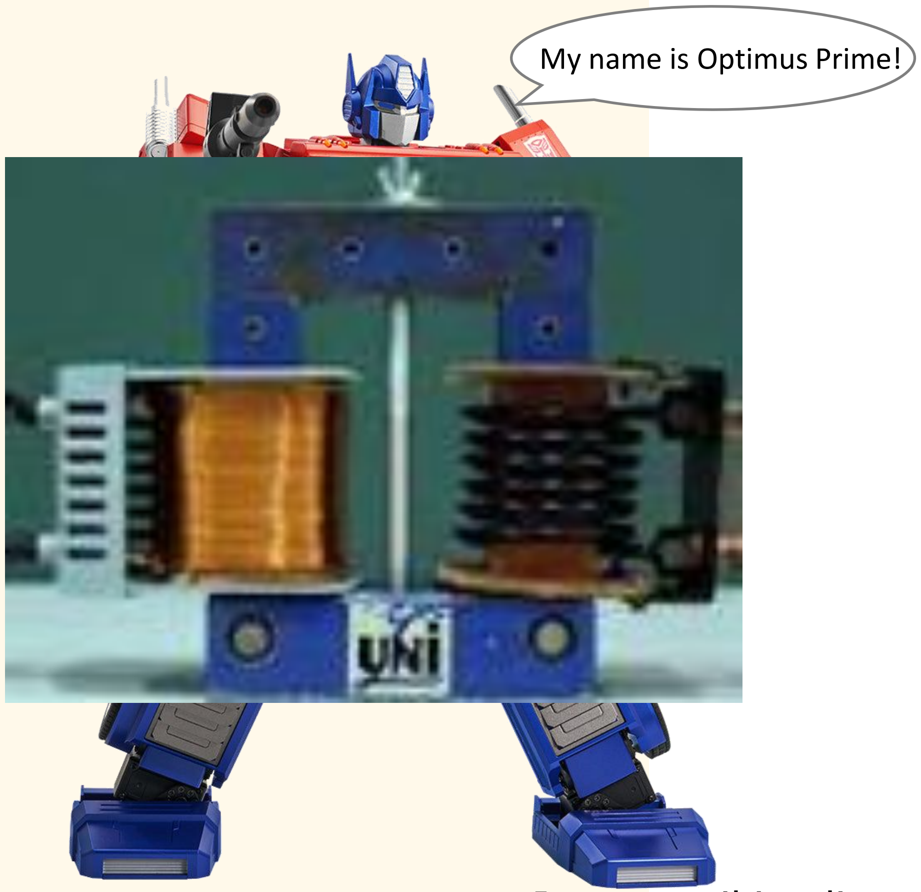


Image: wikipedia.org

The ideal transformer

Primary coil $\mathbf{N_p}$ turns

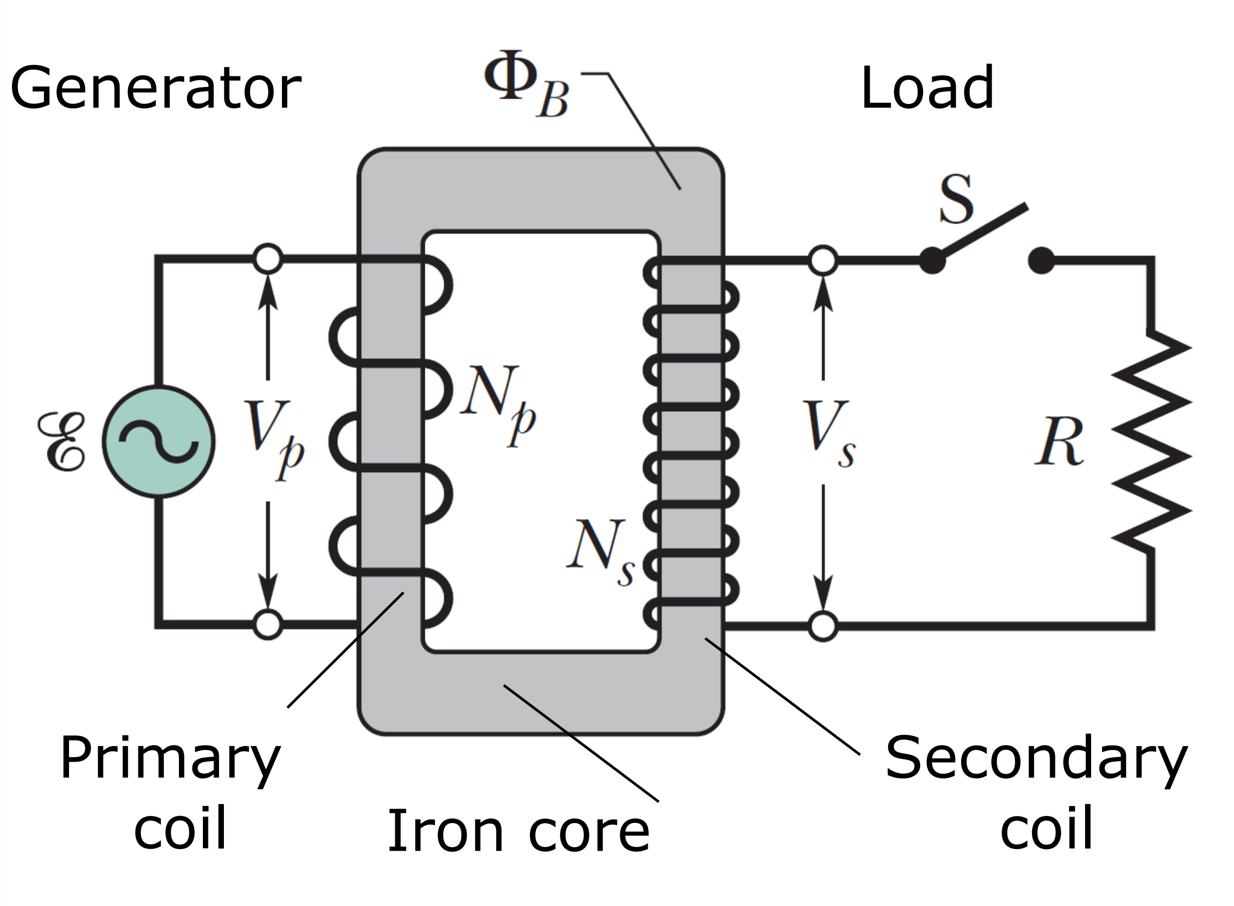
Secondary coil $\mathbf{N_s}$ turns

Iron magnetic core

Switch S open or closed

Resistive load R

Note: The load is simply resistive here for simplification.



How does it works ?

The transformer receive an emf

$$\xi = \xi_m \sin(\omega_d t)$$

The small current in the primary coil is called **magnetizing current i_{mag}**

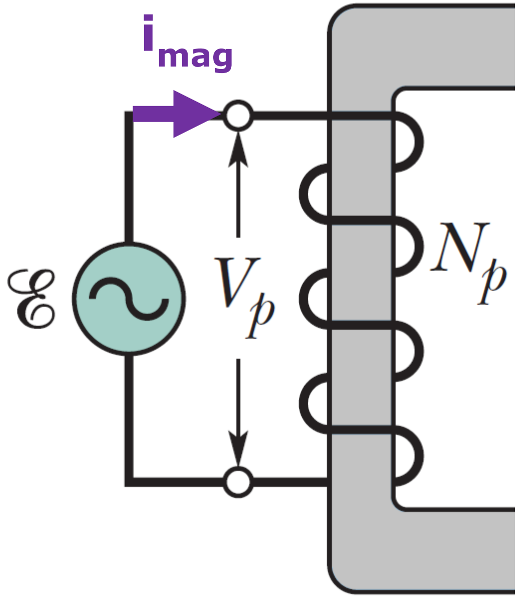
$$i_{mag} = I_{mag} \sin(\omega_d t - 90°)$$

