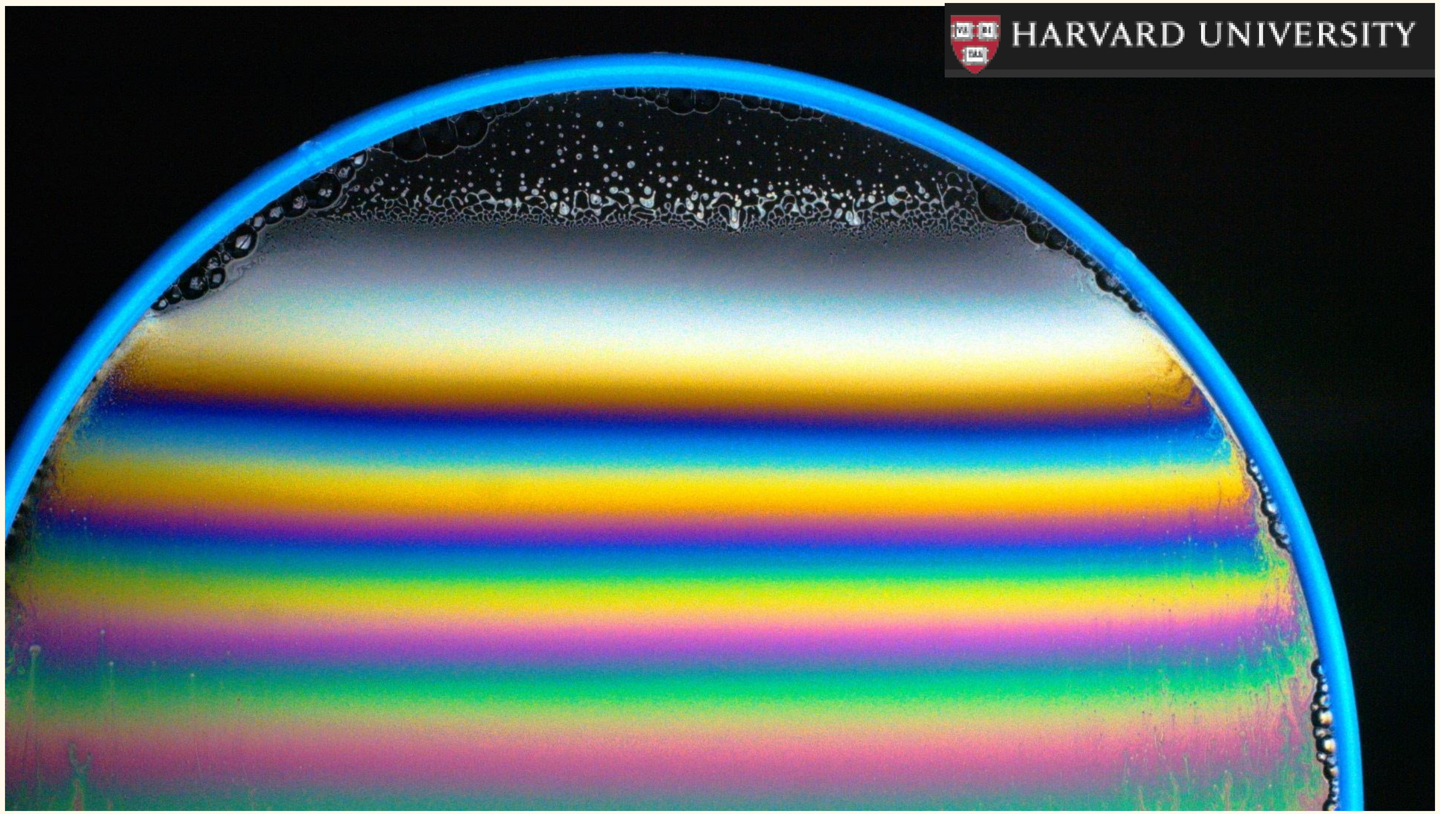
Interference

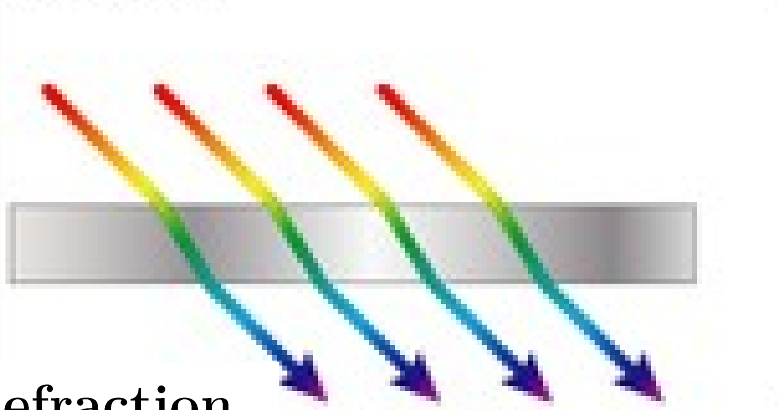


From https://youtu.be/4I34jA1fDp4 and https://youtu.be/QyeN1T1VyF8

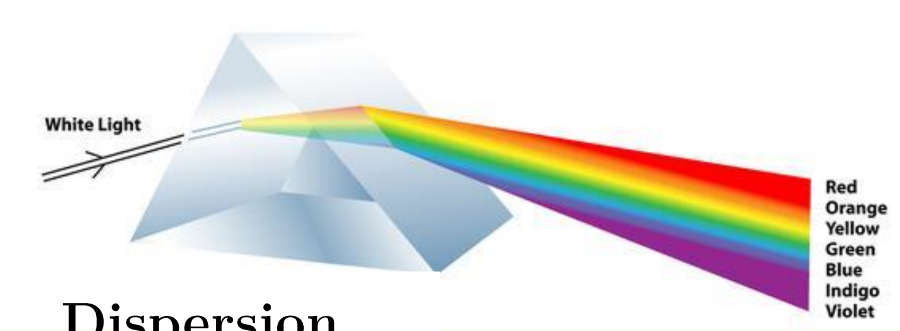
- Electromagnetic Oscillations & Alternating Current
- Maxwell’s Equations & Magnetism of Matter
- Electromagnetic Waves
- Images
- **Interference**
- Diffraction

Reflexion

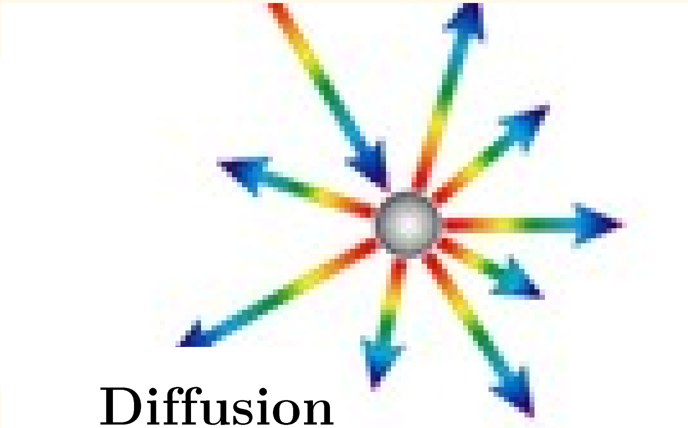
Absorption



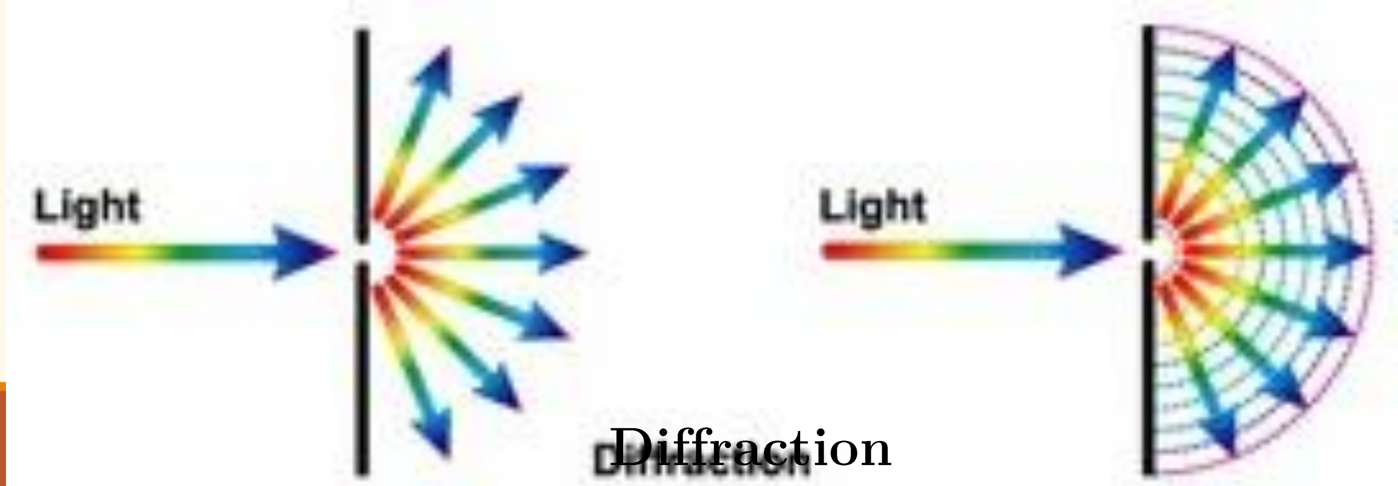
Refraction



Dispersion



Diffusion

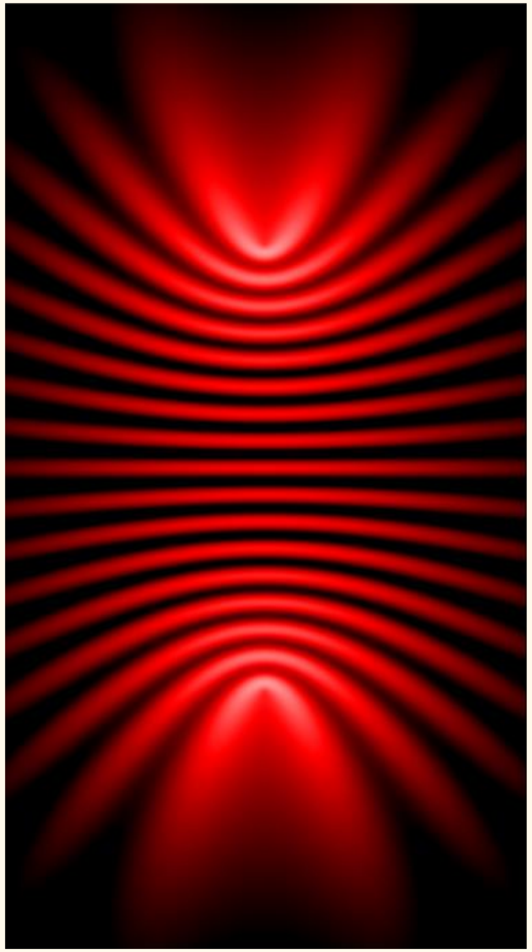


Diffraction

INTERFERENCE

Textbook: Chapter 35

- LIGHT AS A WAVE
- YOUNG’S INTERFERENCE EXPERIMENT
- INTERFERENCE & DOUBLE-SLIT INTENSITY
- INTERFERENCE FROM THIN FILMS
- MICHELSON’S INTERFEROMETER



