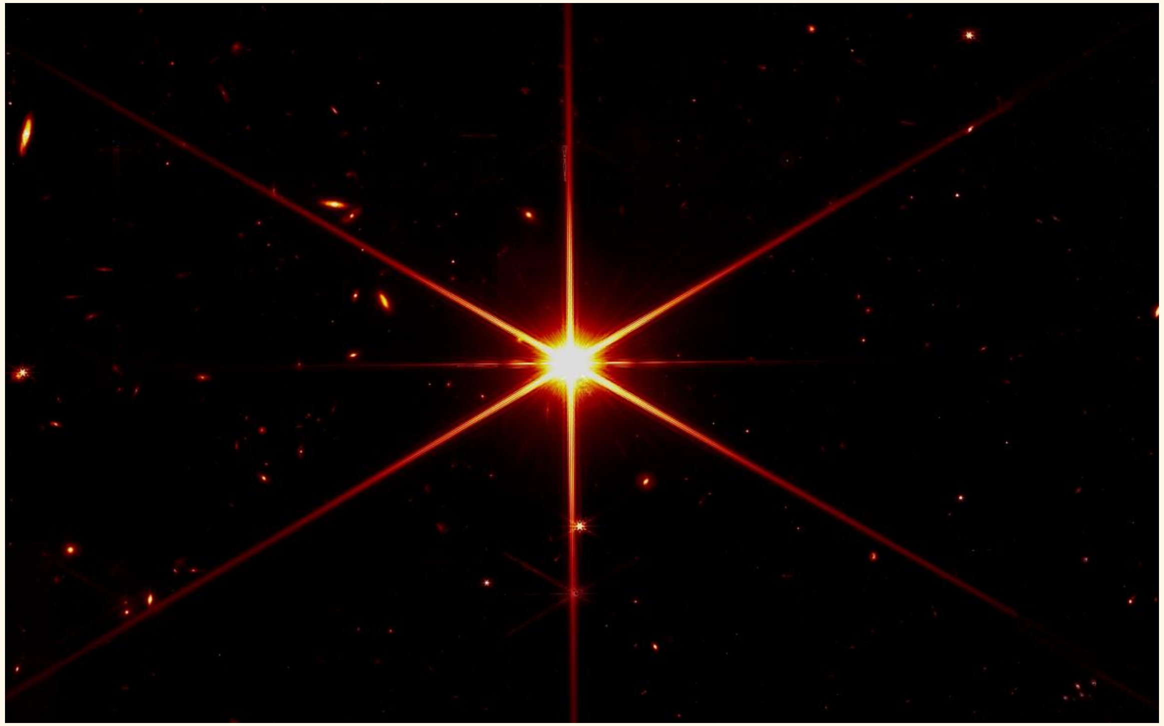
DIFFRACTION

CHAPTER 36



Picture from James Webb telescope

- Electromagnetic Oscillations & Alternating Current
- Maxwell’s Equations & Magnetism of Matter
- Electromagnetic Waves
- Images
- Interference
- **Diffraction**

DIFFRACTION

Textbook: Chapter 36

- SINGLE-SLIT DIFFRACTION
- DIFFRACTION BY A CIRCULAR APERTURE
- DIFFRACTION BY A DOUBLE SLIT
- DIFFRACTION GRATINGS
- X-RAY DIFFRACTION

SINGLE-SLIT DIFFRACTION

Geometrical optics:

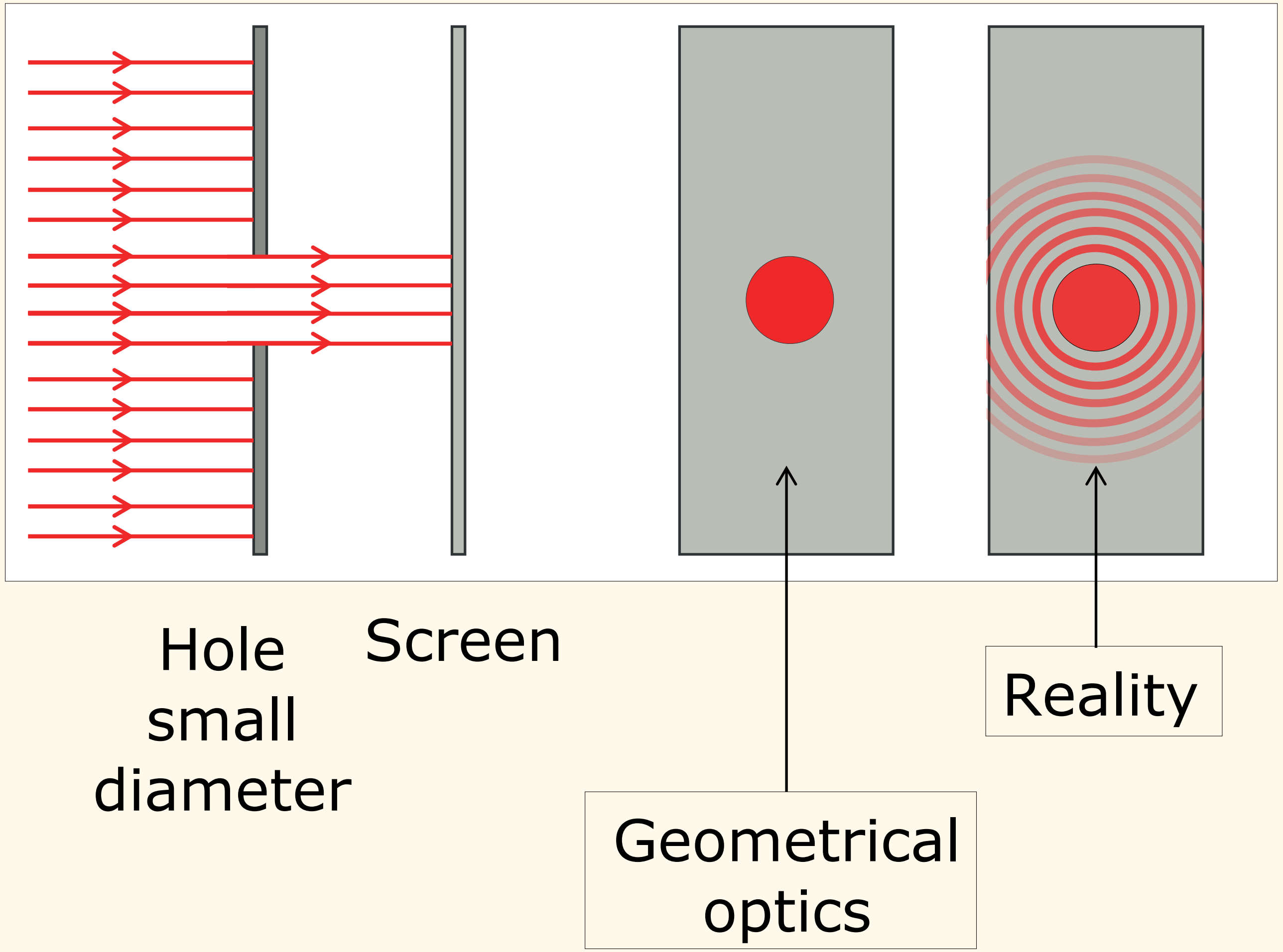
Light = Rays → Straight lines

Beams can be forever as narrow as we want

Reality:

After small apertures or objects **spreading + fringes**

→ Like interference, geometrical optics cannot describe the phenomena



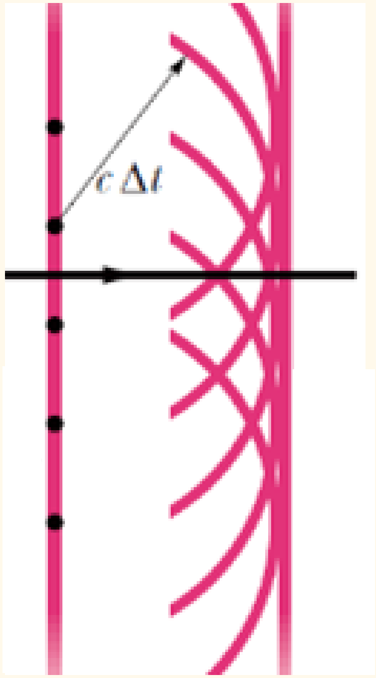
SINGLE-SLIT DIFFRACTION

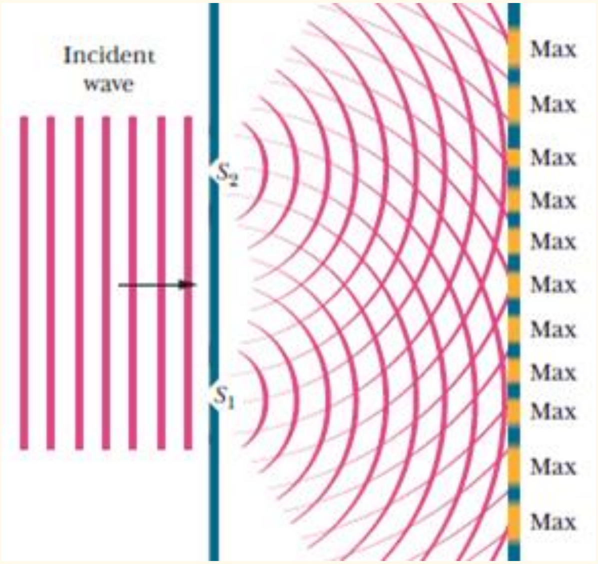
Diffraction

→ Explained by light as **EM wave**

Last chapter: We explained interference with **slits of width comparable to λ**
→ Source of 1 half-spherical wavelet following Huygens' Principle

→ Fringes on **all the screen** (in theory)





Double-slit interference
Width ~ λ

Generalization to objects of **any width**
→ source of **multiple wavelets**
Interference pattern not infinite

Diffraction by a single slit

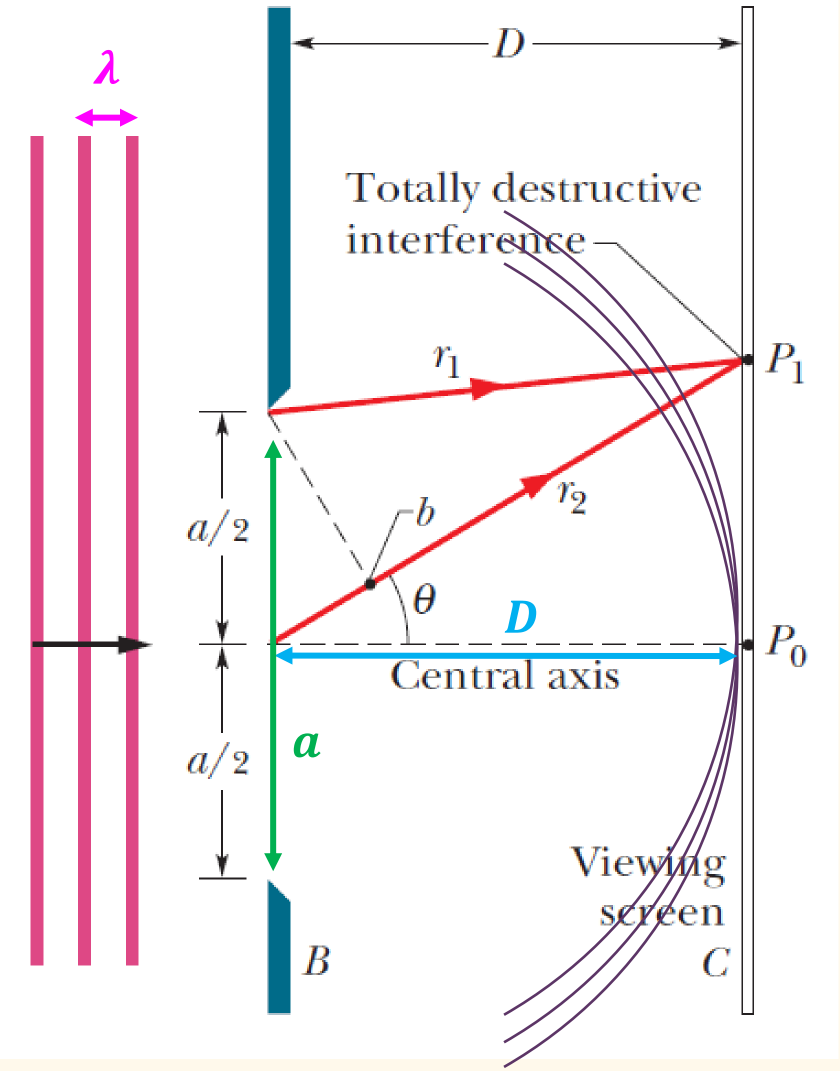
Ken Kay/Fundamental Photographs

How to explain this pattern ?
→ Large bright center
+ dark / bright fringes

Slit of width a screen at distance D
We assume $D >> a > \lambda$

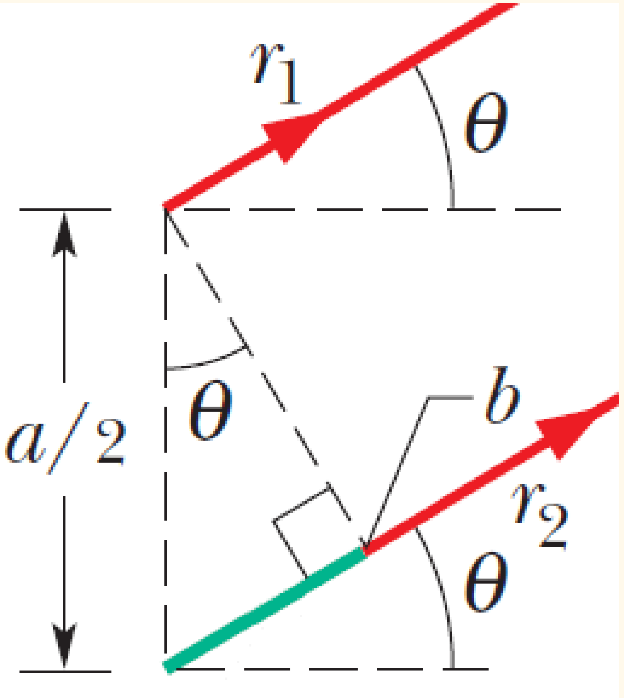
Bright center: wavelets travel ~ the same distance to reach P_0
→ In phase
→ **Constructive** interference