current for pure inductive load

$$\phi = 90°$$

$$\cos(\phi) = 0$$

The power factor is 0
→ no transmitted power



TRANSFORMERS

The transformer receive an emf

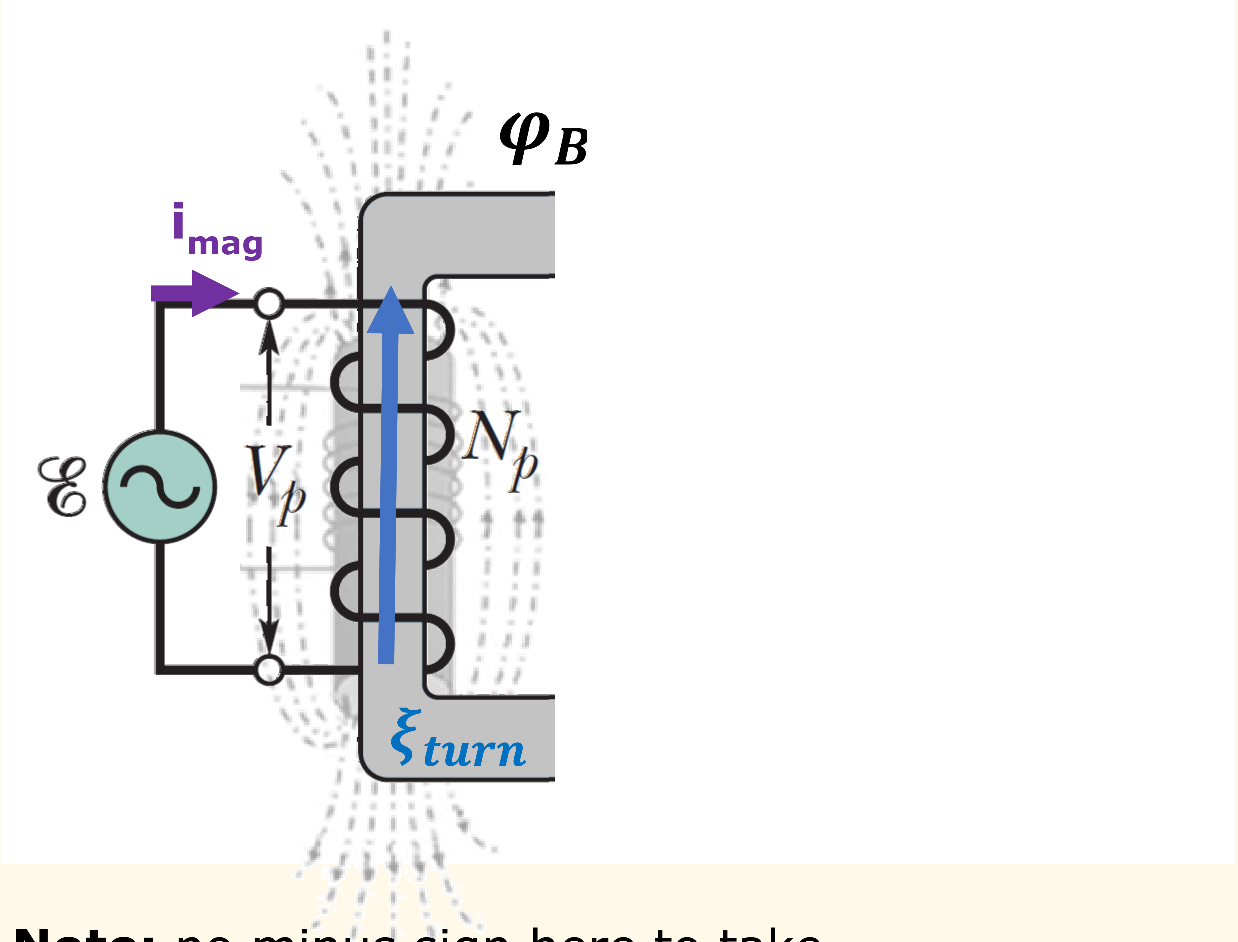
$$\xi = \xi_m \sin(\omega_d t)$$

The small current in the secondary coil is called **magnetizing current i_{mag}**