Note on this chapter:

In the previous chapters, we represented light by rays and postulated:

"There is **no interaction between rays**" - **This is false in many cases**

→ But geometrical optics are still useful to design optical systems

Light is an EM wave $E = E_m \sin(kx - \omega t)$

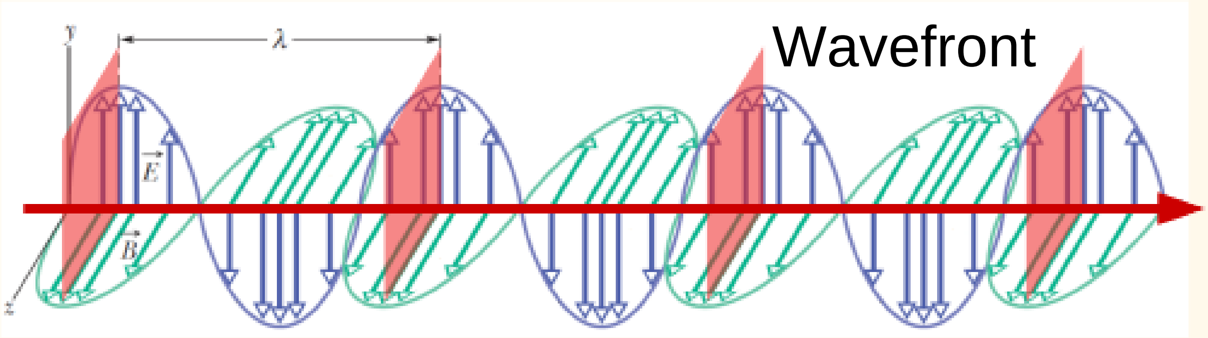
2 incidents wave of E field E_1 & E_2 of the same ω and polarization

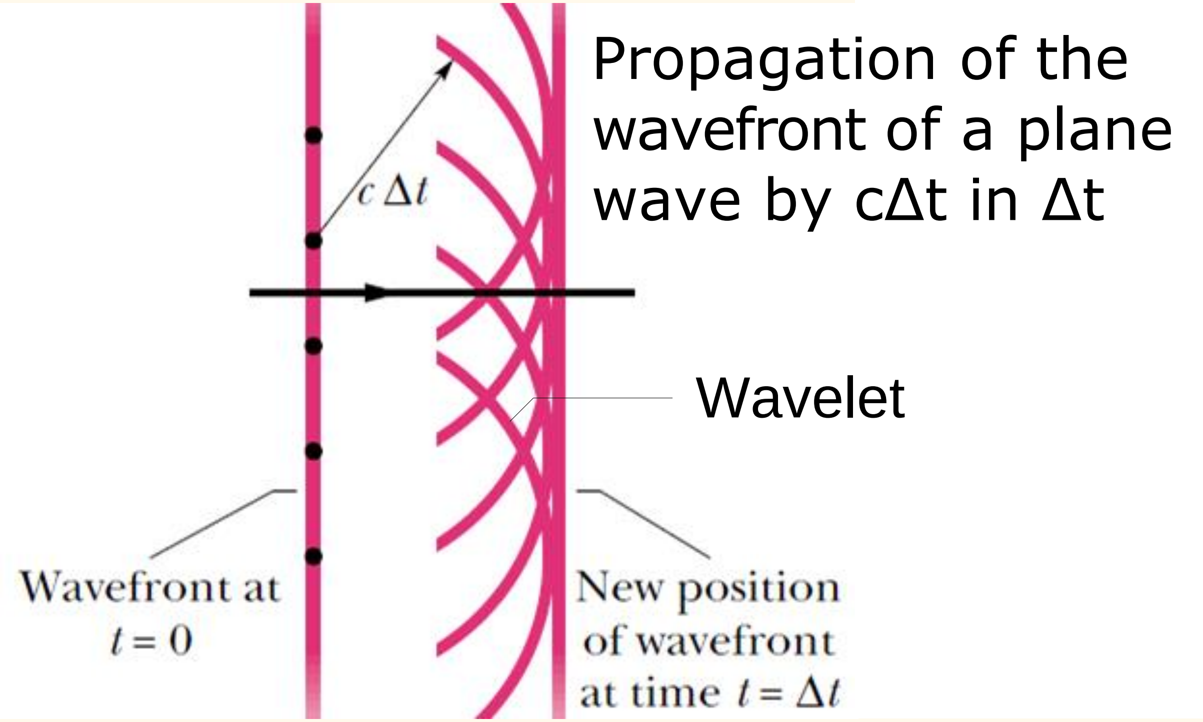
$$\begin{aligned} E_1 &= E_{1m} \sin(kx - \omega t + \phi_1) \\ &+ \\ E_2 &= E_{2m} \sin(kx - \omega t + \phi_2) \end{aligned} \quad = ?$$

Maxwell:
Light = E and B fields
Mutual induction → Propagation

Huygens' Principle for propagation

"All points on a wavefront serve a ***point sources of spherical secondary wavelets****. After a time t, the new position of the wavefront will be that of a surface tangent to these secondary wavelets."*





Refraction with Huygens' Principle

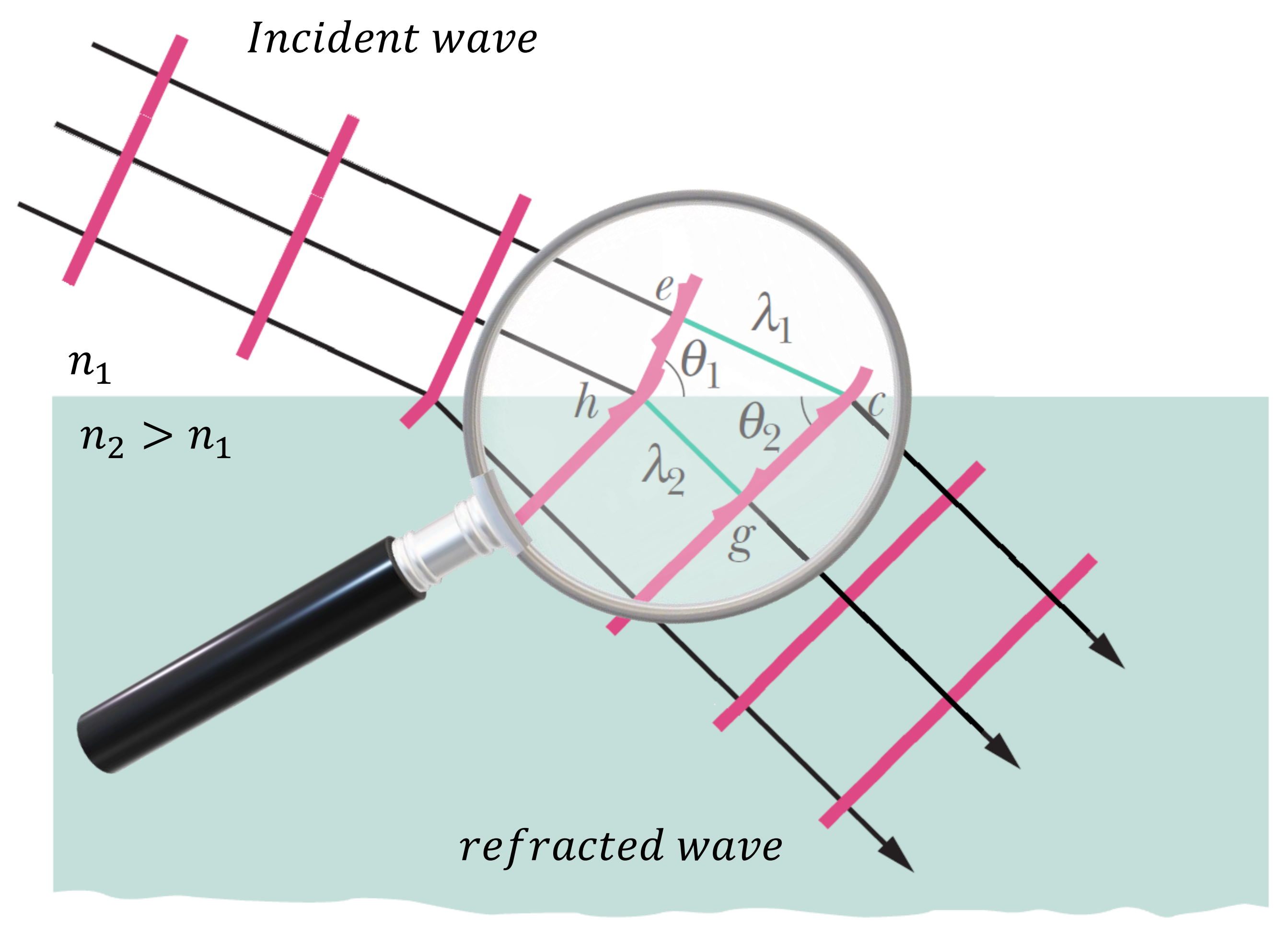
We represent **wavefront spaced by 1 wavelength**
λ_1 & λ_2 in mediums 1 & 2

Speed of light:

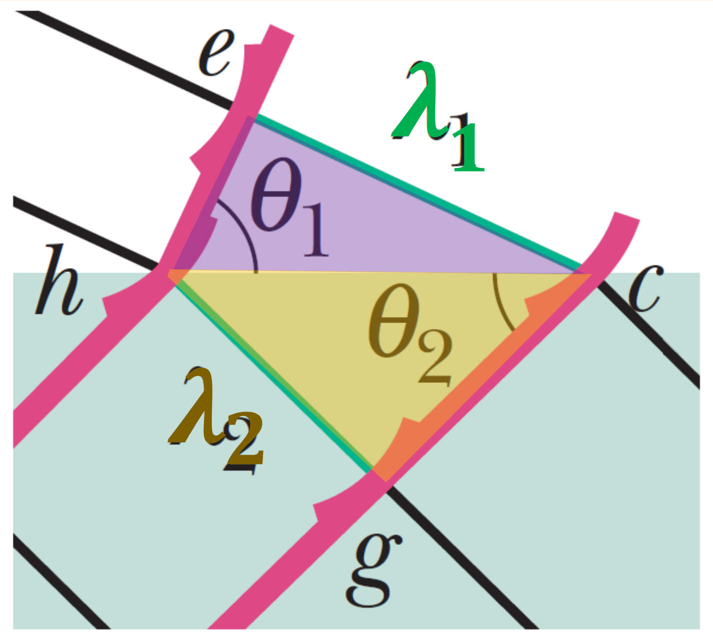
v_1 & v_2 in mediums 1 & 2
We assume $v_1 > v_2$