Dark fringes: Wavelets out of phase
→ **Destructive** interference



Position of the dark fringes: 1st minimum P_1

→ Strategy = Pairing rays spaced by $a/2$ and finding the condition for destructive interference



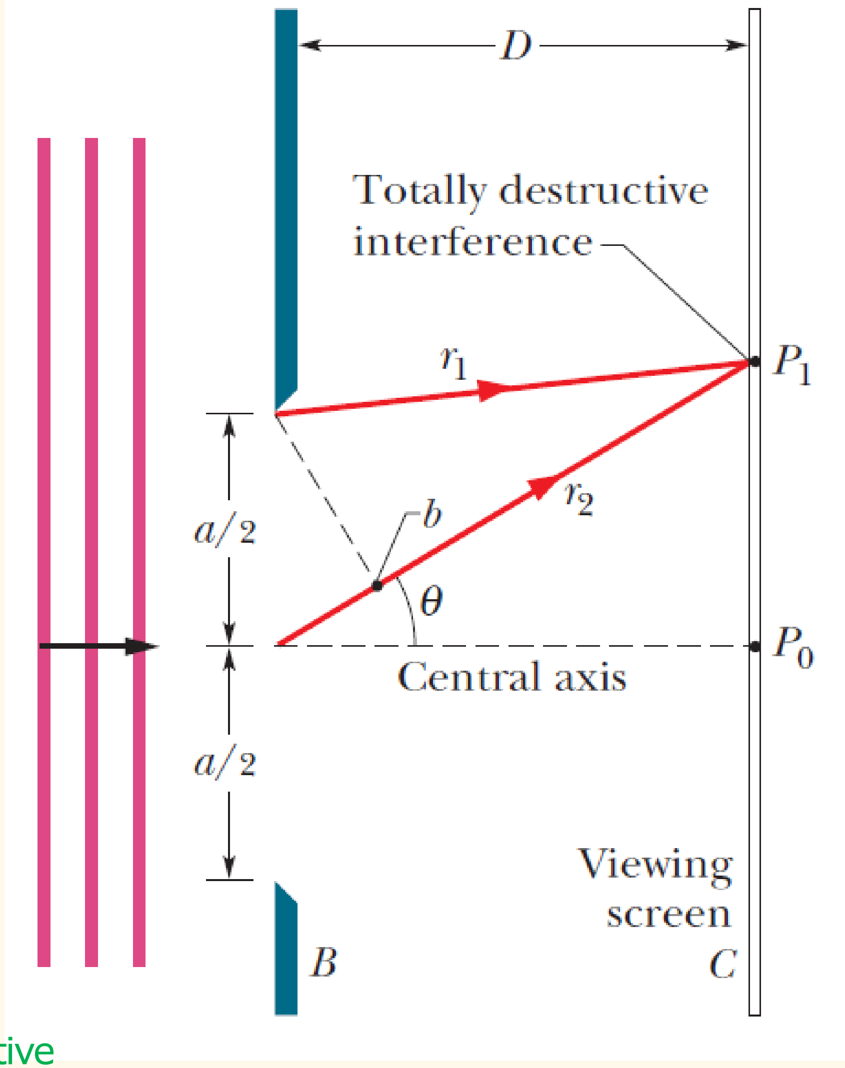
We assume **D >> a**

→ Rays parallels close to the slit – angle θ to reach P_1

→ Difference of path

$$\frac{a}{2}\sin\theta \longrightarrow \frac{\lambda}{2}$$

condition for destructive interference

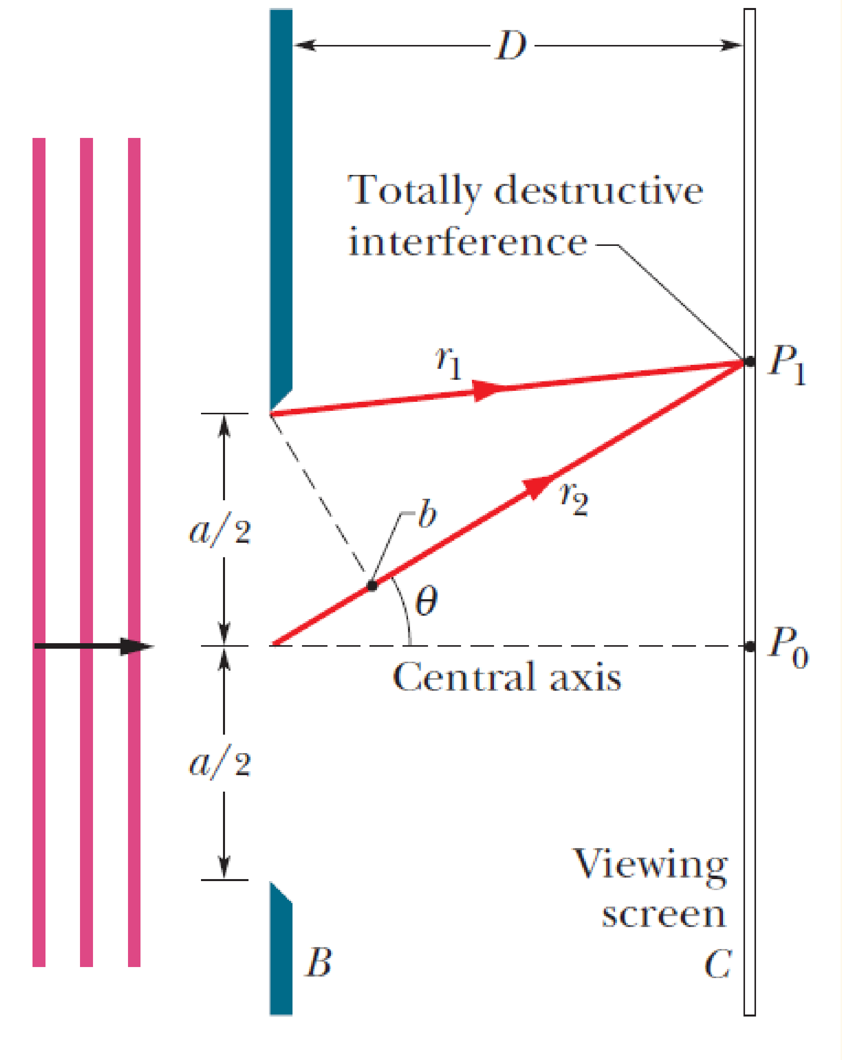


SINGLE-SLIT DIFFRACTION

Position of the dark fringes: 1st minimum P_1

→ Every pair of rays spaced by $a/2$ going to P_1

satisfy: $a \sin\theta = \lambda$



Note: If we decrease a → sin θ increases (λ is cst)
→ θ increases
→ Pattern is enlarged

→ Other minimums ?

SINGLE-SLIT DIFFRACTION

Position of the dark fringes: 2nd minimum P_2

→ Group of 4 rays spaced by $a/4$ going to P_2

$$(a/4)\sin\theta = \lambda/2 \longrightarrow \boxed{a\sin\theta = 2\lambda}$$

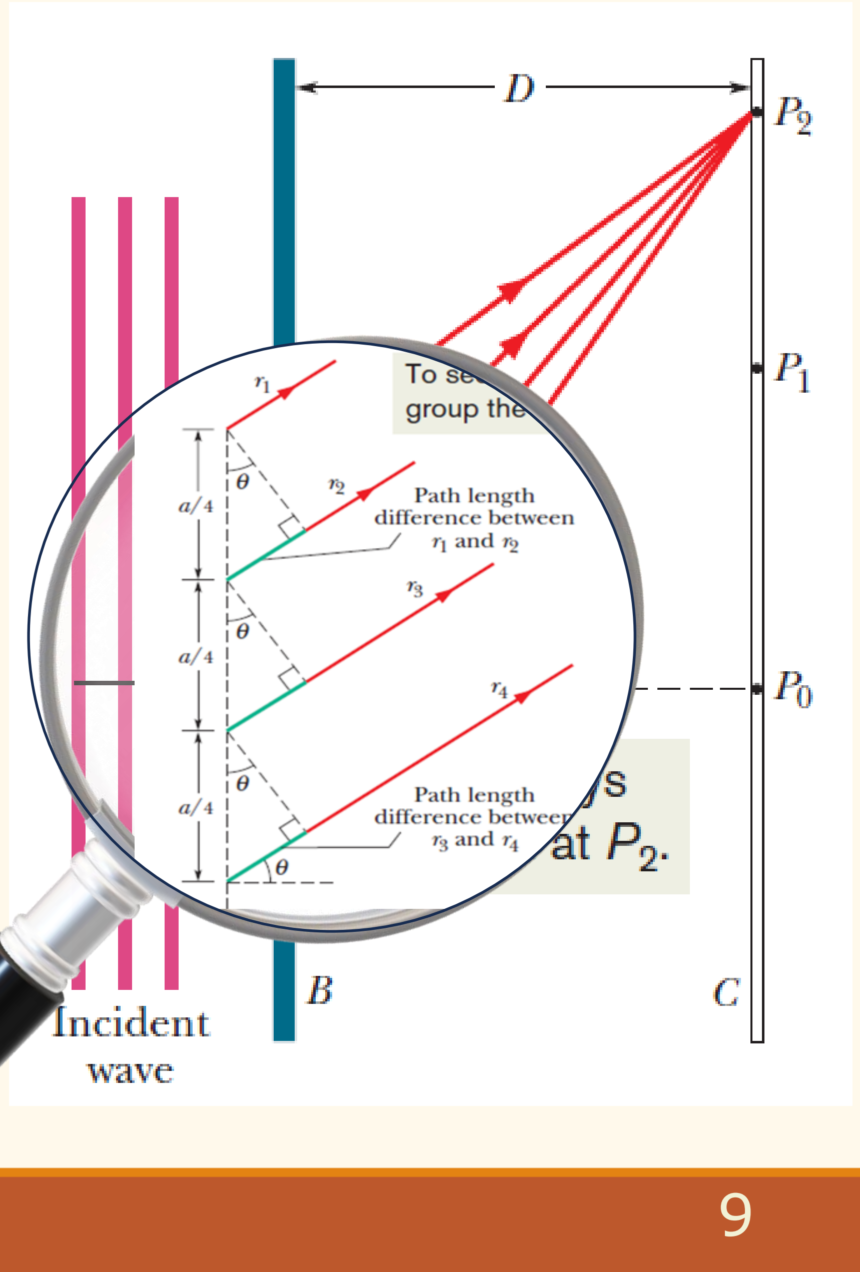
Position of the dark fringes: mth minimum P_m

→ Group of m rays spaced by a / 2m going to P_m

$$(a/2m)\sin\theta = \lambda/2 \longrightarrow \boxed{a\sin\theta = m\lambda}$$

θ satisfying: $a\sin\theta = m\lambda$ (m integer > 1)

→ **Dark fringe at angle θ**



Intensity profile

General problem: finding the intensity at every angle θ to point P

→ Slit divided into N zones of width Δx

→ Each is source of a wavelet

Need to determine E_θ: the field at P to calculate intensity

Phase difference between 2 adjacent wavelets: $\Delta\phi = \frac{2\pi}{\lambda}(\Delta x \sin\theta)$

$$E_\theta = \sum \Delta E \sin(kx - \omega t + \phi)$$

Sum of — Amplitude of each wavelet — projected by the phase factor

Wave number k — Difference of path

Intensity profile

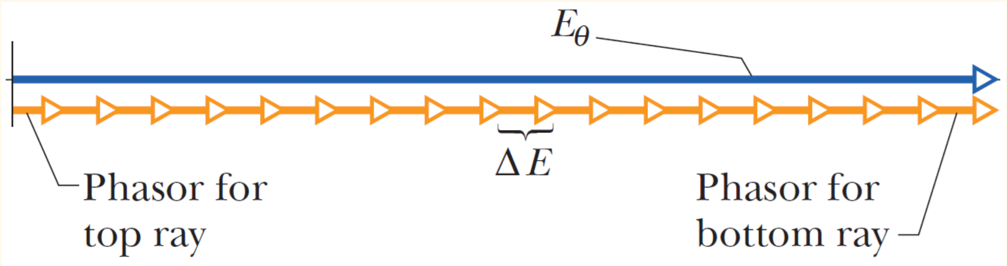
$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x \sin\theta)$$ between 2 adjacent wavelets