I_{mag} produces a sinusoidally varying magnetic field
→ **sinusoidally varying magnetic flux ϕ_B in the iron core**

In each turn of the coils **ϕ_B produces and emf**

$$\xi_{turn} = \frac{d\varphi_B}{dt}$$



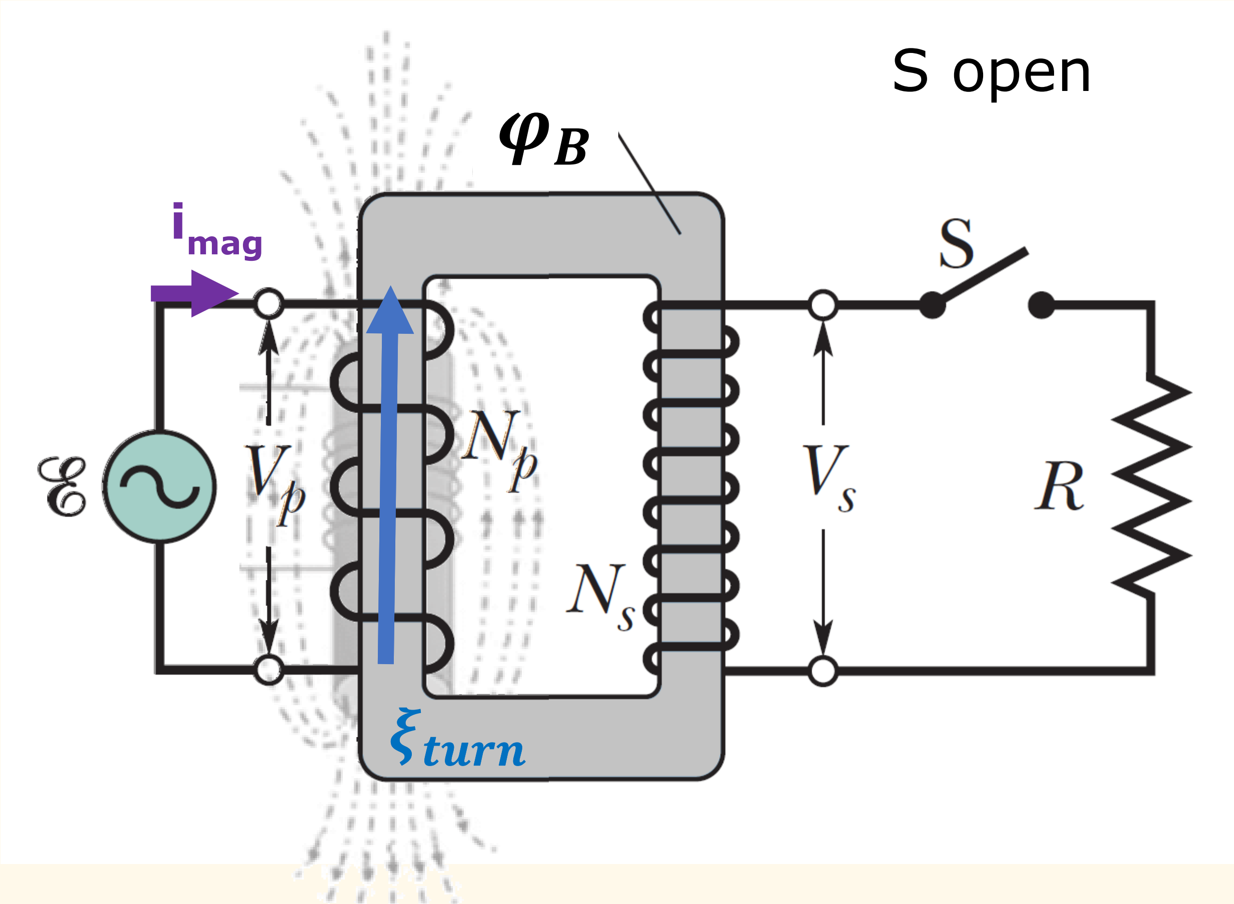
Note: no minus sign here to take into account the orientation of the turns with respect to the core