θ_1: Angle of incidence

θ_2: Angle of refraction



Refraction with Huygens' Principle



$\overline{ec} = \lambda_1$

$\rightarrow$ light travels from e to c in $\Delta t = \lambda_1 / v_1$

$\overline{hg} = \lambda_2$

$\rightarrow$ light travels from h to g in $\Delta t = \lambda_2 v_2$

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

We define refractive indexes as $n = \boldsymbol{c} / v$

$$\frac{\lambda_1}{\lambda_2} = \frac{\boldsymbol{c} / n_1}{\boldsymbol{c} / n_2} = \frac{n_2}{n_1}$$

In $\widehat{hec}$: $\sin(\theta_1) = \overline{ec} / \overline{hc} = \lambda_1 / \overline{hc}$

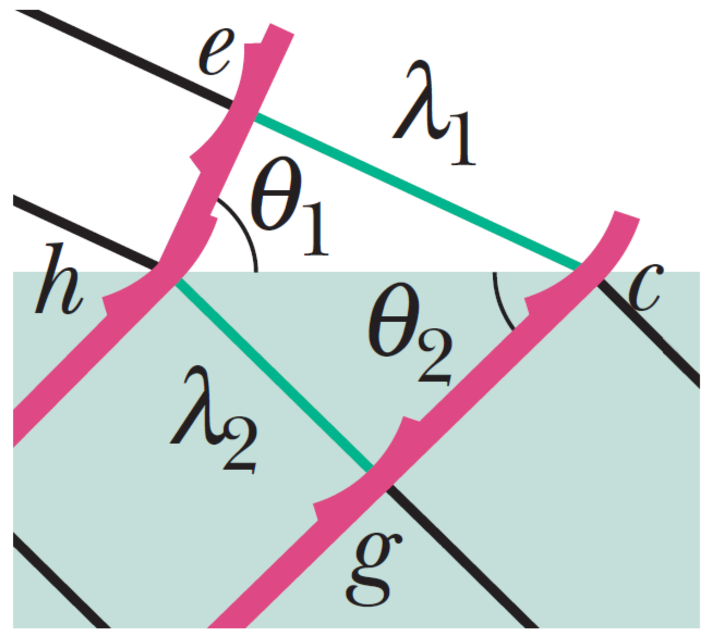
$\rightarrow \overline{hc} = \lambda_1 / \sin(\theta_1)$

In $\widehat{hgc}$: $\sin(\theta_2) = hg / hc = \lambda_2 / hc$

$\rightarrow hc = \lambda_2 / \sin(\theta_2)$

$$\frac{\lambda_1}{\lambda_2} = \frac{\sin(\theta_1)}{\sin(\theta_2)}$$

Refraction with Huygens' Principle

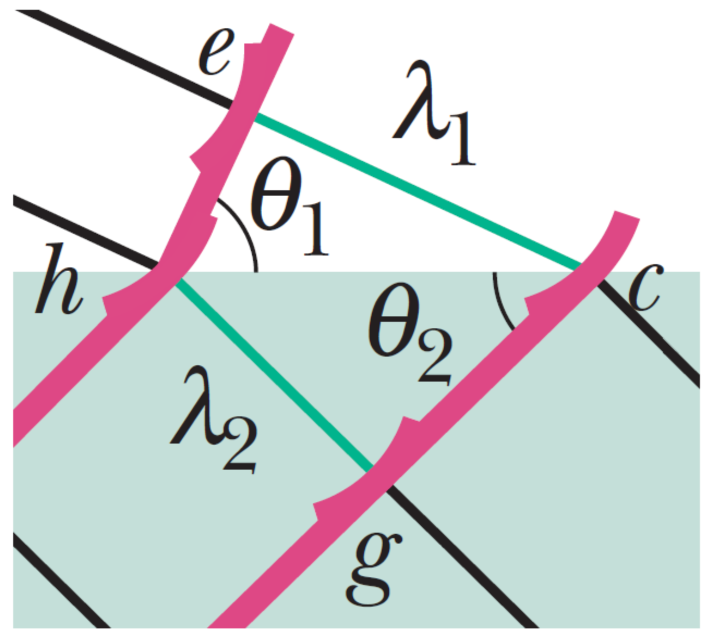


$$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{\sin(\theta_1)}{\sin(\theta_2)}$$

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Note on wavelength



Change of medium
→ Change of wavelength
$\lambda_1 \neq \lambda_2$

For monochromatic light going from vacuum to a medium of index n:

$$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} \longrightarrow \frac{\lambda}{\lambda_n} = \frac{n}{1} \longrightarrow \boxed{\lambda_n = \frac{\lambda}{n}}$$

λ : *wavelength* in *vacuum*

λ_n : *wavelength* in a *medium of index* n

Thus, frequencies are: $\boxed{f = \frac{c}{\lambda} \text{ and } f_n = \frac{v}{\lambda_n}}$

f : *frequency in vacuum*

f_n : *frequency in a medium of index n*

v : *speed of light in a medium of index n*

Note on wavelength

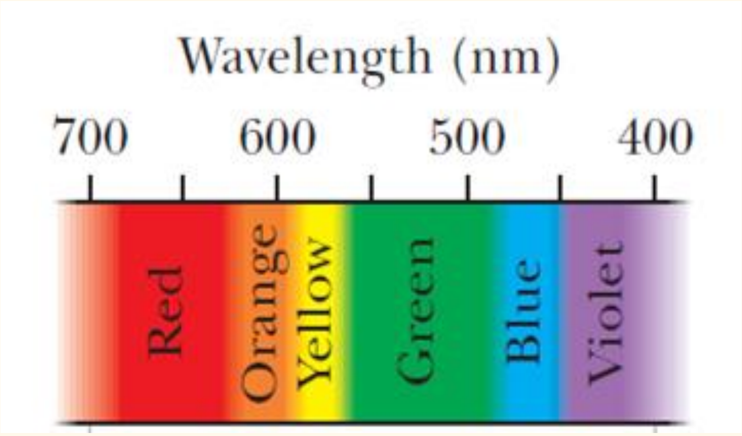
$$\lambda_n = \frac{\lambda}{n}$$

$$f = \frac{c}{\lambda} \quad \text{and} \quad f_n = \frac{v}{\lambda_n}$$

Thus, we have:

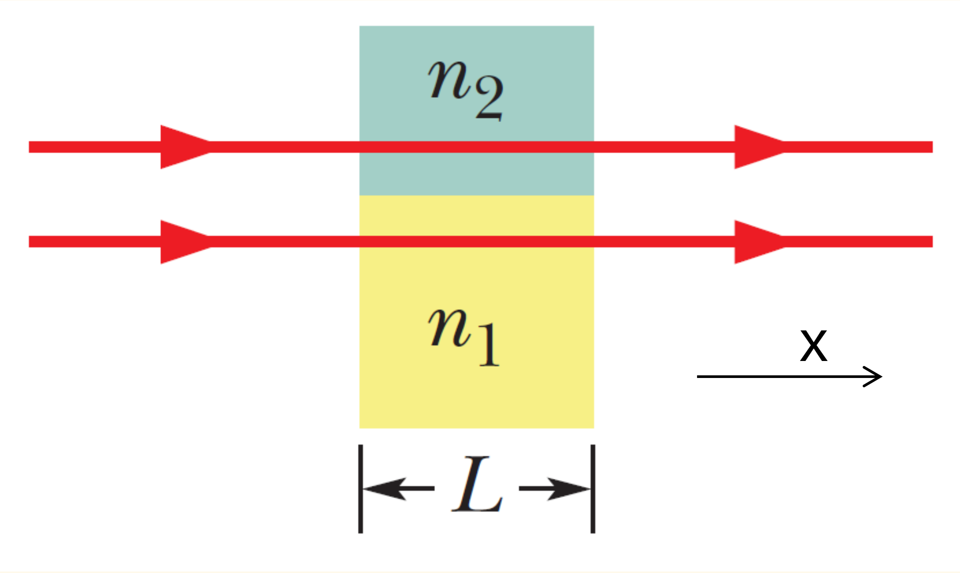
$$\boldsymbol{f_n} = \frac{v}{\lambda_n} = \frac{c\ /\ n}{\lambda\ /\ n} = \frac{c}{\lambda} = \boldsymbol{f}$$

Wavelength change →
Frequency does not change →
(Always the same than in vacuum)



In the visible spectrum, color of light change due to refraction ?

No because $\boldsymbol{f}$ determines color



$$E = E_m \sin(kx - \omega t)$$

$$E = E_m \sin\left(\frac{2\pi}{\lambda_n} x - \omega t\right)$$

Note: here we assume that the waves have the same amplitude

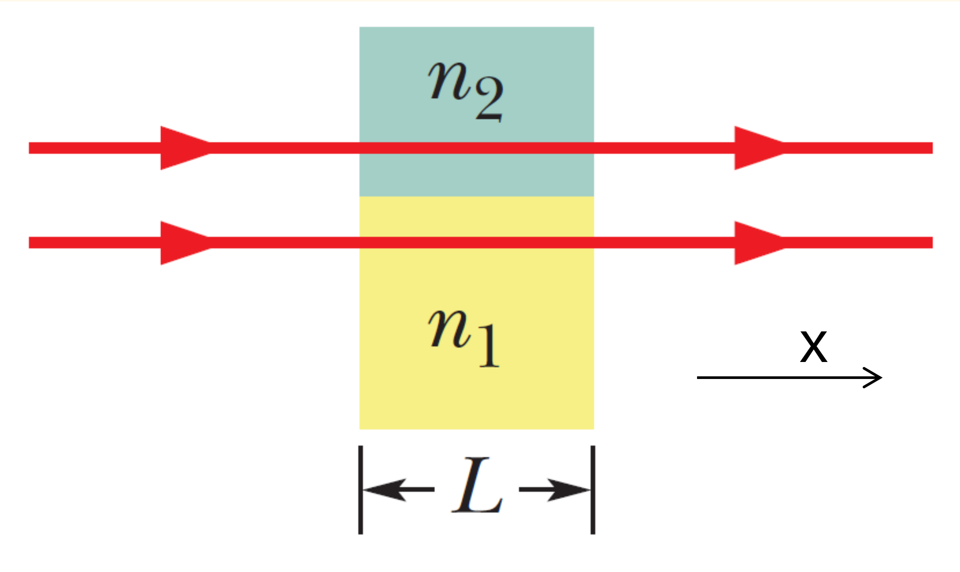
We consider two light waves with the same frequency that travel a distance L in different mediums

Assumed in phase before

→ Not the same wavelength in mediums 1 & 2

Not in phase after

What happens when they reach a common point ?