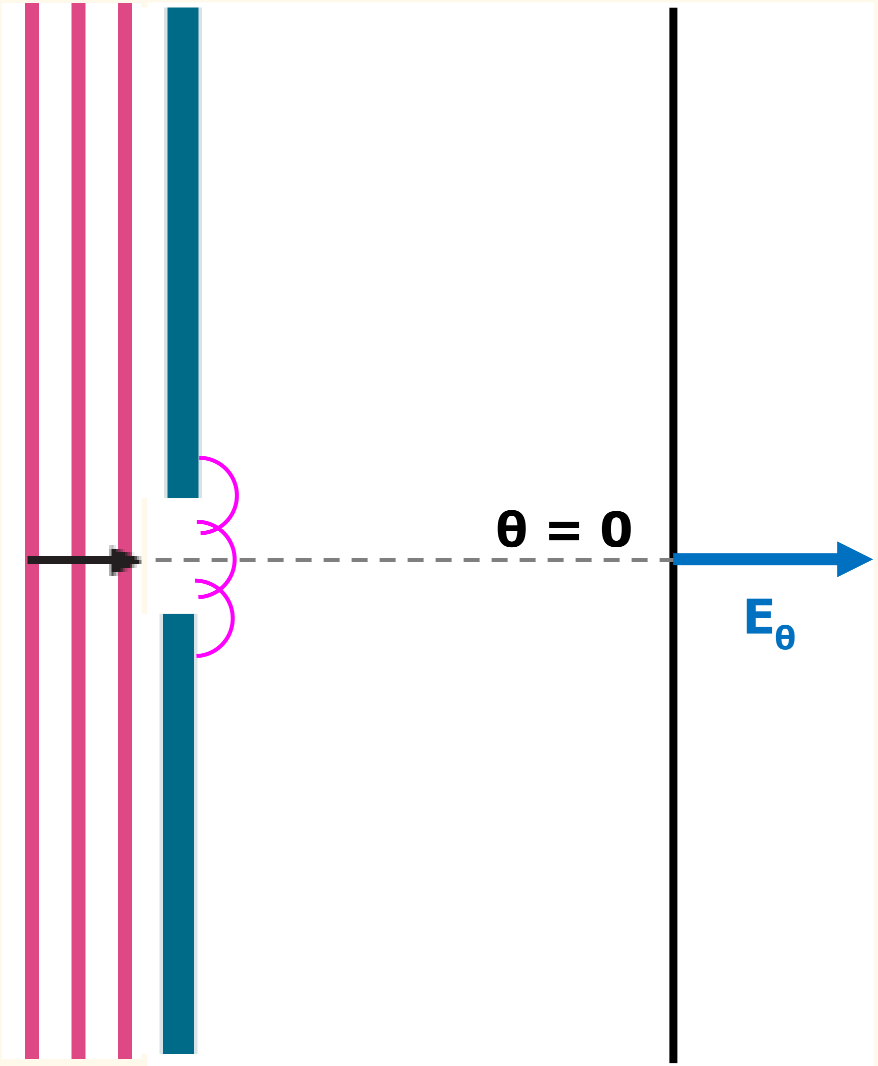
θ = 0 → Δϕ = 0 for every wavelet

→ All wavelets are in phase

→ **Brightest fringe $E_\theta = N\,\Delta E$**

Representation with phasors:





Intensity profile

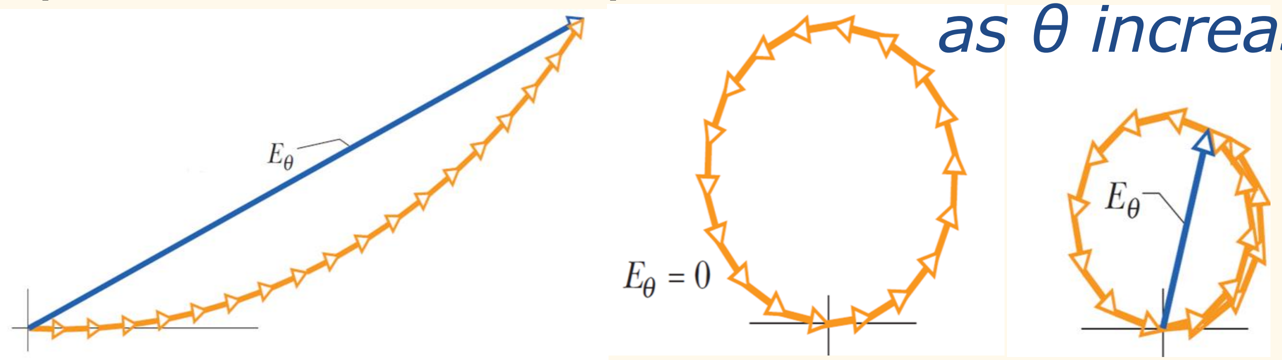
$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x \sin\theta)$$ between 2 adjacent wavelets

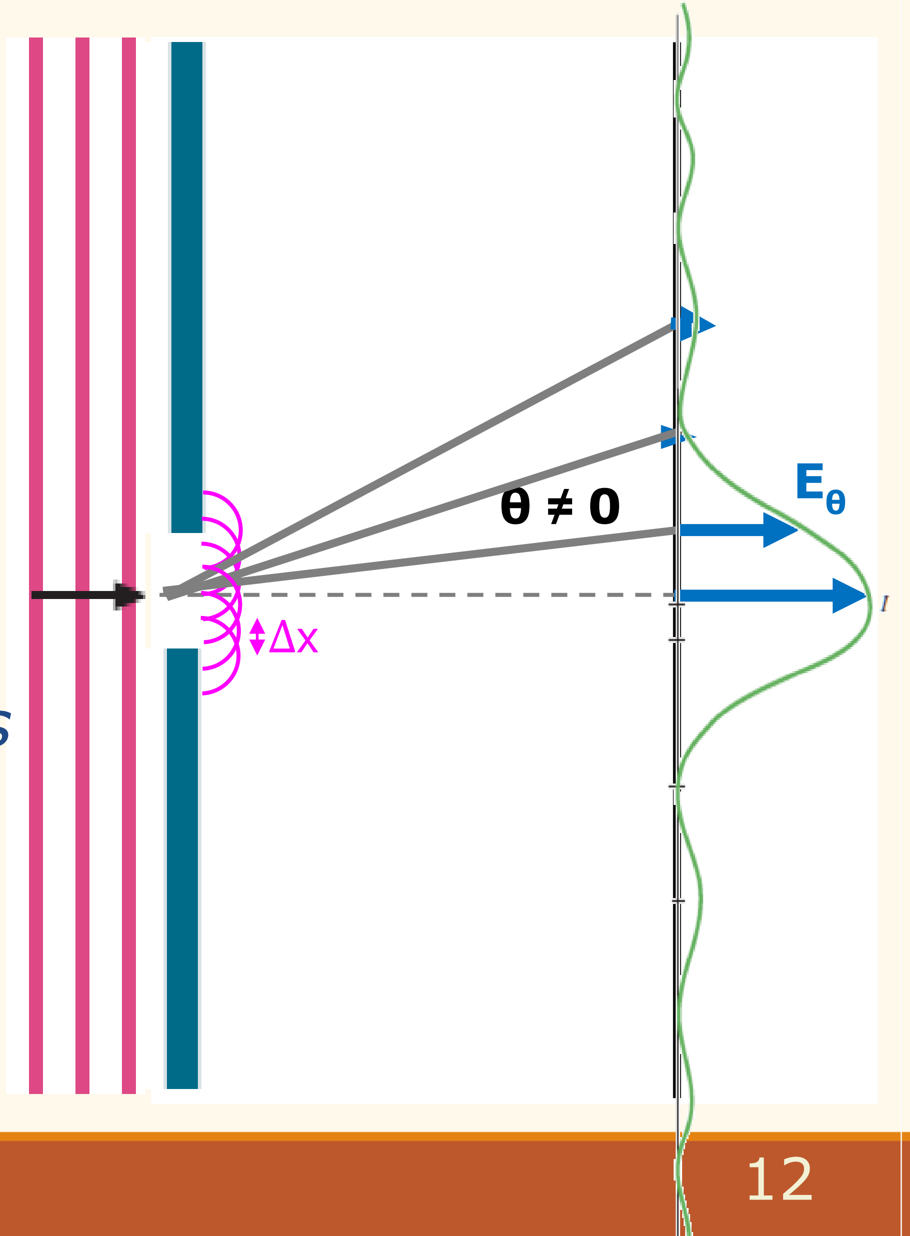
θ ≠ 0 → Δϕ ≠ 0 for every wavelet

→ All wavelets have a phase difference

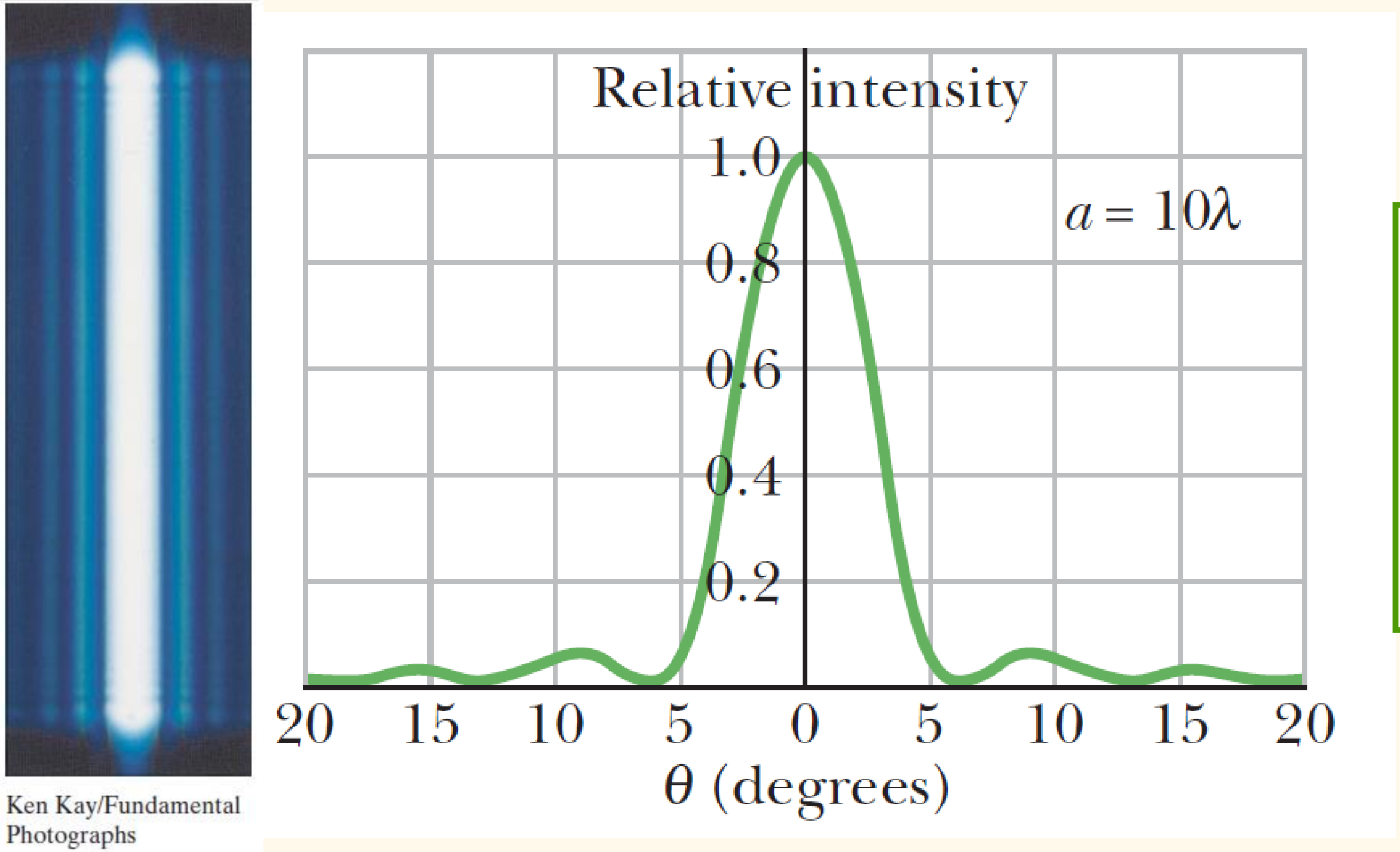
→ **$E_\theta \neq N\Delta E$**

Representation with phasors: *spiral on themselves as θ increases*





Intensity profile



Ken Kay/Fundamental Photographs

Typical intensity profile for single-slit diffraction

As θ increases
E_θ has zeros
and local maxima
→ The same for intensity

→ Analytical expression ?

Intensity profile (optional demonstration)

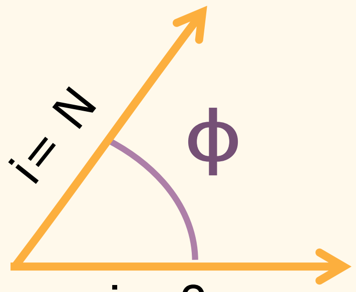
→ **Analytical expression**

For a given angle θ, the i^{th} of the N phasors has a phase difference with the 1^{st}

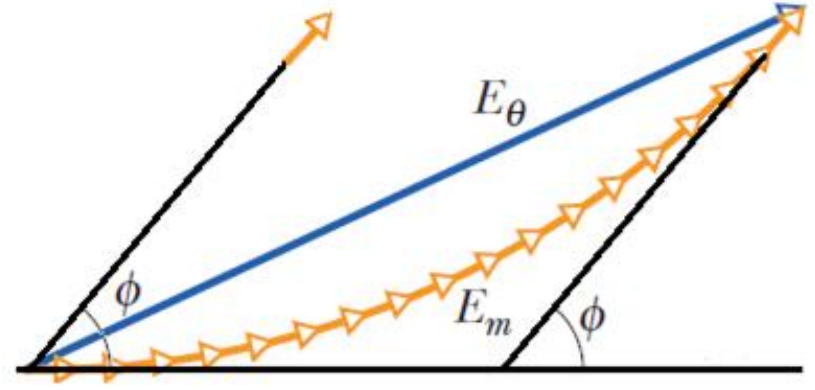
$$\Delta\phi_i = \frac{2\pi}{\lambda}(i\Delta x \sin\theta)$$

We define ϕ as the difference of phase between the 1^{st} (i=0) and the last phasor (i= N)

$$\phi = \Delta\phi_N - \Delta\phi_0$$



We report ϕ at the intersection of the projections of the 1^{st} and the N^{th} phasors