The transformer receive an emf

$$\xi = \xi_m \sin(\omega_d t)$$

The small current in the secondary coil is called **magnetizing current i_{mag}**

I_{mag} produces a sinusoidally varying magnetic field
→ **sinusoidally varying magnetic flux ϕ_B in the iron core**



In each turn of the coils **ϕ_B produces and emf**

$$\xi_{turn} = \frac{d\varphi_B}{dt}$$

Note: no minus sign here to take into account the orientation of the turns with respect to the core

TRANSFORMERS

The transformer receive an emf

$$\xi = \xi_m \sin(\omega_d t)$$

In each turn of the coils
ϕ_B produces and emf

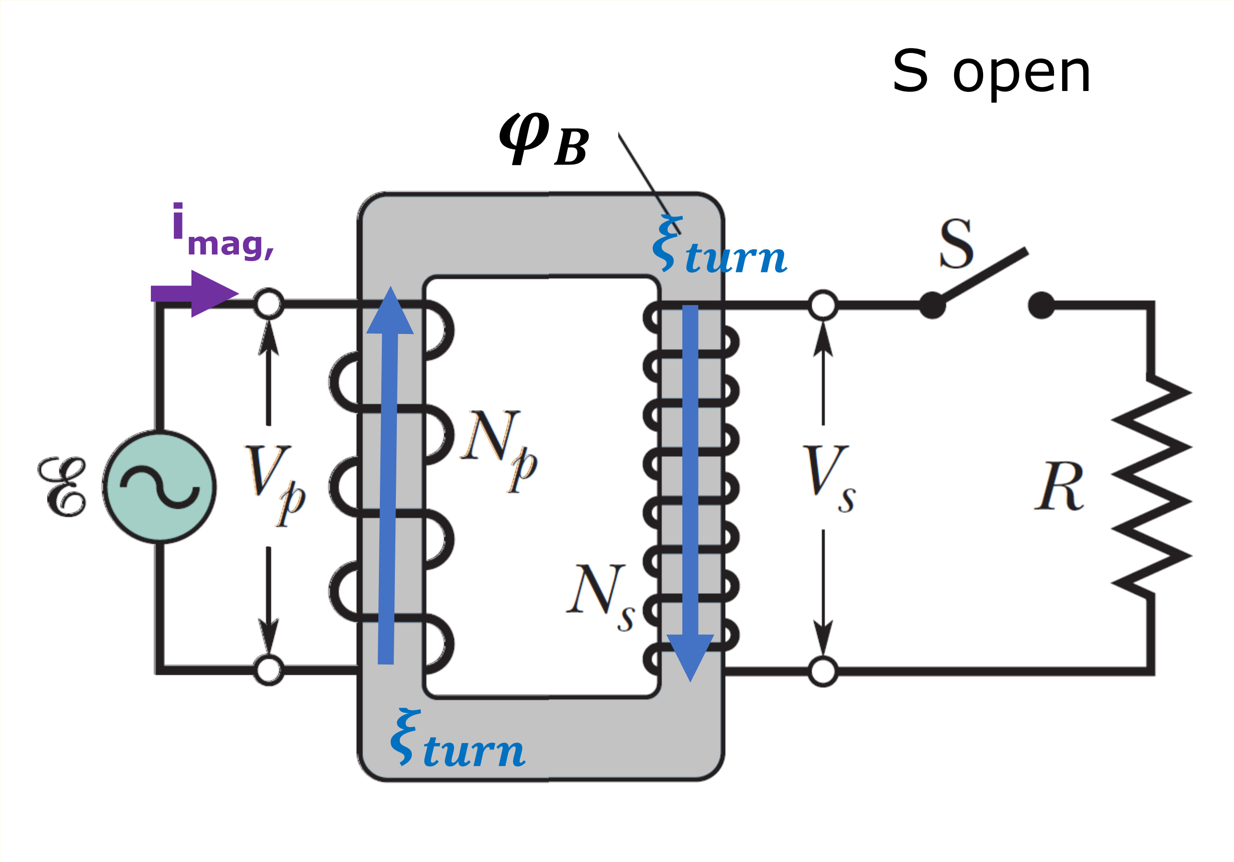
$$\xi_{turn} = \frac{d\phi_B}{dt}$$

So we have:

$$V_p = N_p \, \xi_{turn}$$

$$V_s = N_s \, \xi_{turn}$$

$$\boldsymbol{V_s = V_p \frac{N_s}{N_p}}$$

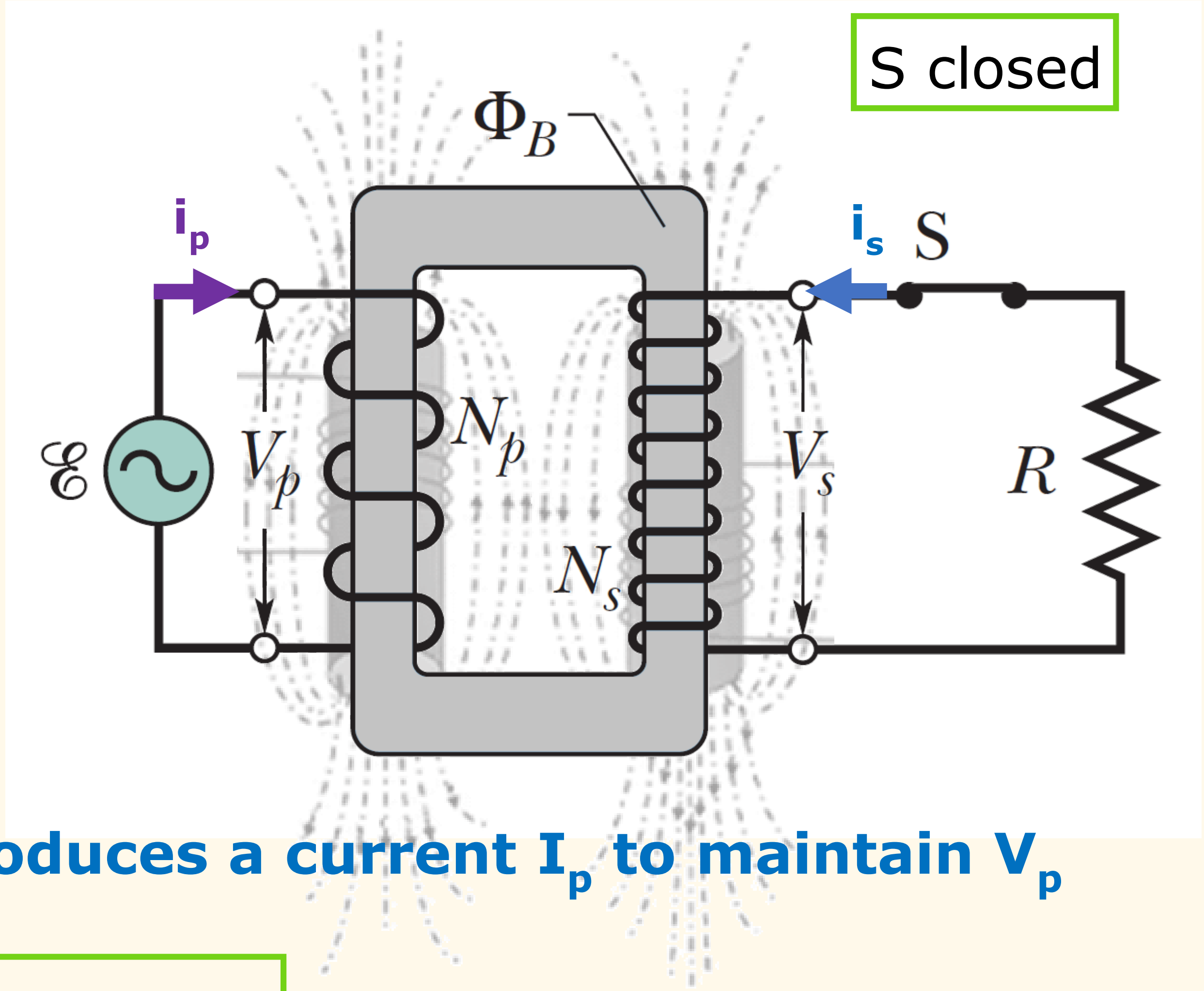


Transformation of voltage

$N_s > N_p \rightarrow V_s > V_p$: **Step-up transformer**

$N_s < N_p \rightarrow V_s < V_p$: **Step-down transformer**

Still no power is transmitted because S is open (no current in the second coil)→ now we close S



The transformer receive an emf