Qualitatively: we look for the ratios $N_{1,\,2}$ of L over the wavelength in mediums 1 and 2

$$N_1 = \frac{L}{\lambda_{n1}} = \frac{L\,n_1}{\lambda} \quad \text{and} \quad N_2 = \frac{L}{\lambda_{n2}} = \frac{L\,n_2}{\lambda}$$

Thus, $$N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1)$$

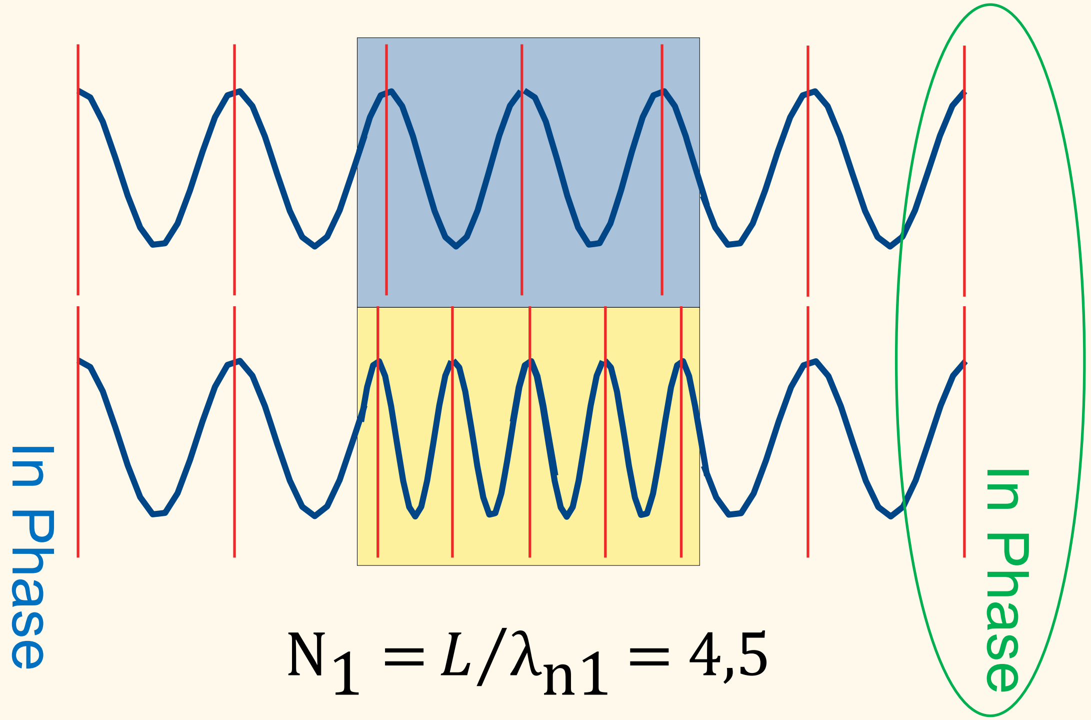
N_1 and N_2: *"the number of wavefronts in mediums 1 and 2"* **(not integers)**

If $N_2 - N_1 = \mathbf{0\ [1]}$ → Phase difference $= 0\ [2\pi]$ Electric fields will add
→ **constructive interference**

If $N_2 - N_1 = \mathbf{1/2\ [1]}$ → Phase difference $= \pi/2\ [2\pi]$ Electric fields will subtract
→ **destructive interference**

Represented schematically as an example:

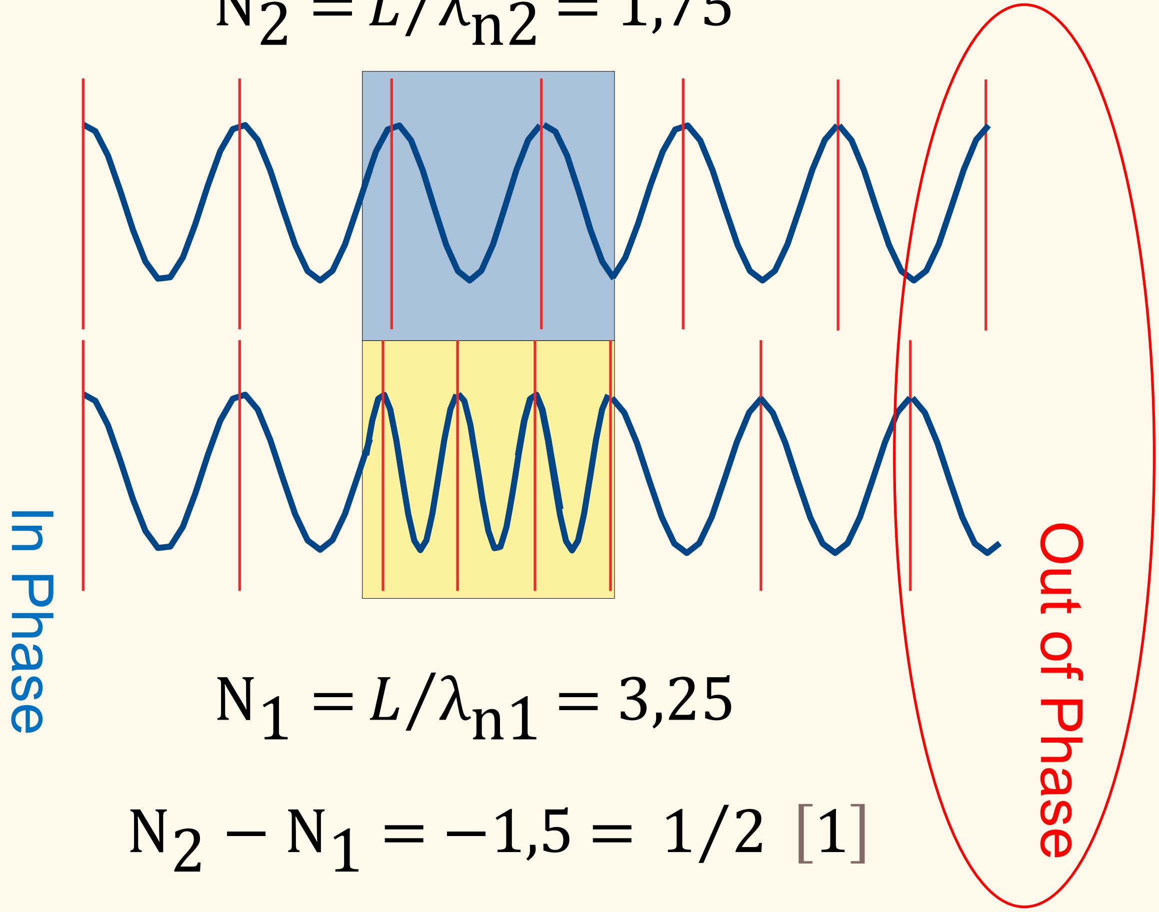
$N_2 = L/\lambda_{n2} = 2{,}5$



$N_1 = L/\lambda_{n1} = 4{,}5$

$N_2 - N_1 = -2 = 0\ [1]$

$N_2 = L/\lambda_{n2} = 1{,}75$

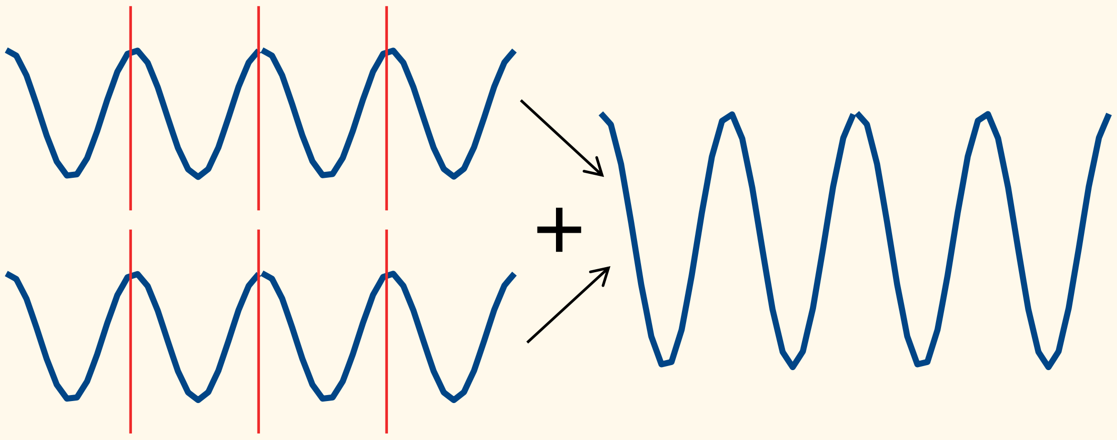


$N_1 = L/\lambda_{n1} = 3{,}25$

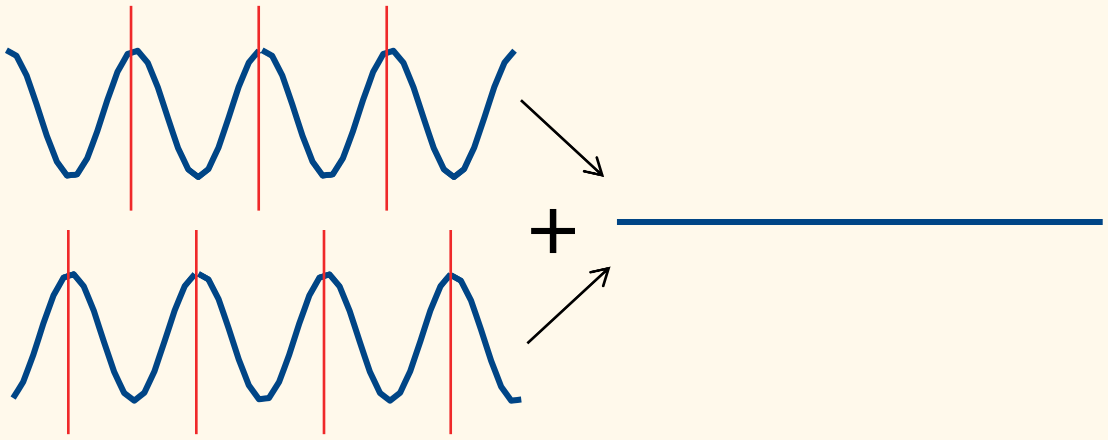
$N_2 - N_1 = -1{,}5 = 1/2\ [1]$

LIGHT AS A WAVE

Back in air after the mediums 1 and 2:



In Phase **constructive interference**



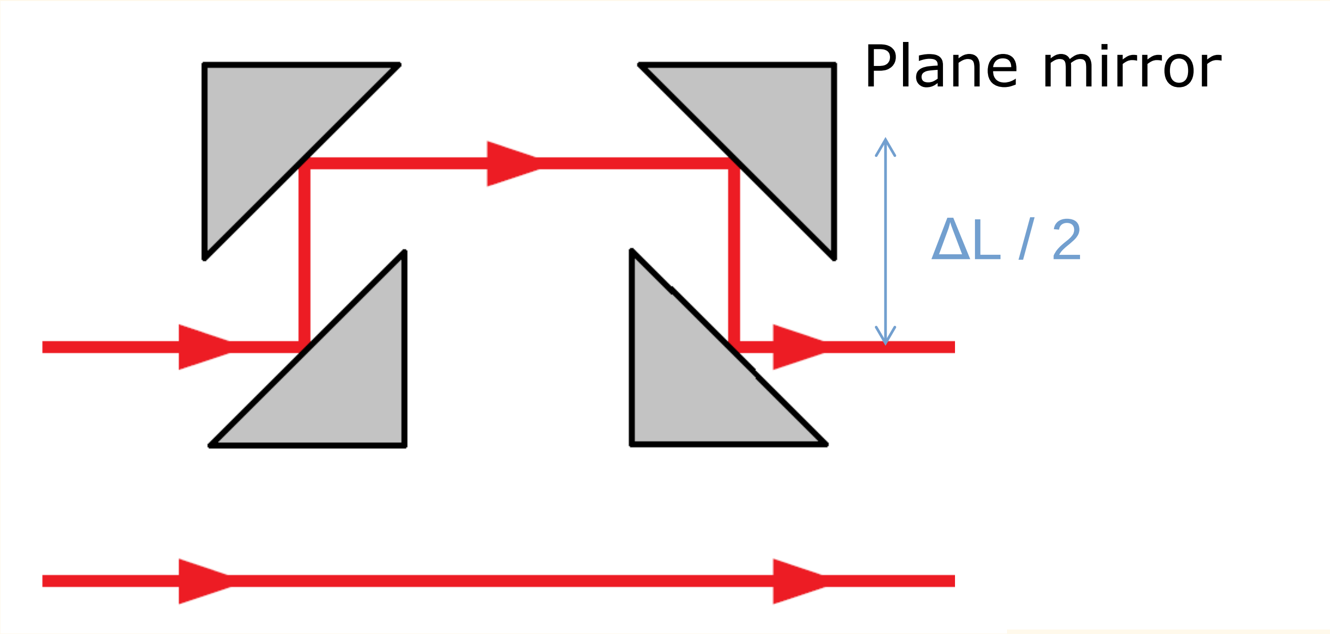
Out of Phase **destructive interference**

If $N_2 - N_1 = \mathbf{0}\ [\mathbf{1}]$ → Phase difference = $0\ [2\pi]$ Electric fields will add
→ **constructive interference**

If $N_2 - N_1 = \mathbf{1/2}\ [\mathbf{1}]$ → Phase difference = $\pi/2\ [2\pi]$ Electric fields will subtract
→ **destructive interference**

Light with the same initial phase that have **propagated in different mediums** will **interfere** when they reach a common point
→ Their phase is now different

The same is true if **path length is different**



$$\frac{\Delta L}{\lambda} = 0, 1, 2, 3\ldots$$

→ Constructive interference

$$\frac{\Delta L}{\lambda} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}\ldots$$

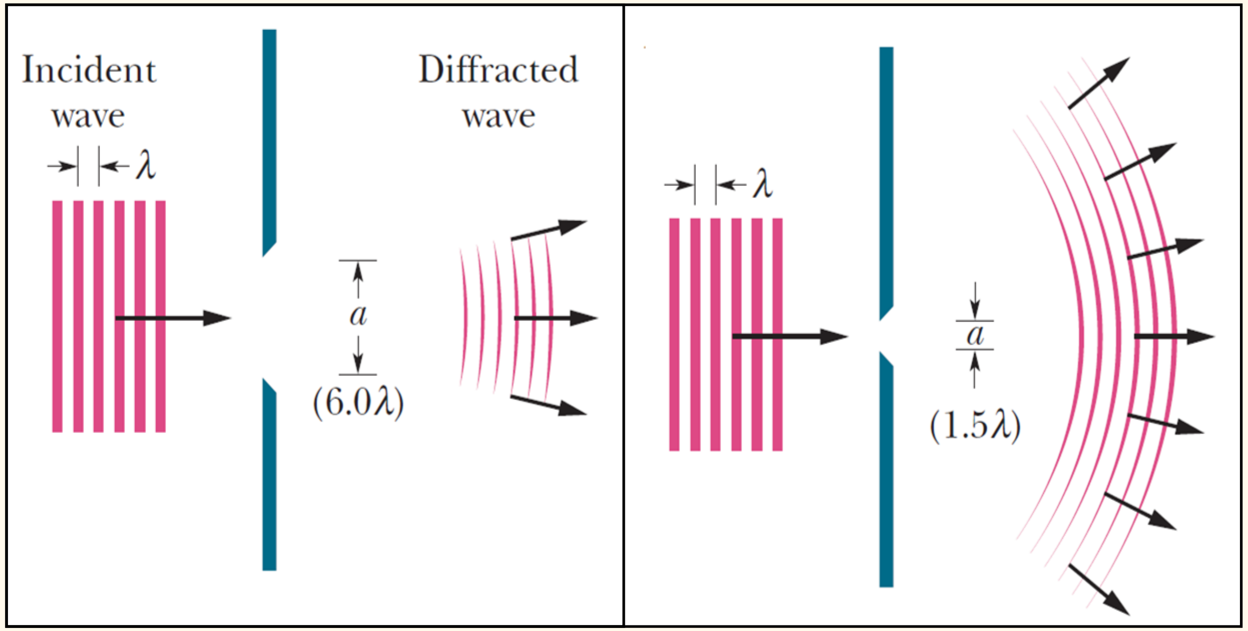
→ Destructive interference

YOUNG'S INTERFERENCE EXPERIMENT

Note on diffraction
→ Topic of the next chapter but we need some concepts now

Consequence of Huygens' Principle
→ When a plane wave encounters a slit, its **width (a)** being of the **order of magnitude of λ**, (a ~ λ) light spreads

Small d → Large spreading



Practically, we cannot have a beam so narrow that is width is comparable to λ
Limitation not described by geometrical optics

YOUNG'S INTERFERENCE EXPERIMENT

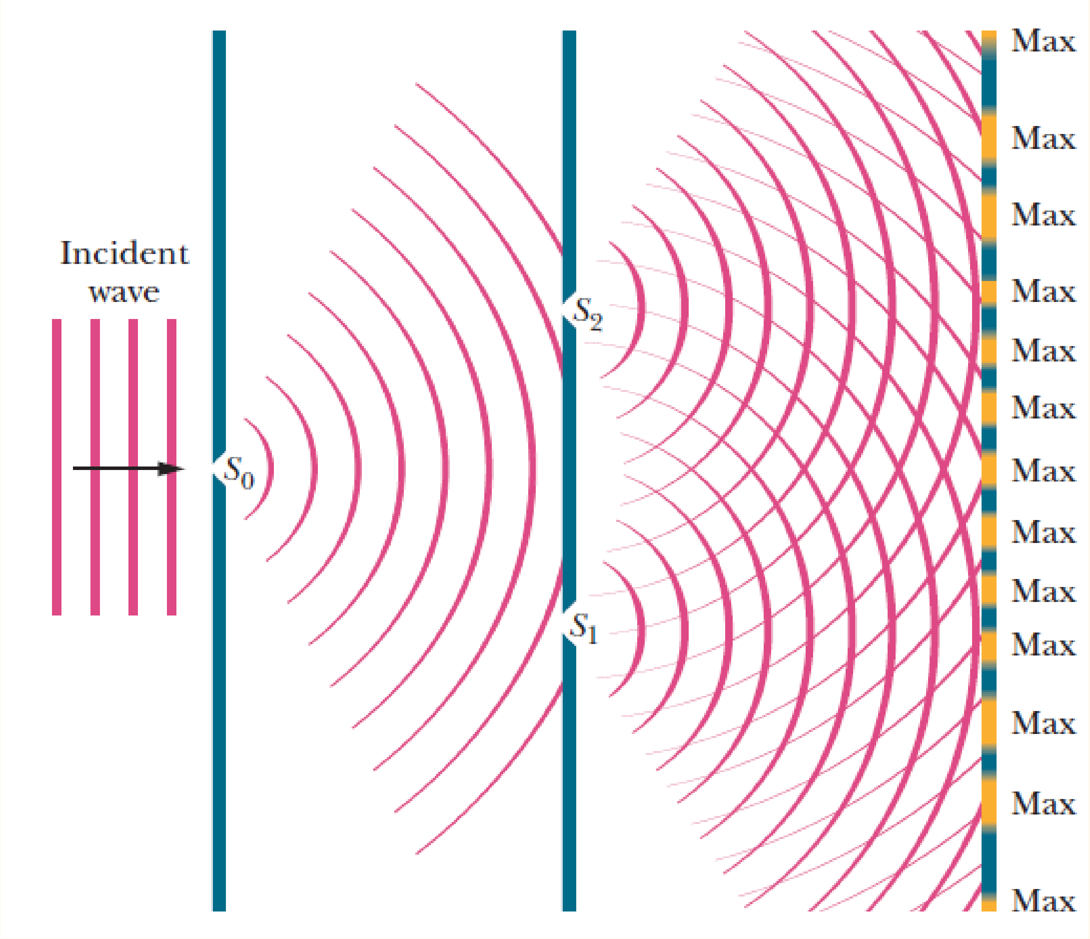
Young's experiment:

An incident plane wave is **diffracted by a first slit**.

The diffracted wave is again **diffracted by two slits**.

Interference pattern is imaged on a screen

Max: **Bright fringes**
Min: **Dark fringes**



Max: Bright fringes – Min: Dark fringes

YOUNG'S INTERFERENCE EXPERIMENT

https://www.youtube.com/@veritasium

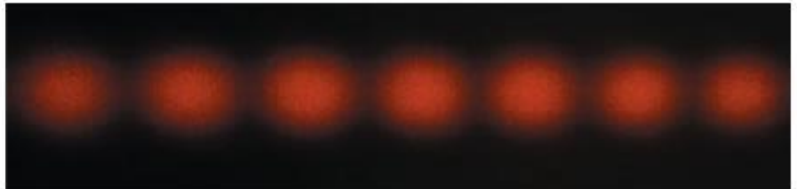
YOUNG'S INTERFERENCE EXPERIMENT

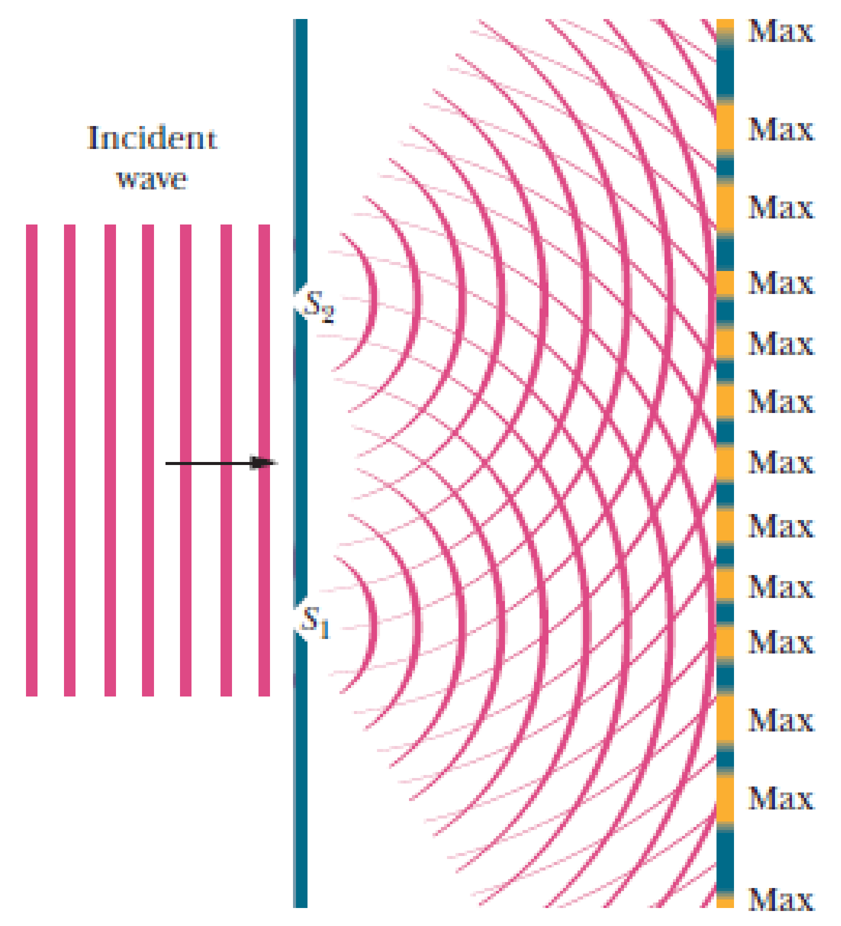
Modern double-slit experiment:

An incident plane wave is **diffracted by two slits**.