SINGLE-SLIT DIFFRACTION

Intensity profile (optional demonstratio

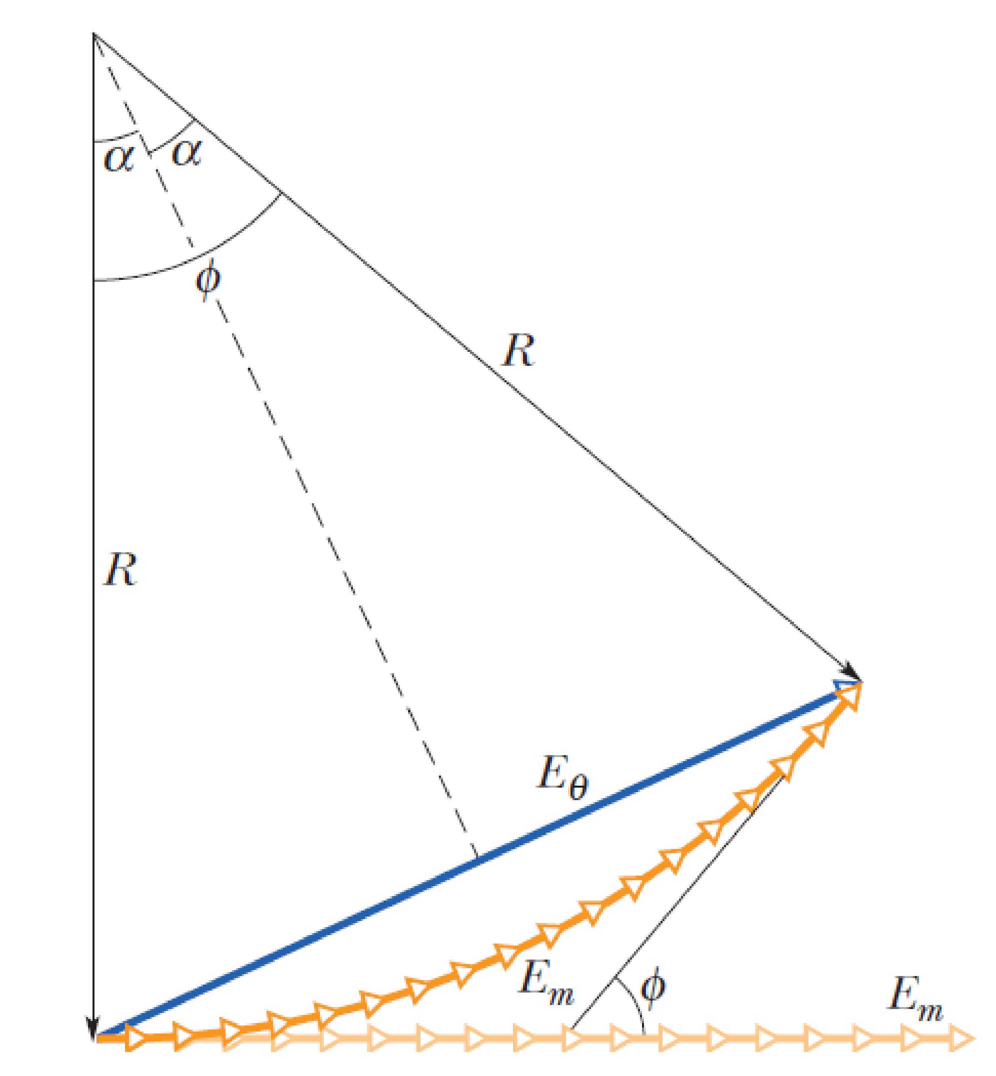
→ **Analytical expression**

$N \to \infty$: The sum of the phasors of the wavelets is an arc of circle of radius R.

The length of the arc is always the same → = to E_m (when $\theta = 0$)

By construction, ϕ is the angle between the radius

We define $\alpha = \phi/2$



SINGLE-SLIT DIFFRACTION

Intensity profile (optional demonstratio

→ Analytical expression

We have: $\sin\alpha = \dfrac{E_\theta / 2}{R}$

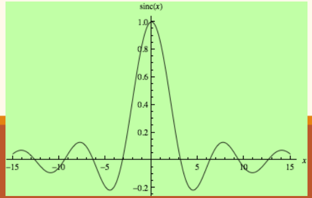
And, in radians: $\phi = 2\alpha = \dfrac{E_m}{R}$

Combining these two expressions:

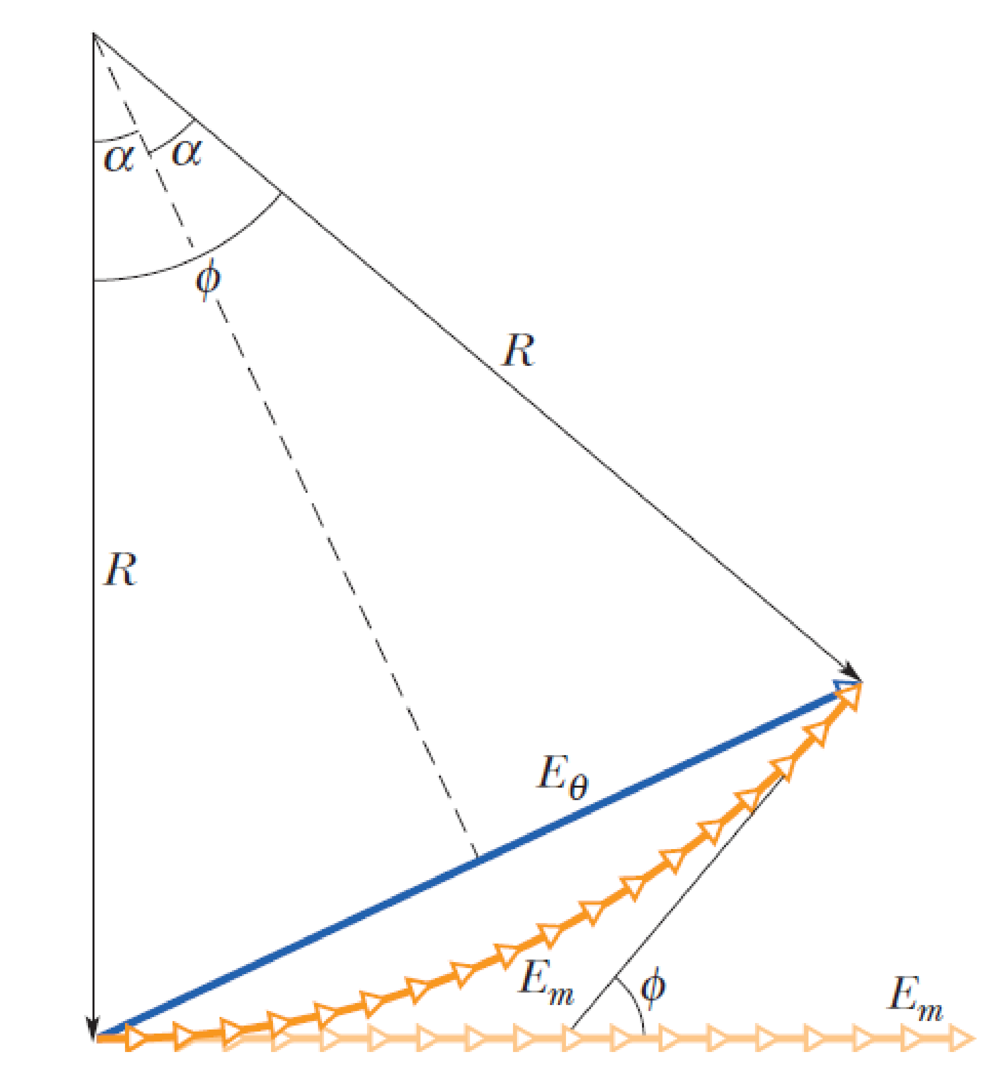
$$\sin\alpha = \frac{E_\theta / 2}{E_m / (2\alpha)} \longrightarrow E_\theta = E_m \frac{\sin\alpha}{\alpha}$$

Note: $\boldsymbol{sin_c}(x) = sin(x) / x$

the sinc function



$$E_\theta = E_m \, \mathrm{sin}_c\alpha$$



Intensity profile (optional demonstration)

→ **Analytical expression**

We define the relative intensity as: $I_r = \dfrac{I_\theta}{I_{\theta = 0}} = \dfrac{I_\theta}{I_m}$

Since the intensity is proportional to the squared length of the phasor:

$$I_r = \frac{E_\theta^2}{E_m^2}$$

So $\dfrac{I_\theta}{I_m} = \dfrac{E_\theta^2}{E_m^2} \longrightarrow I_\theta = I_m \dfrac{E_\theta^2}{E_m^2}$ with $E_\theta = E_m \, \mathrm{sin}_c \alpha \longrightarrow \boxed{I_\theta = I_m \, \mathrm{sin}_c^2 \alpha}$

Last step: relation between α and θ

Intensity profile (optional demonstration)

→ Analytical expression

We have: $\Delta\phi_i = \frac{2\pi}{\lambda}(i\Delta\, x sin\theta)$, $\phi = \Delta\phi_N - \Delta\phi_0$, $\alpha = \frac{\phi}{2}$ and $I_\theta = I_m \sin_c^2 \alpha$

$\Delta\phi_{i=0} = 0$

$\Delta\phi_{i=N} = \frac{2\pi}{\lambda}(N\Delta x \sin\theta)$

a = width of the slit,
divided in N intervals Δx,
so we have: $N\Delta x = a$

$\Delta\phi_N = \frac{2\pi}{\lambda}(a \sin\theta)$

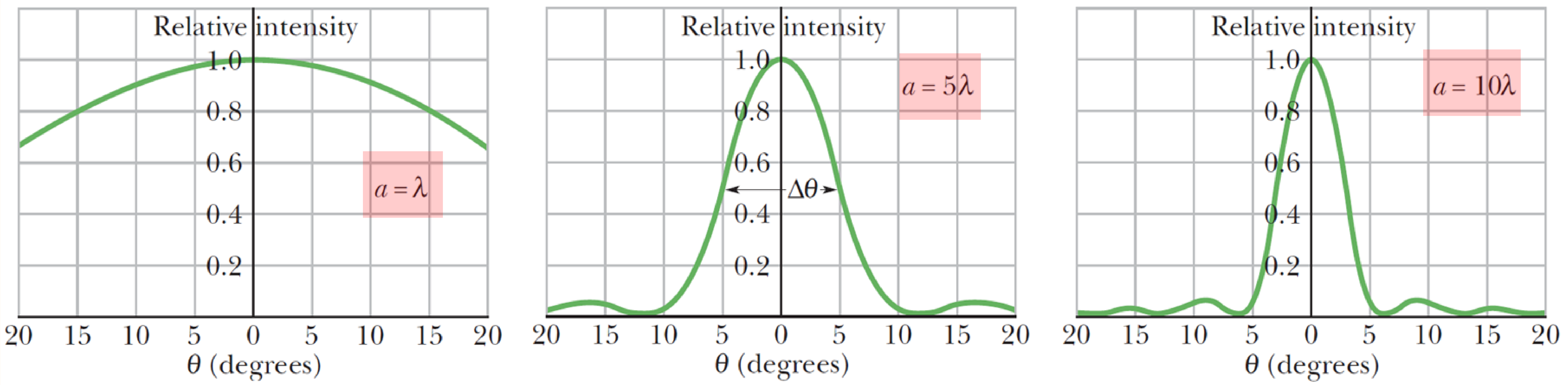
$\phi = \frac{2\pi}{\lambda}(a \sin\theta)$

$\alpha = \frac{\pi}{\lambda}(a \sin\theta)$

$$I_\theta = I_m \sin_c^2\left(\frac{a\pi}{\lambda}\sin\theta\right)$$

Intensity profile

→ **Analytical expression**



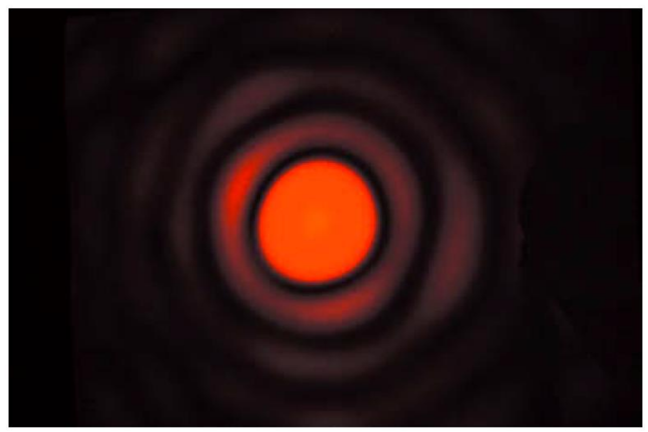
Narrower slit → More spreading
Larger slit →Less spreading

$$I_\theta = I_m \sin_c^2\left(\frac{a\pi}{\lambda}\sin\theta\right)$$

Circular apertures

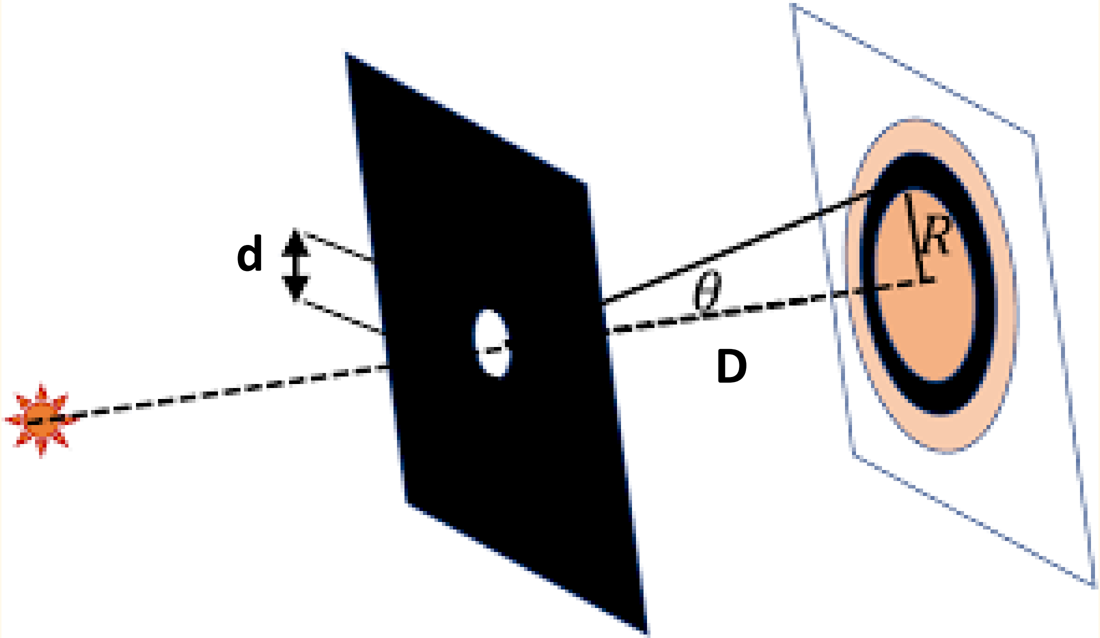
→ A little more analytically complex than slits

→ We will admit some results



Courtesy Jearl Walker

Pattern = central disk + fringes



First minimum at: $\sin\theta = 1{,}22\,\frac{\lambda}{d}$

The main difference with single slit $\sin\theta = \frac{\lambda}{d}$ is the factor 1.22, which enters because of the circular shape of the aperture (not demonstrated).