$\xi = \xi_m \sin(\omega_d t)$

Transformation of voltage $V_s = V_p \dfrac{N_s}{N_p}$

Current in the secondary circuit: $I_s = \dfrac{V_s}{R}$

I_s creates its own varying magnetic flux
Opposes to ϕ_B → **"lowering of V_p"**

But V_p is kept at ξ by the generator → **Produces a current I_p to maintain V_p**

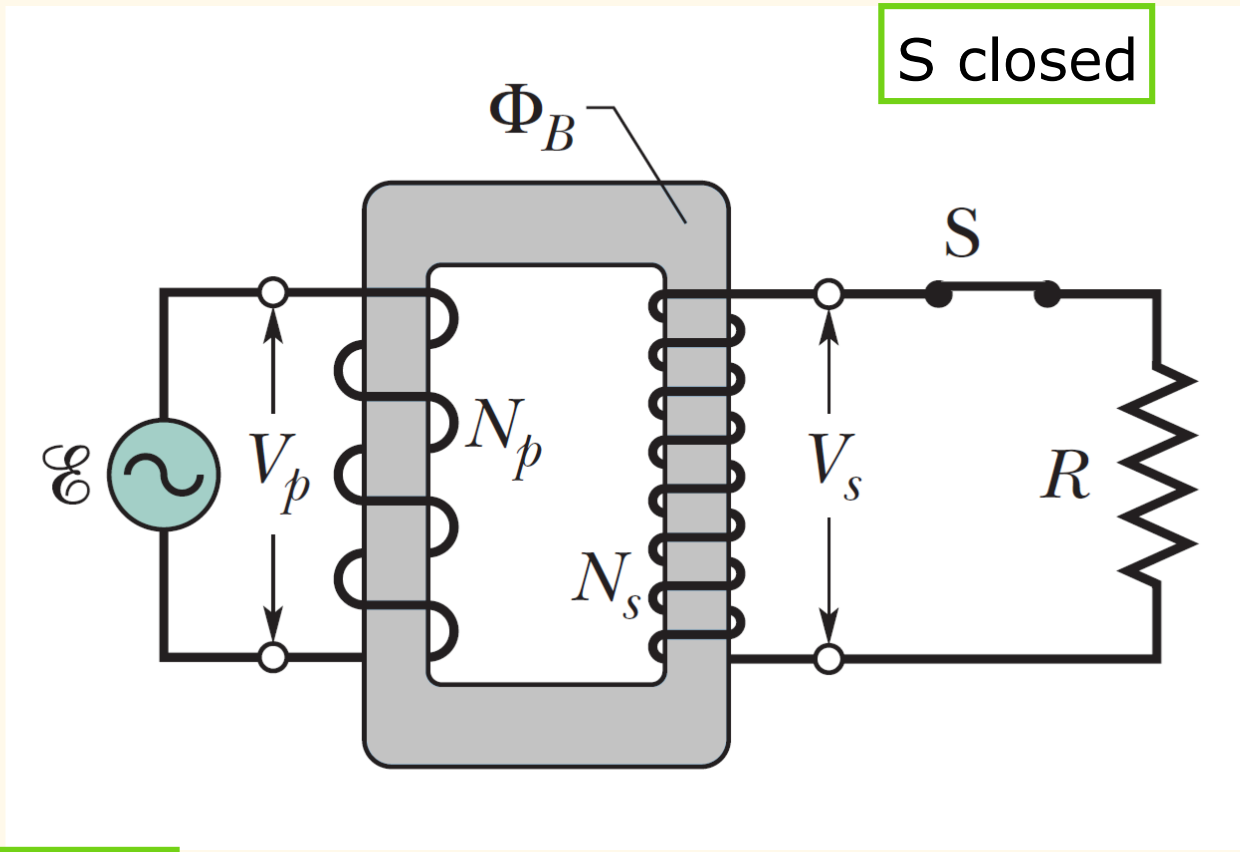
Power factor of $I_p \neq 0$ → Power is transferred