Resolvability

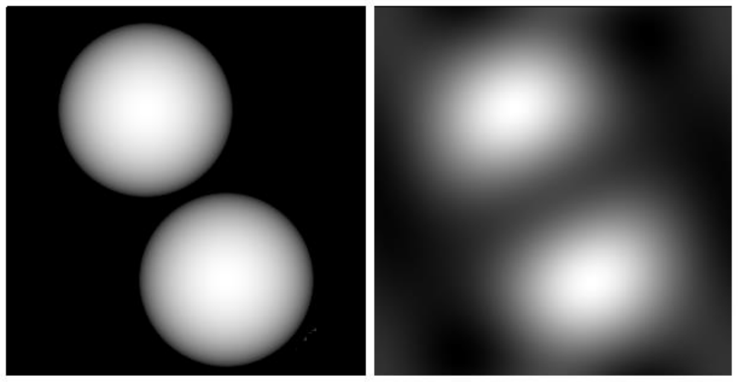
The same result is true for lenses of diameter d

First minimum at: $\boldsymbol{\sin\theta = 1{,}22\,\frac{\lambda}{d}}$

If one wants to image objects with **small angular separation**
diffraction = problem

e.g.Two stars close to each other with a telescope
or two near micro-objects with a microscope

Expectation (from geometrical optics)

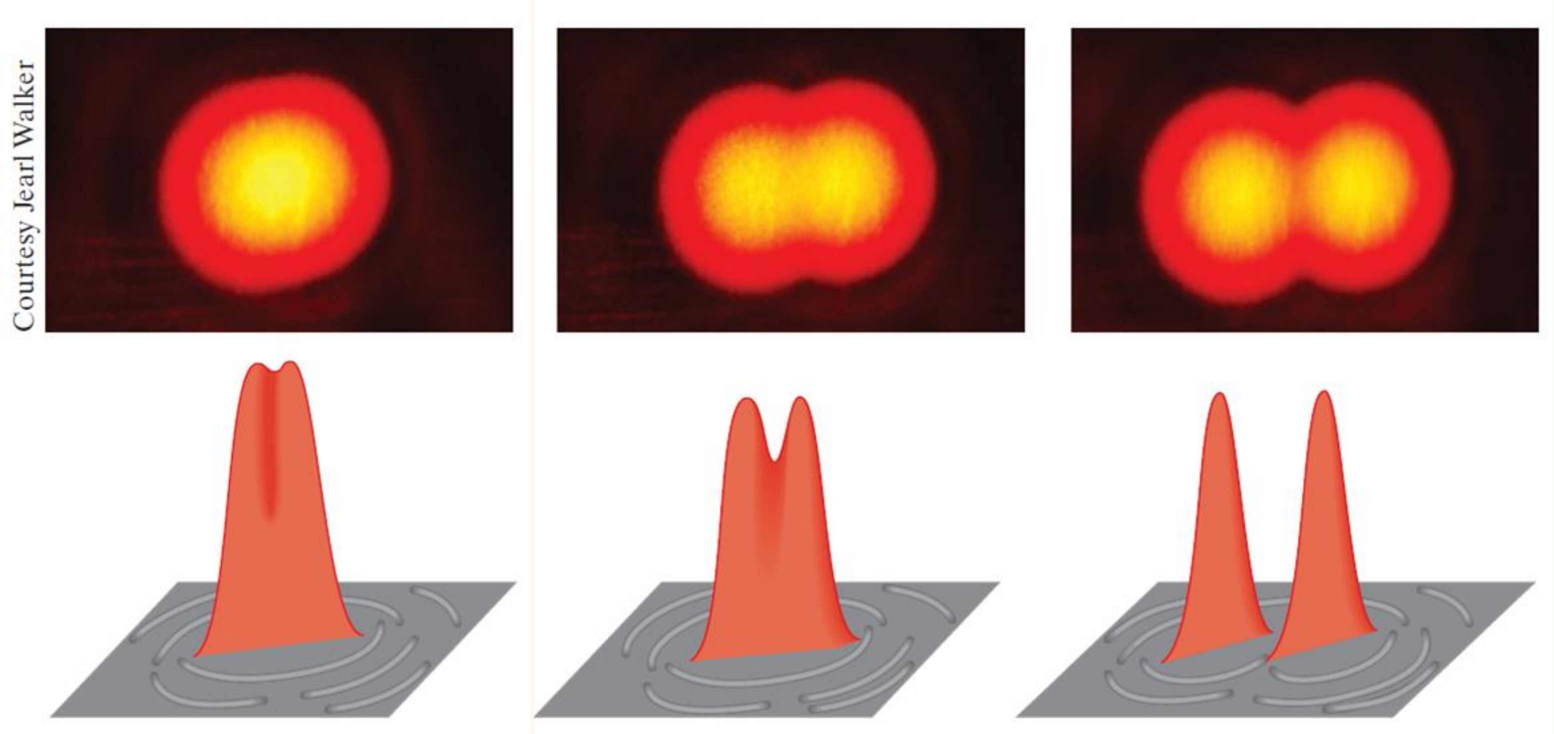


Reality

Resolvability

Closer are the objects, the more difficult they are to **resolve**

→ distinguish as 2 separate objects



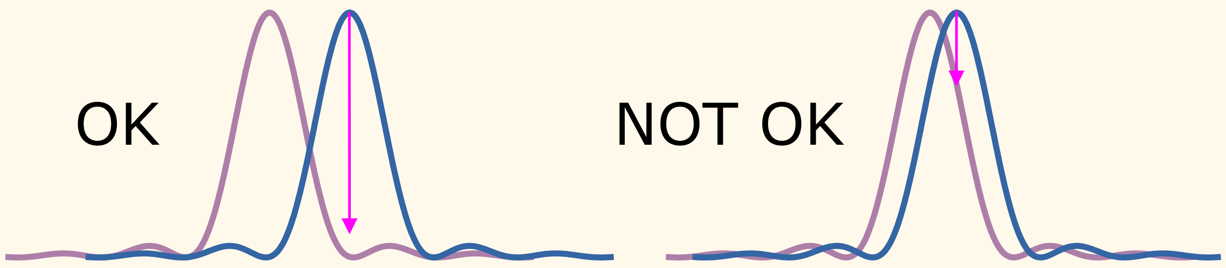
Diffraction patterns overlap

→ Need **objective criteria** to claim that we see two objects

Resolvability → Rayleigh's criterion (other criteria exist)

First minimum at: $\sin\theta = 1,22\,\dfrac{\lambda}{d}$

***Minimum condition** to resolve 2 objects when the 1st minimum of the 1st object overlaps the maximum of the 2nd object*



(Intensity profiles for single objects)

Since the angles are small, we can replace $\sin\theta_R$ with θ_R

In other words, we define the angular separation θ_R as:

$$\theta > \theta_R = 1,22\,\frac{\lambda}{d}$$

The smallest angular separation between two resolved objects

Resolvability → **Rayleigh's criterion** (other criteria exist)

Notes:

θ_R depends of the instrument

→ here d is for a single lens

→ if more lenses then more complex

θ_R depends of the wavelength

→ mean λ of visible light ~ 550 nm

→ smaller λ (*e.g.* UV < 400 nm) means smaller θ_R and better resolution

Valid for particles of smaller λ *e.g.* e^- (quantum mechanics)

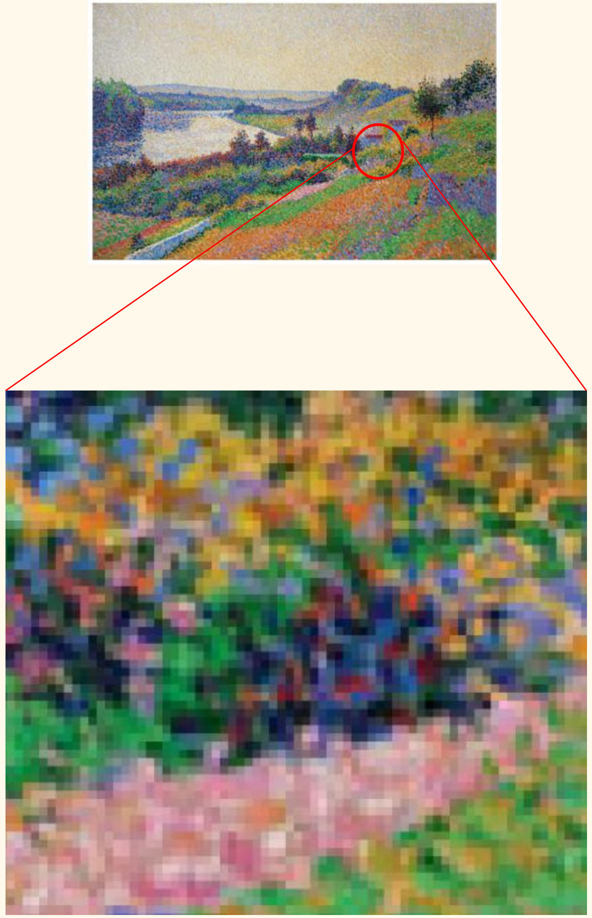
→ Scanning Electron Microscopy (SEM)

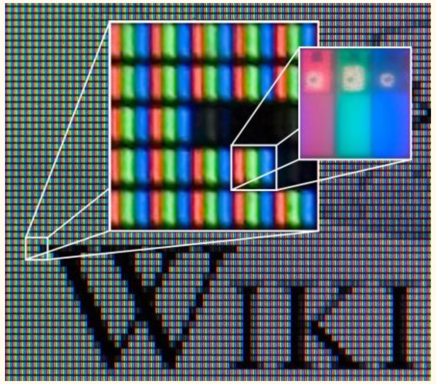
In other words, we define the angular separation θ_R as:

$$\theta > \theta_R = 1,22\,\frac{\lambda}{d}$$

The smallest angular separation between two resolved objects

DIFFRACTION BY A CIRCULAR APERTURE





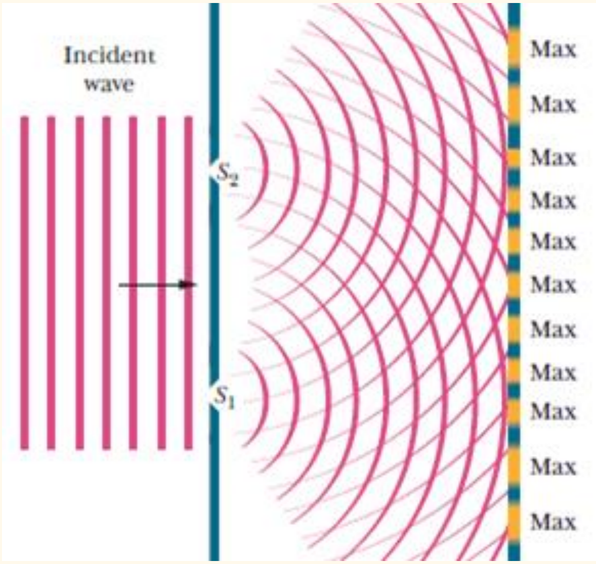
At normal viewing distances, the dots are irresolvable and thus blend.

So far, we investigated diffraction by **single objects**:
- single slit
- single circular aperture

What would happen if **several objects** diffract light ?
→ More interference !

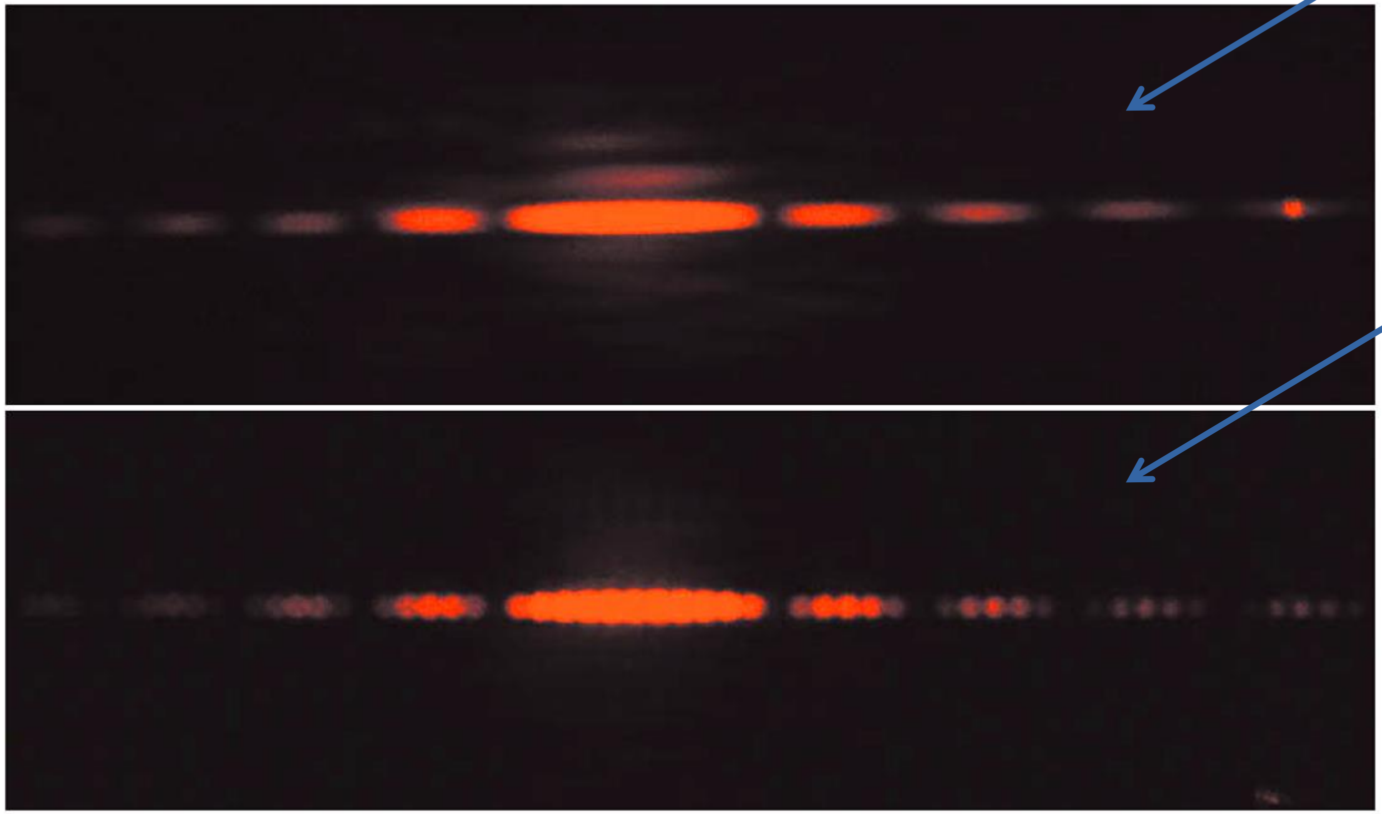
First, diffraction by **double slits of width not** $<< \lambda$

When we studied interference by double-slits, we assumed $a << \lambda$
→ a <<, so diffraction pattern >>
→ Interference on all the screen
Bright fringes with equal intensity



Typical experimental results:

Courtesy Jearl Walker

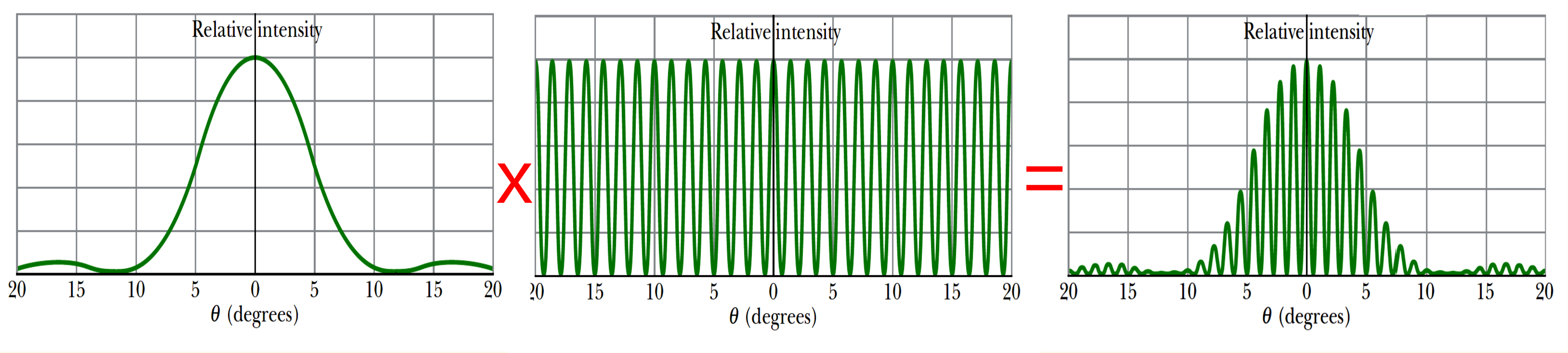


Single slit of width a:
"sin_c^2" pattern

Double-slit of width a spaced by distance d:
"sin_c^2" pattern
multiplied by
"cos^2" pattern

→ "cos^2" is the interference pattern of 2 slits

With graphs:



Diffraction pattern of 1 slit (a > λ)

Interference pattern of 2 slits (a ~ λ) spaced by d

Diffraction pattern of 2 slits (a > λ) spaced by d

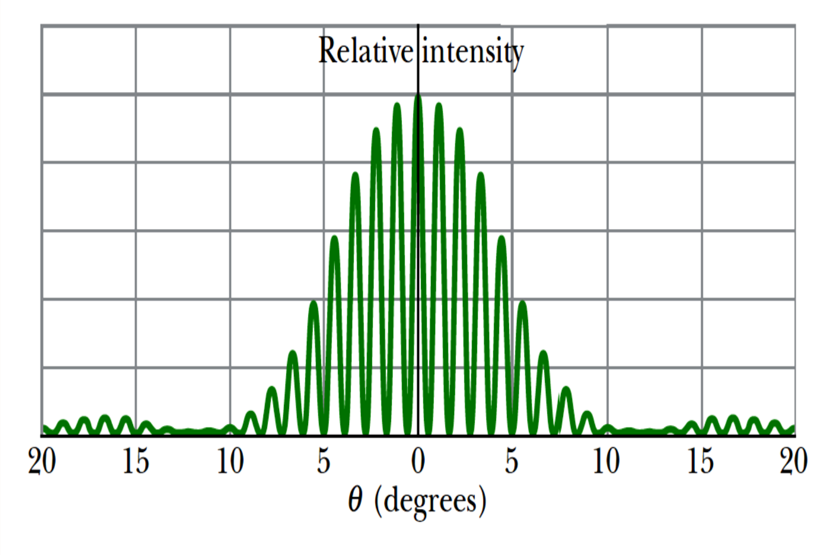
With equations:

$$I_\theta = I_m \cos^2\left(\frac{d\pi}{\lambda}\sin\theta\right) \sin_c^2\left(\frac{a\pi}{\lambda}\sin\theta\right)$$

Interference   Diffraction

If $a \rightarrow 0$ then $sin_c(\ldots) \rightarrow 1$
Formula for double-slit ($a << \lambda$) interference

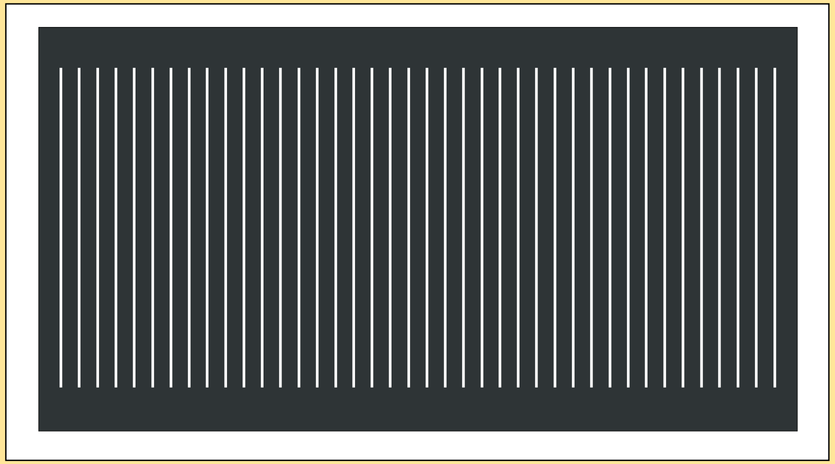
If $d \rightarrow 0$ then $\cos(\ldots) \rightarrow 1$
Formula for single slit ($a > \lambda$) diffraction

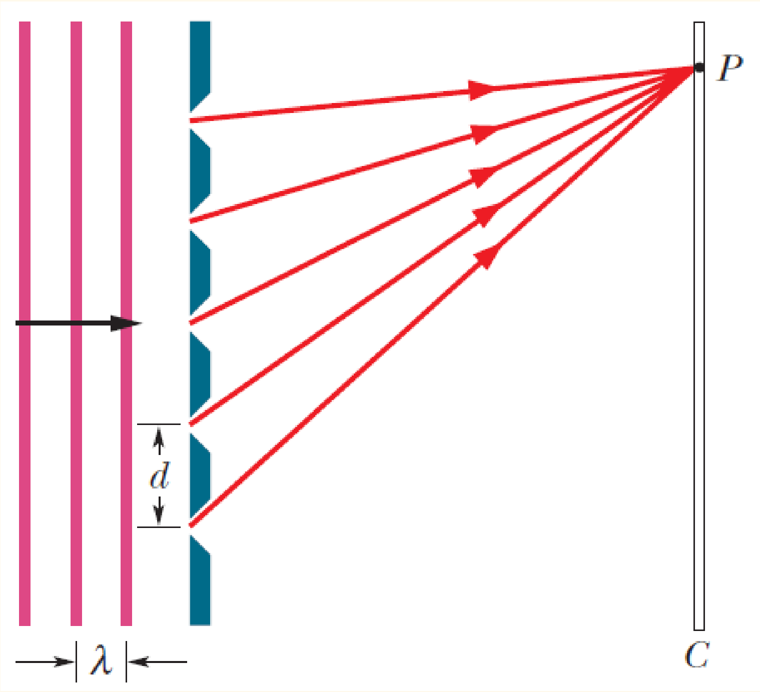


Diffraction pattern of 2 slits (a > λ) spaced by d

DIFFRACTION GRATINGS

Diffraction grating:
Alignment of **N (large number)** slits called **rulings** of **width** $\sim \boldsymbol{\lambda}$ and spaced by $\boldsymbol{d} > \boldsymbol{\lambda}$
Width of the grating: w = Nd





Rulings $a \sim \lambda$ cause **interference**
Spacing $d > \lambda$ causes **diffraction**

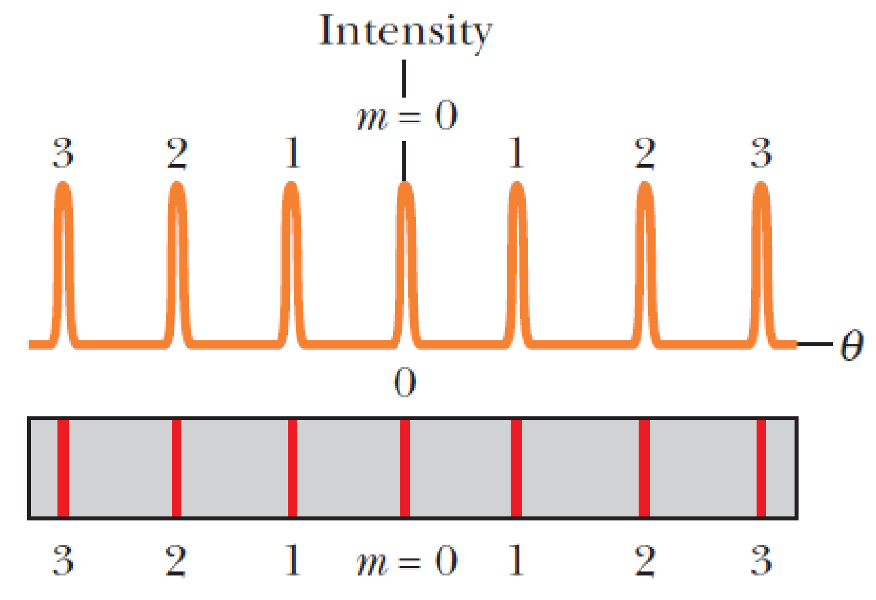
DIFFRACTION GRATINGS

Diffraction grating:

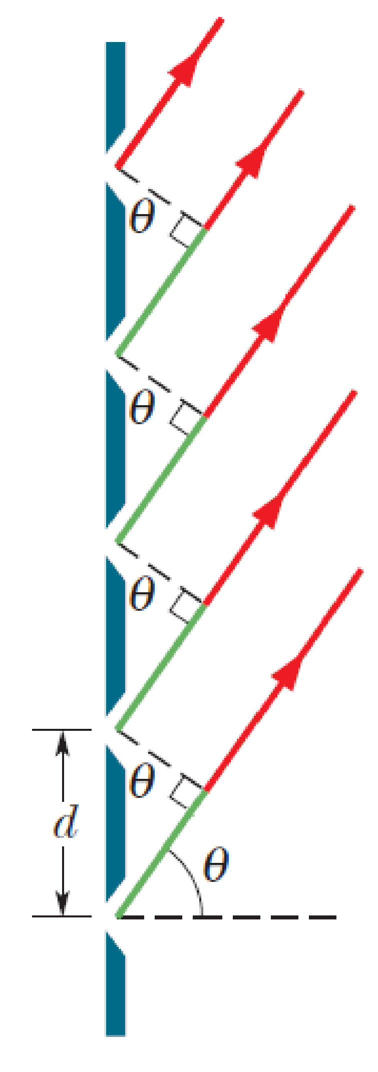
Pattern for a monochromatic incident wave:
Very narrow maxima = **lines regularly spaced**

Difference of path between adjacent rays: $d \sin\theta$

Constructive interference at lines
$\boldsymbol{d} \sin \boldsymbol{\theta} = \boldsymbol{m} \boldsymbol{\lambda}$ (m integer)