The transformer receive an emf

$$\xi = \xi_m \sin(\omega_d t)$$

Transformation of voltage $V_s = V_p \dfrac{N_s}{N_p}$

Current in the secondary circuit: $I_s = \dfrac{V_s}{R}$



Conservation of energy:

$$I_s V_s = I_p V_p$$

Conservation of energy and transformation of voltage lead to: $I_s = I_p \dfrac{N_p}{N_s}$ Transformation of current

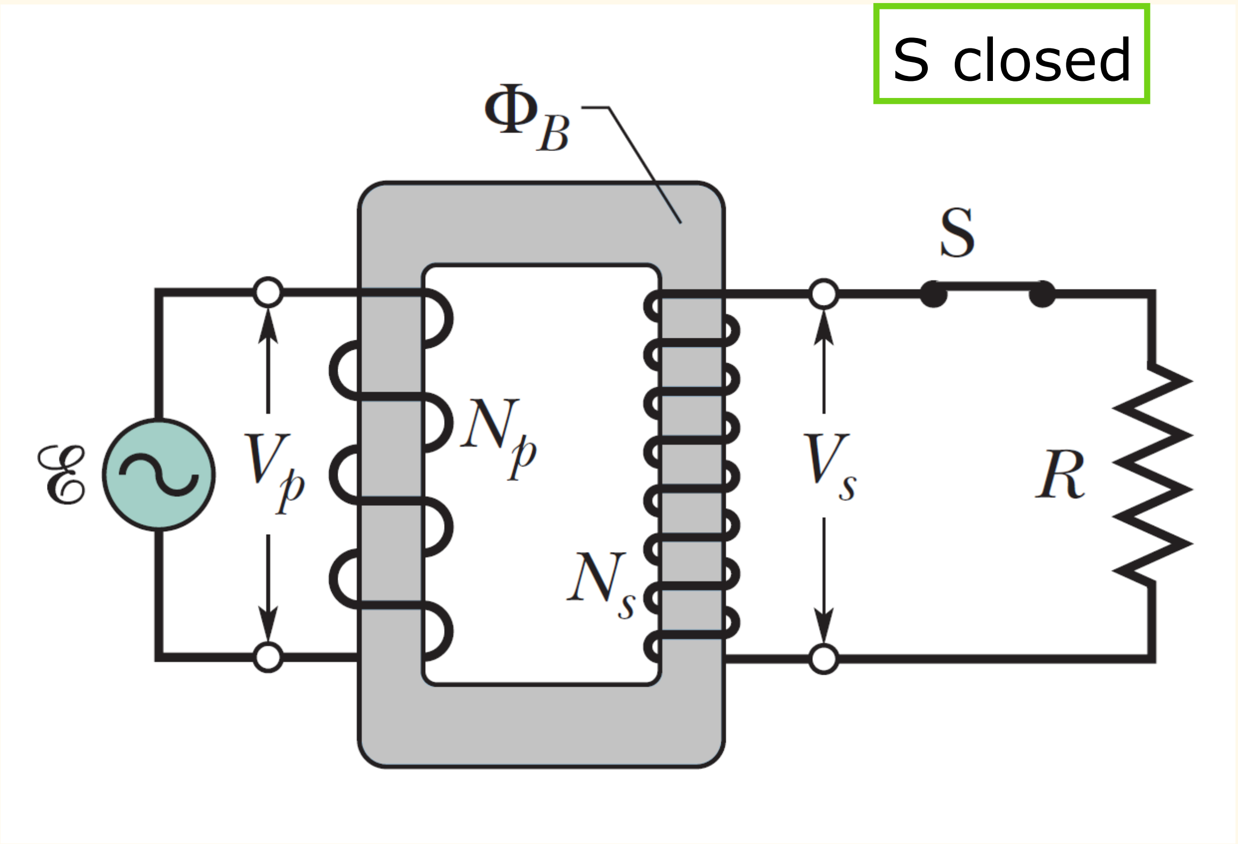
The transformer receive an emf

$\xi = \xi_m \sin(\omega_d t)$

Transformation of voltage $V_s = V_p \frac{N_s}{N_p}$

Current in the secondary circuit: $I_s = \frac{V_s}{R}$

Transformation of current $I_s = I_p \frac{N_p}{N_s}$

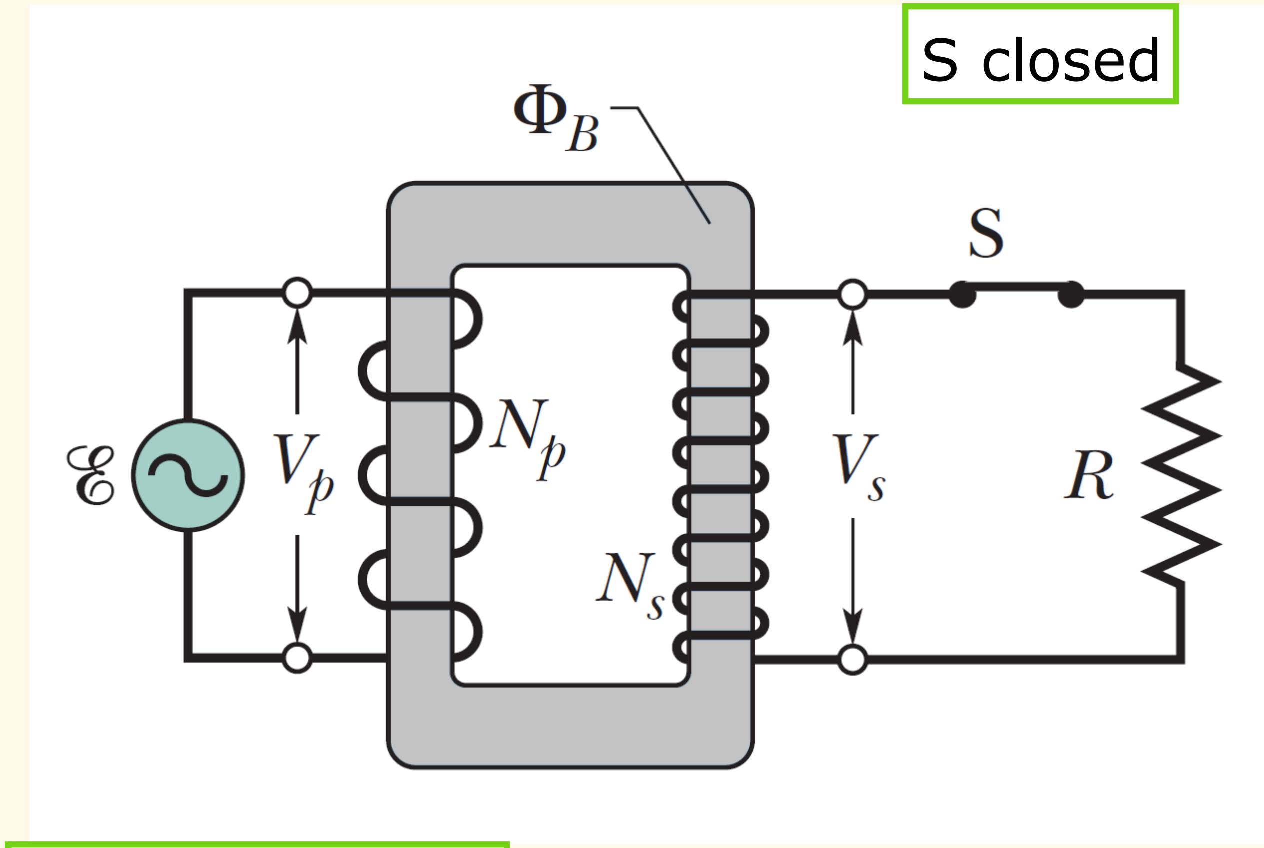


If $V_s < V_p$ then $I_s > I_p$

The transformer receive an emf

$$\xi = \xi_m \sin(\omega_d t)$$

Transformation of voltage $V_s = V_p \dfrac{N_s}{N_p}$



Current in the secondary circuit: $I_s = \dfrac{V_s}{R}$

Transformation of current $I_s = I_p \dfrac{N_p}{N_s}$

From these 3 equations we deduce I_p

$$I_p = \frac{1}{R}\left(\frac{N_s}{N_p}\right)^2 V_p$$

Note on impedance matching

$$I_p = \frac{1}{R}\left(\frac{N_s}{N_p}\right)^2 V_p$$

→ Equivalent load for the primary circuit

$$R_{eq} = R\left(\frac{N_p}{N_s}\right)^2$$

For efficient power transfer, the small impedance of the emf generator must equals R_{eq} → use of step-up transformer

KEY POINTS

Inductors and Capacitors respectively store energy as: $U_B = \frac{Li^2}{2}$ and $U_E = \frac{q^2}{2C}$

LC and RLC (R<<) circuits oscillate freely at their natural angular frequency $\omega = \frac{1}{\sqrt{LC}}$

Oscillations of RLC are damped → Energy loss rate Ri^2

Forced oscillations of circuits by an external emf $\xi = \xi_m \sin(\omega_d t)$

Forced RLC circuits by an external emf at ω_d are at resonance for $\omega = \omega_d$

Alternating current can be tuned by transformers $I_s = I_p \frac{N_p}{N_s}$

READING ASSIGNMENT

Chapter 32 of the textbook