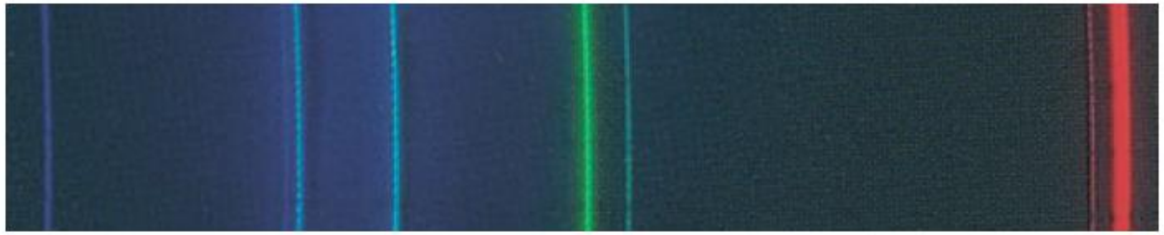


Lines labeled by their order with respect to the central axis (angle θ)



DIFFRACTION GRATINGS

$$d \sin\theta = m\lambda \longrightarrow \theta = \operatorname{asin}\left(m\frac{\lambda}{d}\right)$$



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Experimentally:

- We can measure θ & count m
- d is a known parameter

→ **Determining unknown λ**

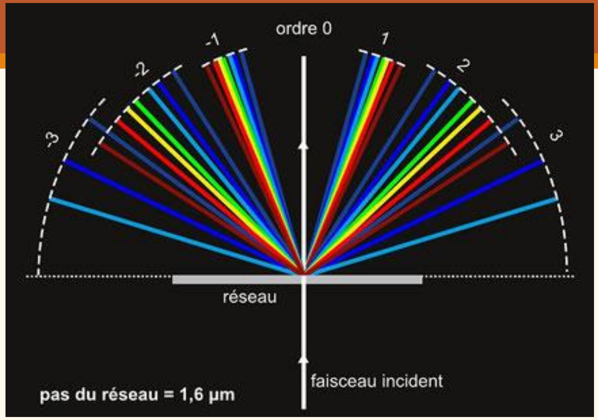
Non-monochromatic incident light:

- Each λ produce a pattern of lines

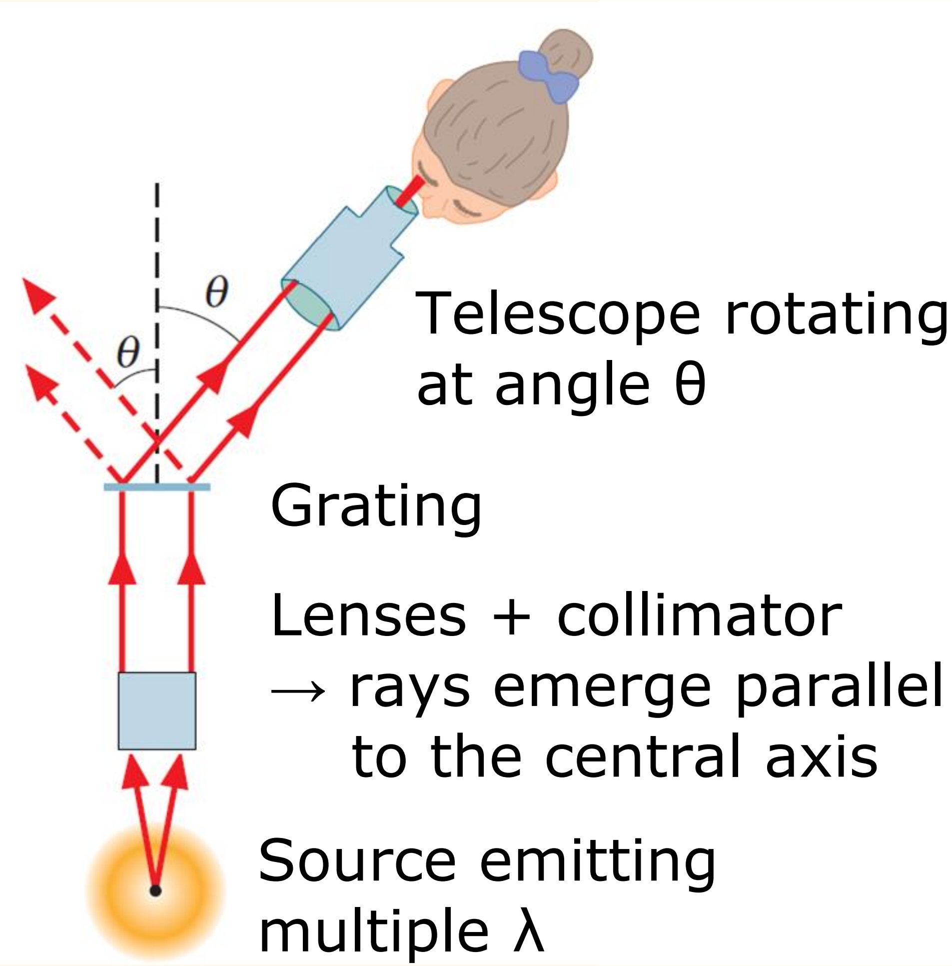
→ **Separating and determining wavelengths**

Composition of an unknown sample emitting light can be determined with a **grating spectroscope**

DIFFRACTION GRATINGS



Grating spectroscope:

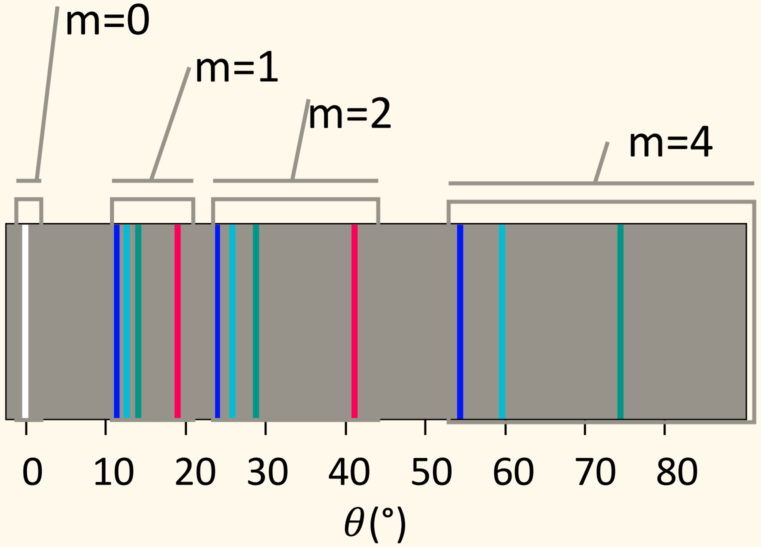


Example: Hydrogen lamp

→ Emits white light

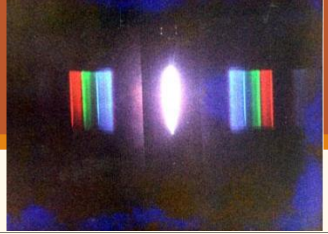
→ Decomposed into multiple λ

→ Emission lines



Notes:

- For $\theta = 0$ lines superposed
- $m = 3$ not show for clarity
- 4th red line cannot be observed ($\theta > 90°$)

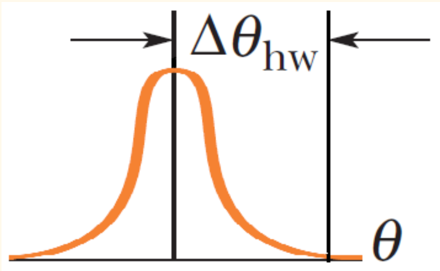


Grating spectroscope:

Lines must be clearly separated to be resolved:

We admit that the **half-width of a line ($\Delta\theta_{hw}$)** is:

$$\Delta\theta_{hw} = \frac{\lambda}{N d \cos\theta}$$



N and d are characteristics of the grating

We define the **dispersion (D)** of the grating as:

$$D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta}$$

If $\Delta\lambda$ <<, D must be large to measure $\Delta\theta \geq \Delta\theta_{hw}$

Grating spectroscope:

$$\frac{m\Delta\lambda}{d\cos\theta} = \frac{\lambda}{N\,d\cos\theta}$$

We define the **resolving power R** between two wavelength λ_1 & λ_2

$$R = \frac{1/2\,(\lambda_1 + \lambda_2)}{\lambda_2 - \lambda_1} = \frac{\lambda_{avg}}{\Delta\lambda}$$

Since two lines are resolved if, at least : $\Delta\theta = \Delta\theta_{hw} = \dfrac{\lambda}{N d\cos\theta}$

Since: $D = \dfrac{\Delta\theta}{\Delta\lambda} = \dfrac{m}{d\cos\theta}$ We have: $\dfrac{\left(\frac{\lambda}{N\,d\cos\theta}\right)}{\Delta\lambda} = \dfrac{m}{d\cos\theta} \longrightarrow mN = \dfrac{\lambda}{\Delta\lambda}$

So $2mN = \dfrac{\lambda_1}{\Delta\lambda} + \dfrac{\lambda_2}{\Delta\lambda} \longrightarrow mN = \dfrac{1/2\,(\lambda_1 + \lambda_2)}{\Delta\lambda} \longrightarrow$ $R = mN$

DIFFRACTION GRATINGS

Grating spectroscope:

Resolving power R:

$$R = \frac{\lambda_{avg}}{\Delta\lambda} = m\,N$$

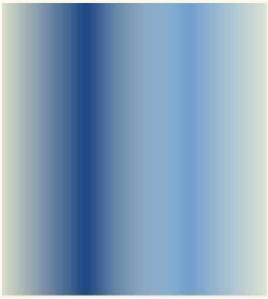
R >> → Narrow lines

Dispersion D:

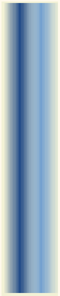
$$D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta}$$

D >> → Spaced lines

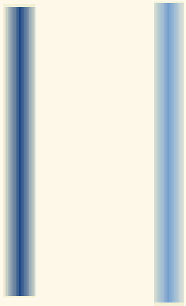
$R <<$ & $D <<$



$R <<$ & $D >>$

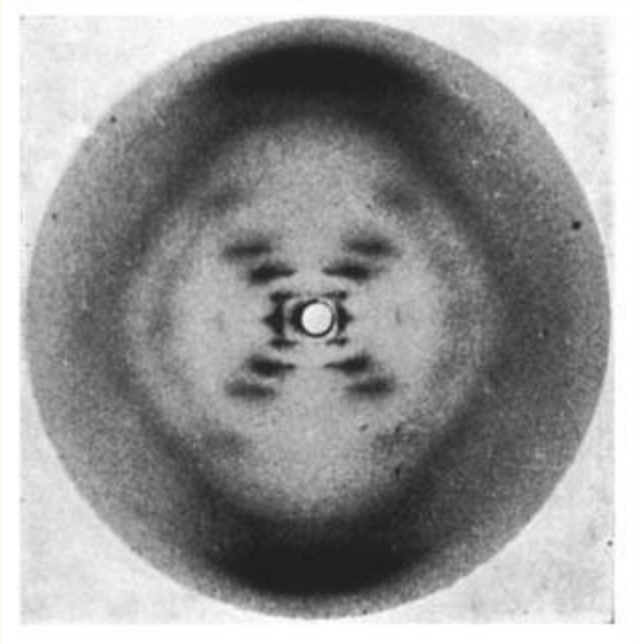


$R >>$ & $D <<$



$R >>$ & $D >>$

Guess what I'm?



X-RAY DIFFRACTION

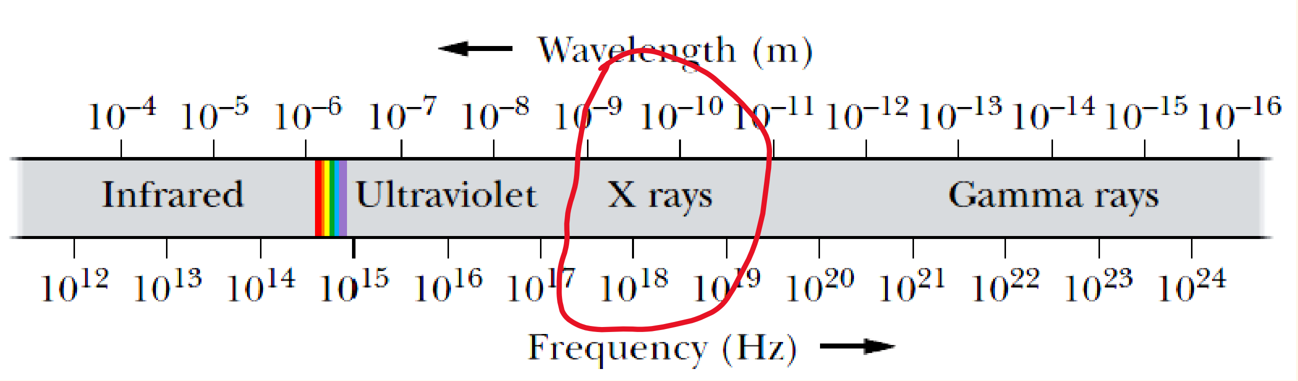
Rosalind Elsie Franklin (25 July 1920 – 16 April 1958) was a British chemist and X-ray crystallographer whose discovered the molecular structures of DNA

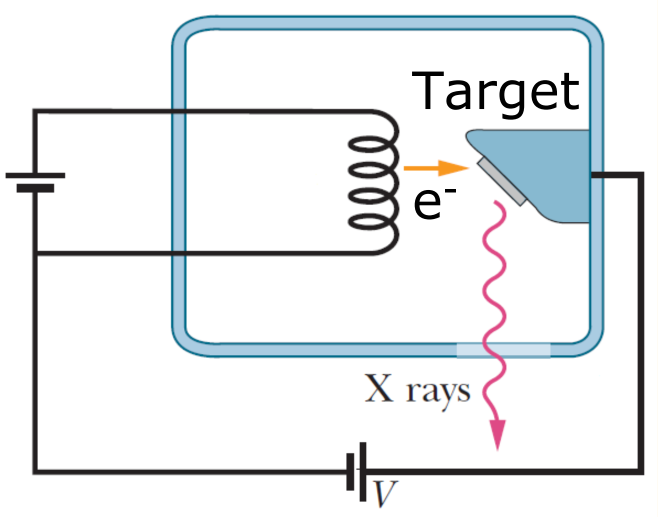
X-RAY: EM waves with λ <<

$\lambda \sim 0.1\, nm\ =\ 1\, \text{Å}$

Atoms in a metal target bombarded by accelerated e⁻ emits X-Rays

/!\ X-rays are high energy radiation
→ must be handled with caution





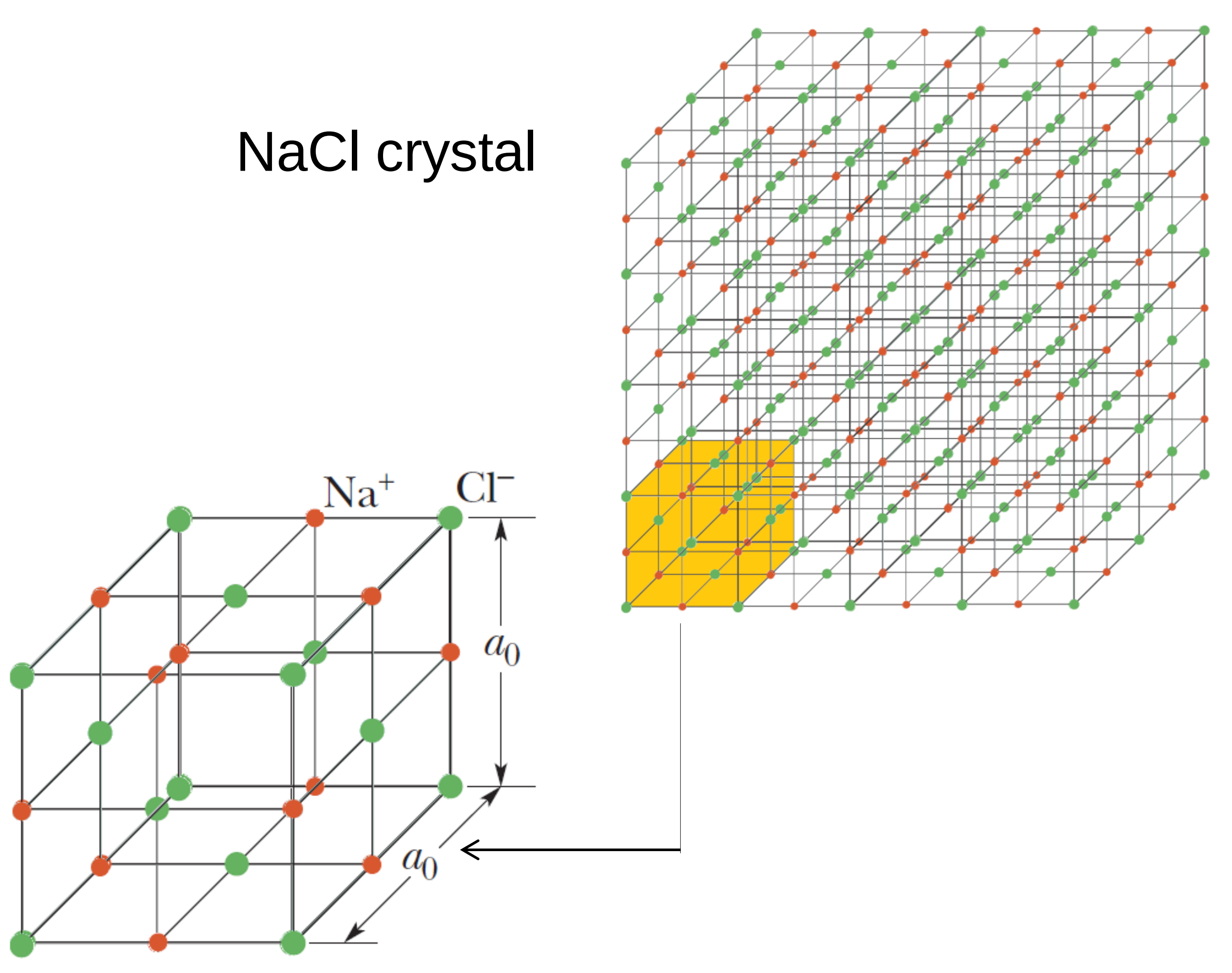
X-RAY: EM waves with λ <<

$\lambda \sim 0.1\, nm \;=\; 1\,\text{Å}$

Crystal = 3d translation of the unit cell

Inter-atomic distances in crystals and atomic radius are small enough to **diffract X-rays**

Crystal = 3d grating for x-rays



NaCl conventional unit cell - Face-centered cubic

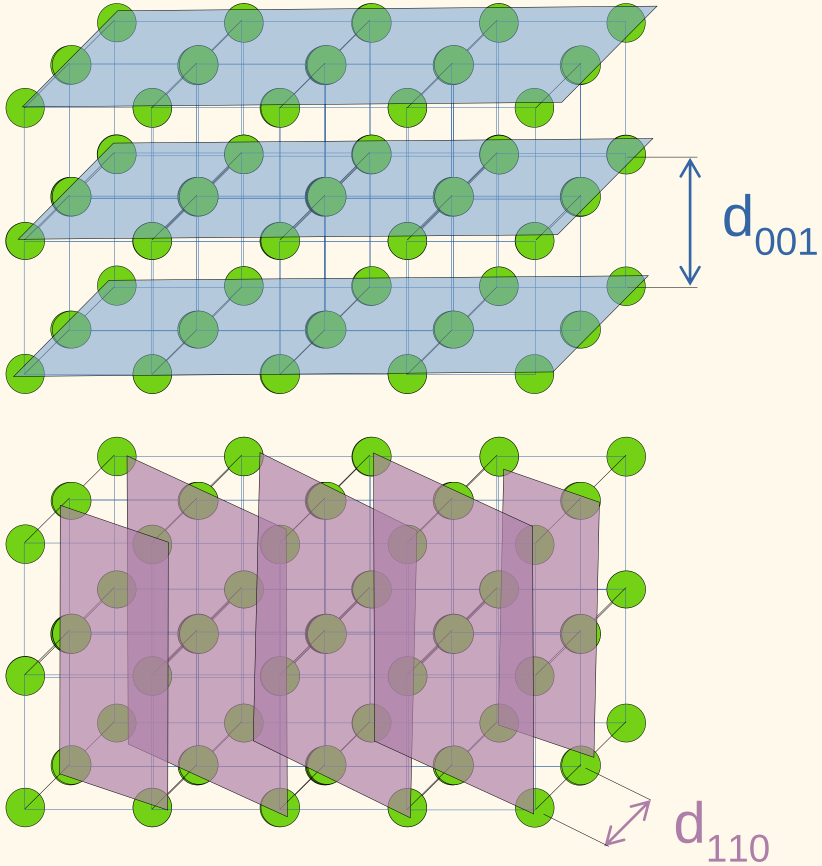
Note:

Previously, we explained how a **spectroscope** with a grating decompose **unknown radiation** to **determine wavelengths**
Grating parameters are known

For X-Ray diffraction by crystals **the situation is reversed**:
An **unknown crystal** diffracts radiation to **characterize the crystal**
λ **is known**

We obtain information on the **geometry of the unit cell**, its **dimensions** and atomic **composition** from the diffraction pattern

X-rays **scattered** by the crystal structure → Redirected in all directions
Scattered waves interfere destructively or **constructively**



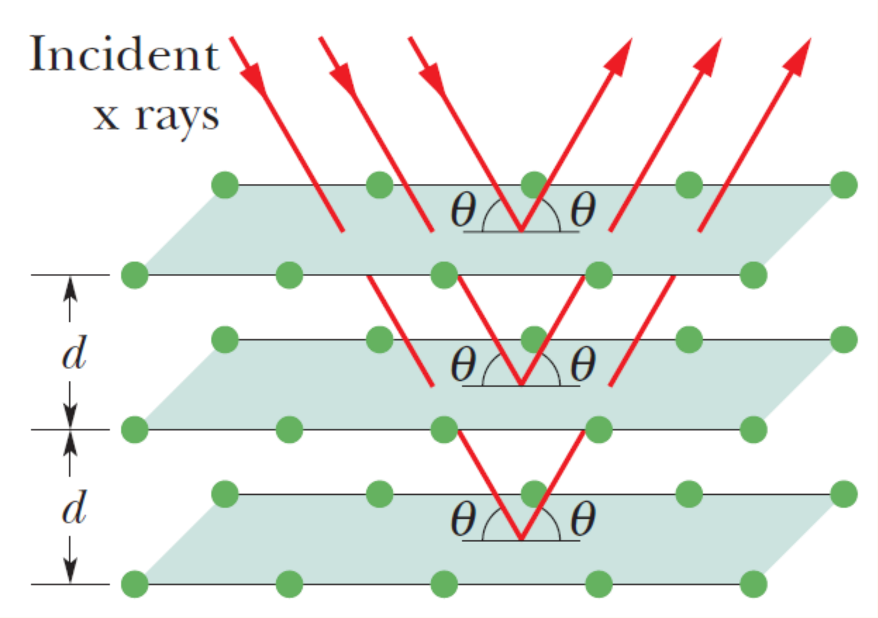
↓

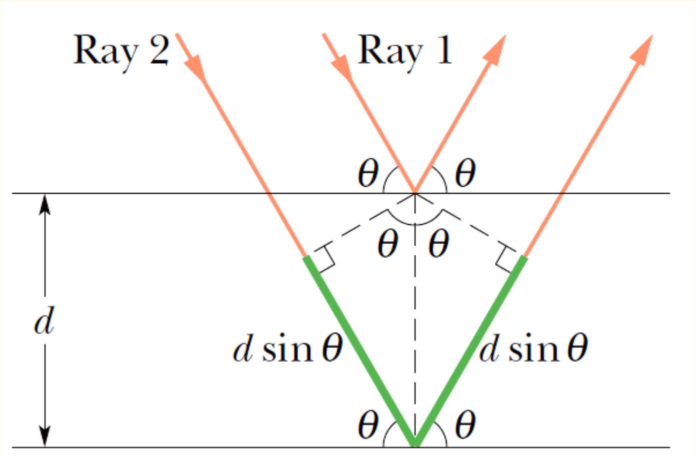
Intensity maxima

As if X-Rays were **reflected** at angle θ
by a family of **parallel imaginary planes**
→ Identical (same atoms at the same positions)
→ spaced by a distance d

Example: two families of planes with
different interplanar distances

Intensity maxima when the reflected waves are in phase





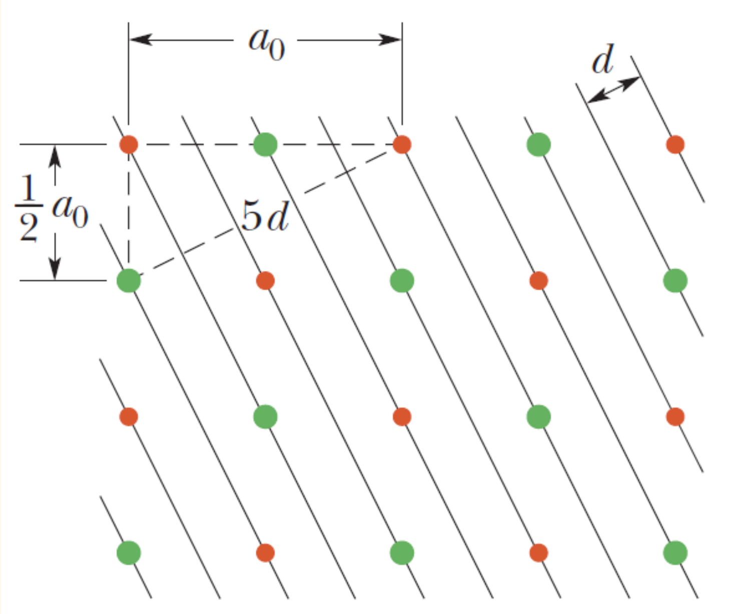
Note: θ with respect to the plane here not the normal

Difference of path length
2 d sinθ

Bragg's law for intensity maxima: $2\,d\,sin\theta = m\lambda$

Intensity maxima when the reflected waves are in phase

Bragg's law for intensity maxima: $\mathbf{2\,d\,\boldsymbol{sin\theta} = \boldsymbol{m\lambda}}$



Example of NaCl:
Diffraction pattern indicates face-centered cubic
Maxima of order m at angle θ

We have: $(5d)^2 = \left(\frac{a_0}{2}\right)^2 + a_0^2 \longrightarrow a_0 = d\sqrt{20}$

If I know $\boldsymbol{\lambda}$ and the angle $\boldsymbol{\theta}$ formed on the screen between the O^th and the 1^rst order (m=1), I can know the interatomic distance

KEY POINTS

Single-slit diffraction $I_\theta = I_m \sin_c^2\left(\frac{a\pi}{\lambda}\sin\theta\right)$

Diffraction by a circular aperture

Rayleigh's criteria $\theta_R = 1,22\,\frac{\lambda}{d}$

Double-slit diffraction $I_\theta = I_m \cos^2\left(\frac{d\pi}{\lambda}\sin\theta\right)\sin_c^2\left(\frac{a\pi}{\lambda}\sin\theta\right)$

Diffraction gratings $d\sin\theta = m\lambda$ $R = \frac{\lambda_{avg}}{\Delta\lambda} = m\,N$ $D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta}$

Diffraction of X-rays by crystals and Bragg's law $2\,d\sin\theta = m\lambda$

https://www.youtube.com/watch?v=NazBRcMDOOo

Diffraction interference patterns with phasor diagrams

Physics Videos by Eugene Khutoryansky

973 k abonnés

Abonné

https://youtu.be/cep6eECGtw4

Why Does Light REALLY Bend?

The Science Asylum

638 k abonnés

Re