Interference pattern is imaged on a screen

Max: **Bright fringes**
Min: **Dark fringes**


Courtesy Jearl Walker



Next → Prediction of the pattern

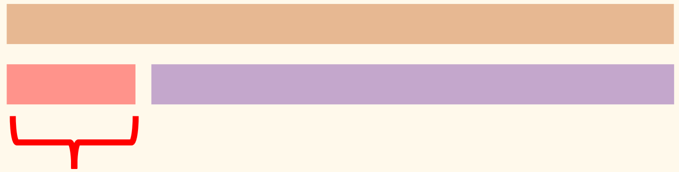
Incident monochromatic plane wave of wavelength λ

Diffraction at points S_1 & S_2 spaced by d/2 from the central axis

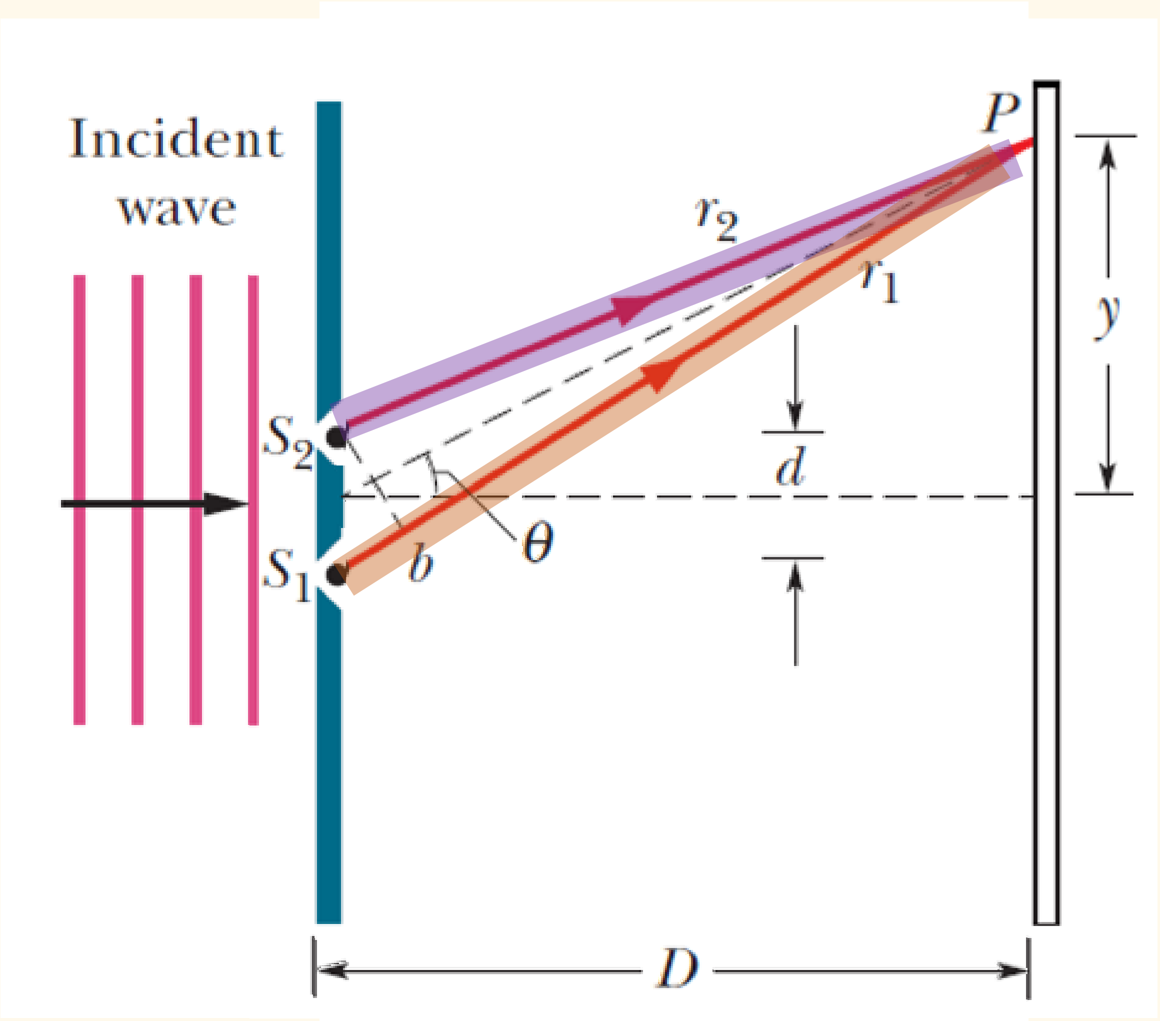
We look the intensity at point P on the screen at distance D and at y from the central axis

θ is the angle from the central axis to P

From the spherical wavefronts we define the rays r_1 and r_2



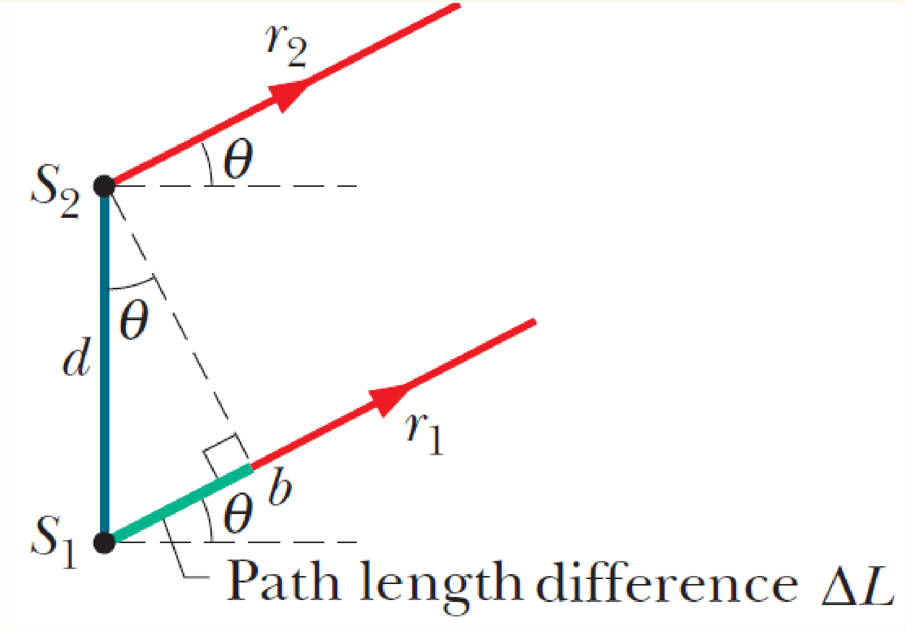
Different path
→ Difference of phase

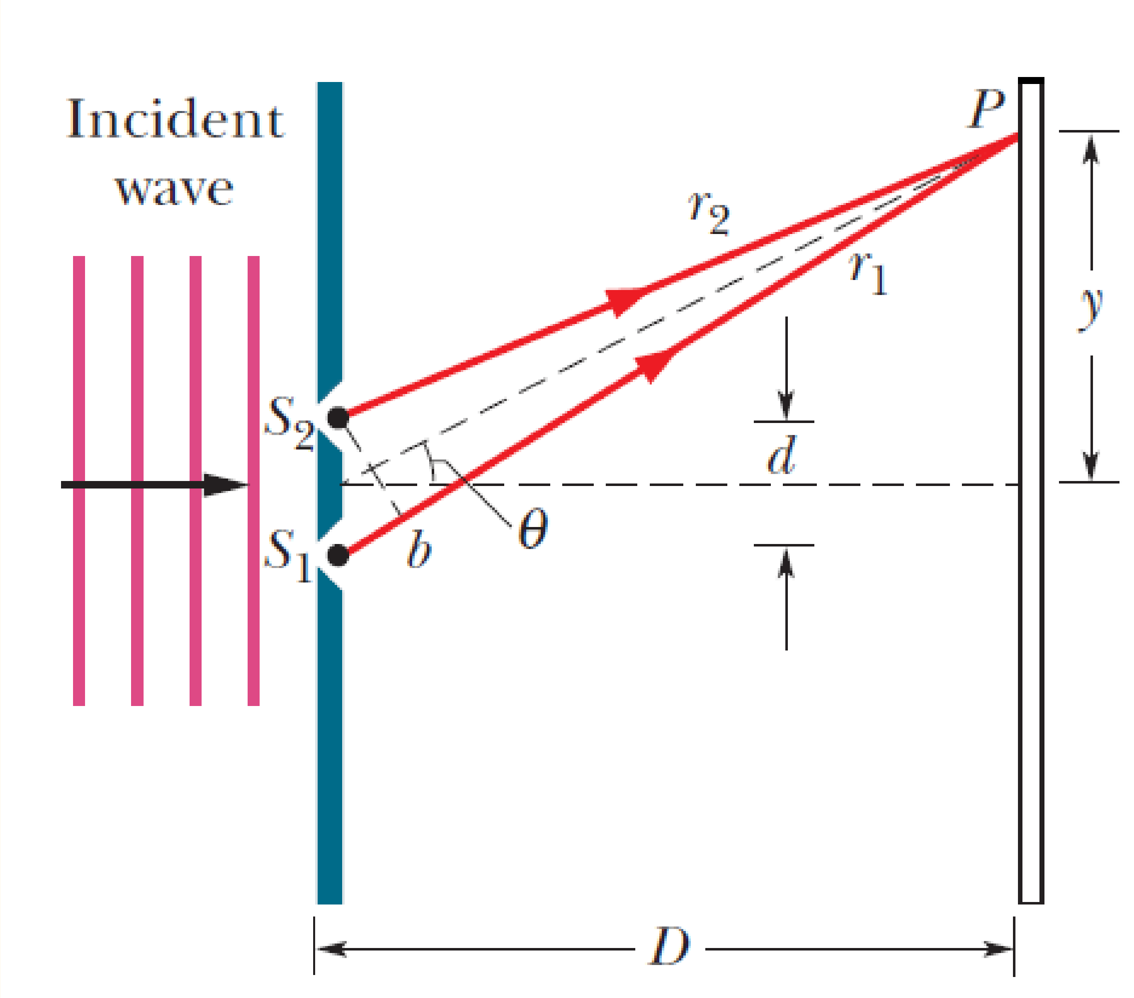


Different path → Difference of phase

Geometrical representation:

We assume r_1 and r_2 parallel close to S_1 and S_2 → d << D

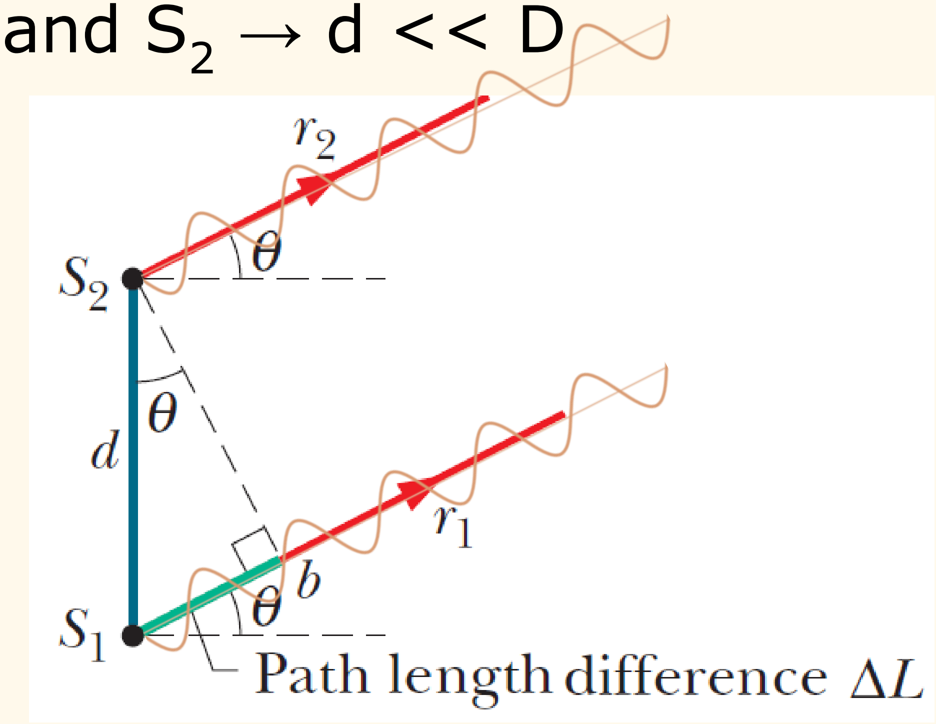




Different path → Difference of phase

Geometrical representation:

We assume r_1 and r_2 parallel close to S_1 and S_2 → d << D



We have: $\Delta L = d\sin\theta$

For $\Delta L = m\lambda$ (m integer)
→ **Constructive** interference
→ **Bright fringes**

For $\Delta L = (m + 1/2)\lambda$ (m integer)
→ **Destructive** interference
→ **Dark fringes**

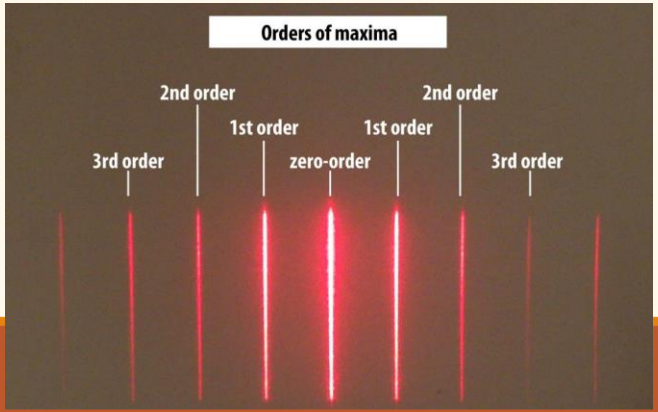
Different path → Difference of phase

Bright fringes at $\theta = \text{asin}\left(\frac{m\lambda}{d}\right)$

Dark fringes at $\theta = \text{asin}\left(\frac{(m+1/2)\lambda}{d}\right)$

Classification of fringes:
Bright / dark and order of the fringe

m = 0: 1st order
m = 1: 2nd order
m = 2: 3rd order



We have: $\Delta L = d\sin\theta$

For $\boldsymbol{\Delta L = m\lambda}$ (m integer)
→ **Constructive** interference
→ **Bright fringes**

For $\boldsymbol{\Delta L = (m+1/2)\lambda}$ (m integer)
→ **Destructive** interference
→ **Dark fringes**

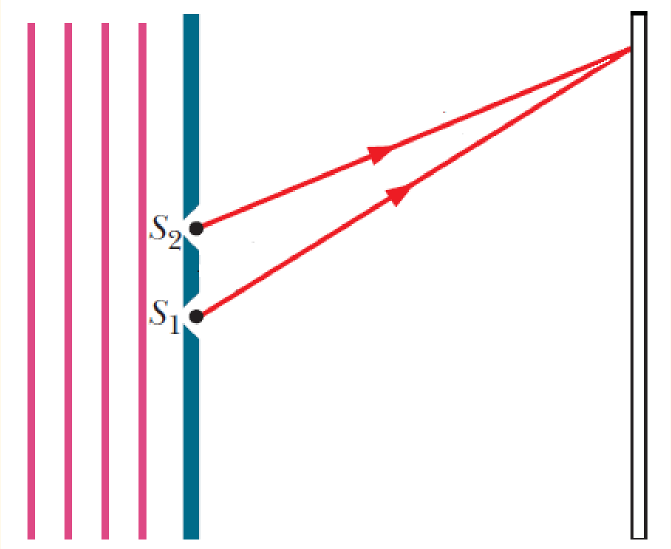
Note on coherence

To observe interference, there must be phase difference between waves

→ This phase difference must **remain the same** at a given point **over time**

→ In a double-slit experiment the same source must illuminate S_1 & S_2

Light from S_1 and S_2 is coherent

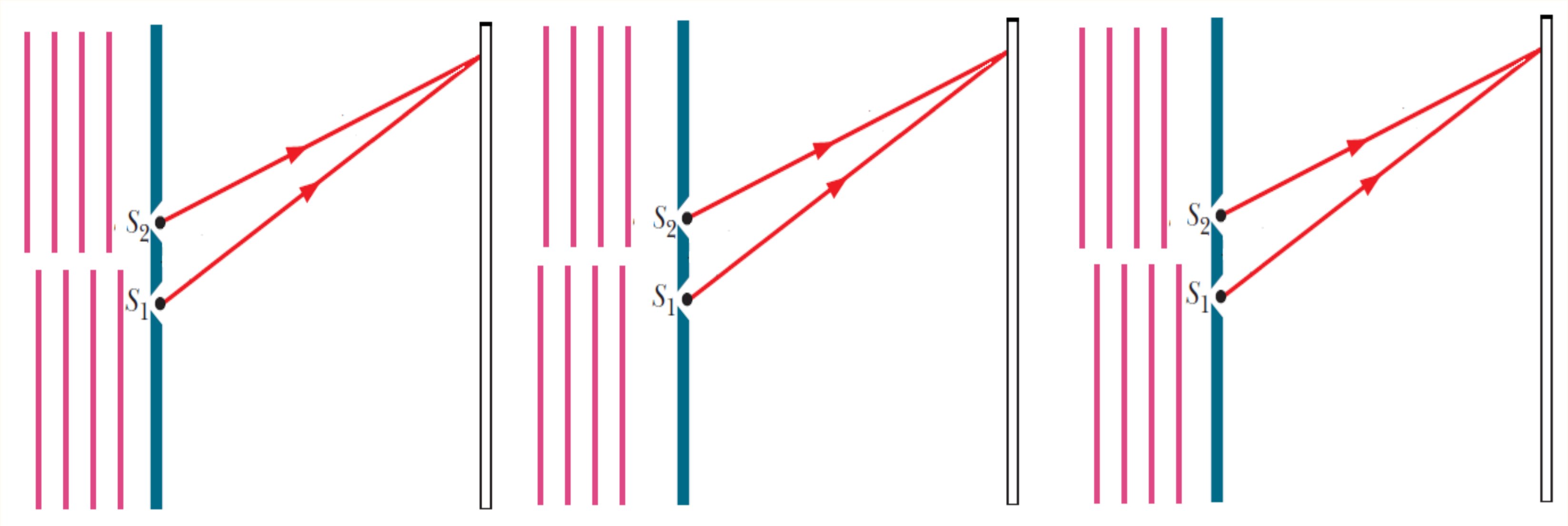


Note on coherence

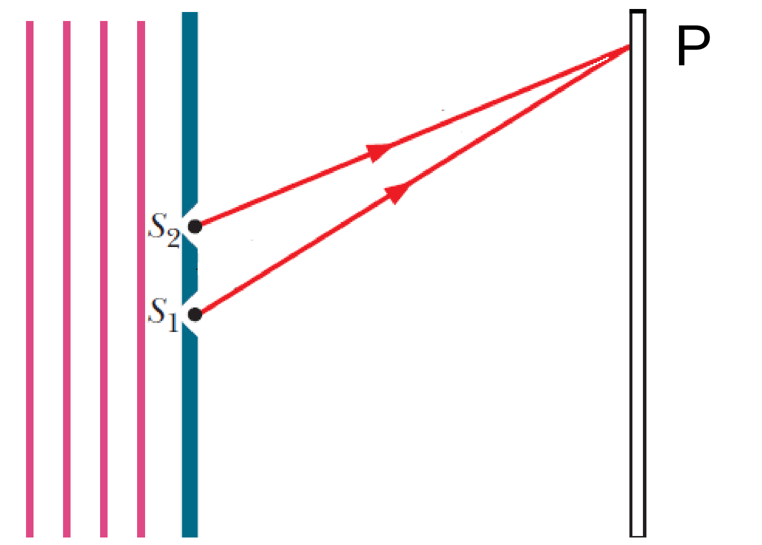
→ If we used 2 sources, we could **not see interference** because **the phase of a given source vary over time**

→ **Light from S_1 and S_2 is incoherent** and the phase difference is **not the same** at a given point **over time**

The interference pattern will change rapidly as the phase of the sources change independently
→ **Cannot see fringes**



Intensity in double-slit experiment



Two EM waves arrive at P with a phase difference:

General expression: $E = E_m \sin(kx - \omega t + \psi)$

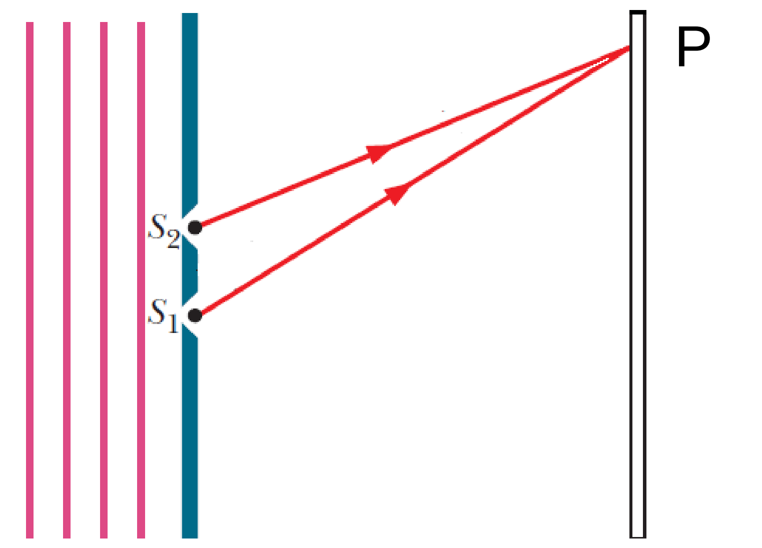
E_1 from S_1: $E_1 = E_0 \sin(\omega t)$

E_2 from S_2: $E_2 = E_0 \sin(\omega t + \phi)$

Same amplitude E_0 and **ϕ contains the phase difference** between E_1 and E_2 at P

Note: the sign before ωt has changed for clarity

→ π phase shift introduced in both fields



Intensity in double-slit experiment

E total at P:

$$E_p = E_1 + E_2$$

Intensity at P:

$$I = S_{avg} = \left(\frac{1}{c\mu_0} E_p^2\right)_{avg} = \frac{1}{c\mu_0}\left(E_p^2\right)_{avg}$$

→ Need to calculate the average value of E_p^2

Intensity in double-slit experiment

$$E_p = E_1 + E_2$$

$$E_p = E_0 \sin(\omega t) + E_0 \sin(\omega t + \phi)$$

$$\longrightarrow E_p^2 = \left(E_0 \sin(\omega t) + E_0 \sin(\omega t + \phi)\right)^2$$

$$E_p^2 = E_0^2 \sin^2(\omega t) + E_0^2 \sin^2(\omega t + \phi) + 2E_0^2 \sin(\omega t)\sin(\omega t + \phi)$$

$$E_p^2 = E_0^2 \left(\sin^2(\omega t) + \sin^2(\omega t + \phi) + 2\sin(\omega t)\sin(\omega t + \phi)\right)$$

$\sin^2(\omega t)$ → Avg value: 1/2

$\sin^2(\omega t + \phi)$ → Avg value: 1/2

$2\sin(\omega t)\sin(\omega t + \phi)$ → Avg value: ?

Intensity in double-slit experiment

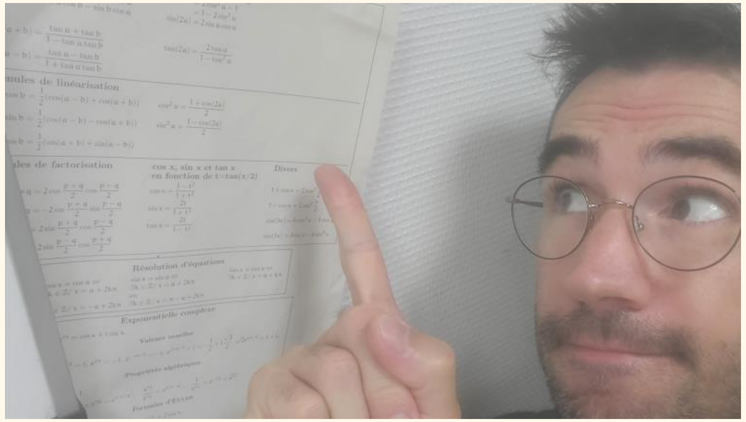
Linearization: $\sin a \sin b = \frac{1}{2}\left(\cos(a-b) - \cos(a+b)\right)$

$$\sin(\omega t)\sin(\omega t + \phi) = \frac{1}{2}\left(\cos(\omega t - \omega t - \phi) - \cos(\omega t + \omega t + \phi)\right)$$

$$= \frac{1}{2}\left(\cos(-\phi) - \cos(2\omega t + \phi)\right)$$

$$= \frac{1}{2}\left(\cos(\phi) - \cos(2\omega t + \phi)\right)$$

Tips from a physicist:



Never be far from a trigonometric formula sheet!

Intensity in double-slit experiment

$$E_p^2 = E_0^2\left(\sin^2(\omega t) + \sin^2(\omega t + \phi) + 2\sin(\omega t)\sin(\omega t + \phi)\right)$$

Avg value: 1/2

Avg value: 1/2

Avg value: ?

$$E_p^2 = E_0^2\left(\sin^2(\omega t) + \sin^2(\omega t + \phi) + \cancel{2}\,\frac{1}{\cancel{2}}\left(\cos(\phi) - \cos(2\omega t + \phi)\right)\right)$$

Avg value: 1/2

Avg value: 1/2

Avg value: cos(ϕ)

Avg value: 0

Intensity in double-slit experiment

$$E_p^2 = E_0^2\left(\sin^2(\omega t) + \sin^2(\omega t + \phi) + \cos(\phi) - \cos(2\omega t + \phi)\right)$$

- $\sin^2(\omega t)$ → Avg value: 1/2
- $\sin^2(\omega t + \phi)$ → Avg value: 1/2
- $\cos(\phi)$ → Avg value: cos(ϕ)
- $\cos(2\omega t + \phi)$ → Avg value: 0

$$\left(E_p^2\right)_{avg} = E_0^2\left(\frac{1}{2} + \frac{1}{2} + \cos(\phi) - 0\right)$$

$$\left(E_p^2\right)_{avg} = E_0^2\left(1 + \cos(\phi)\right)$$

Linearization: $\cos^2 a = \dfrac{1 + \cos 2a}{2}$ or $2\cos^2\frac{b}{2} = 1 + \cos b$

$$\left(E_p^2\right)_{avg} = E_0^2\, 2\left(\frac{1 + \cos\left(2\,\frac{\phi}{2}\right)}{2}\right) = 2\,E_0^2\cos^2\left(\frac{\phi}{2}\right)$$

Intensity in double-slit experiment

$$\left(E_p^2\right)_{avg} = 2\,E_0^2 \cos^2\left(\frac{\phi}{2}\right) \longrightarrow I = \frac{1}{c\mu_0}\left(E_p^2\right)_{avg} = \frac{1}{c\mu_0}\,2\,E_0^2\cos^2\left(\frac{\phi}{2}\right)$$

We define I_0, the intensity at P produced by E_1 without E_2 (or inversely)

$$I_0 = \frac{1}{c\mu_0}\left(E_1^2\right)_{avg} = \frac{1}{c\mu_0}\left(\left(E_0 \sin(\omega t)\right)^2\right)_{avg} = \frac{1}{c\mu_0}\frac{E_0^2}{2}$$

Avg value: 1/2

$\times\, 4\cos^2\left(\frac{\phi}{2}\right)$

Thus, $$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

Intensity in double-slit experiment (optional demonstration)

Demonstration also possible with phasors

$$2\beta + (180 - \phi) = 180 \longrightarrow \beta = \frac{\phi}{2}$$

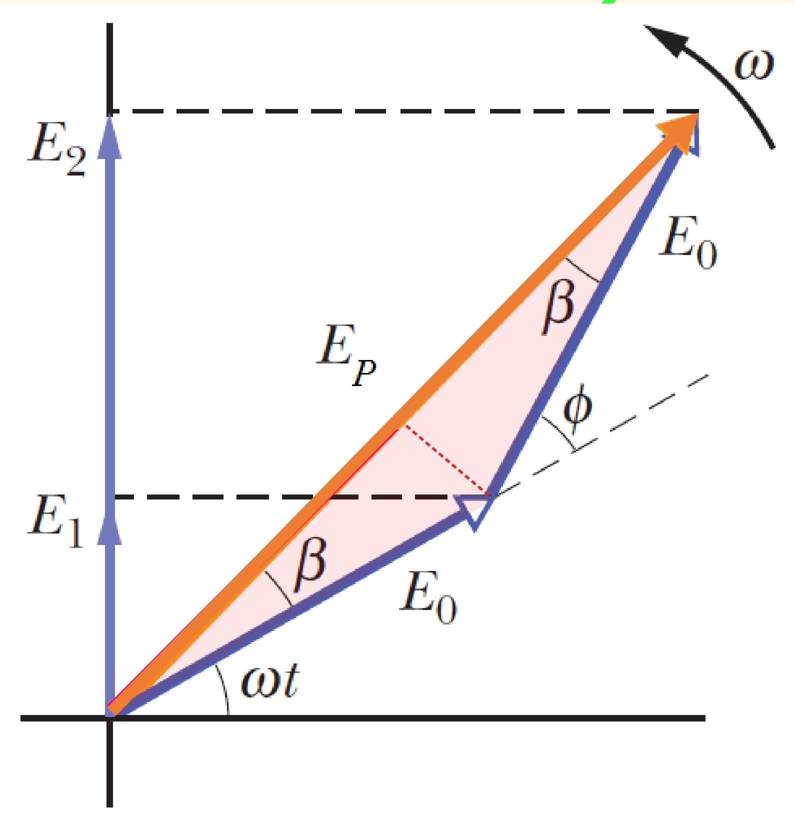
The half-length of the phasor of E_p equals

$$\frac{1}{2}E_p = E_0 \cos\beta$$

So the length of the phasor is $2\, E_0 \cos\frac{\phi}{2}$

That leads to the same result

$$I = \frac{1}{c\mu_0}\, 2\, E_0^2 \cos^2\left(\frac{\phi}{2}\right)$$



Intensity in double-slit experiment

We have: $$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

And $\Delta L = d sin\theta$

Φ correspond to the phase difference due to propagation along different paths → ΔL

$$\phi = k\Delta L = \frac{2\pi}{\lambda}\Delta L = \frac{2\pi}{\lambda} d \sin\theta$$

So $$I = 4I_0 \cos^2\left(\frac{\pi}{\lambda} d \sin\theta\right)$$

Max. $I = 4I_0$ if $\cos^2(\frac{\phi}{2}) = 1$

→ $\phi = 2m\pi$ (m integer)

Min. I=0 if $cos^2\left(\frac{\phi}{2}\right) = 0$

→ $\phi = (2m+1)\,\pi$

Max. I if $\frac{\pi}{\lambda} d \sin\theta = m\pi$

→ $d \sin\theta = m\lambda$

Min. I if $\frac{\pi}{\lambda} d \sin\theta = \left(m + \frac{1}{2}\right)\pi$

→ $d \sin\theta = (m + 1/2)\,\lambda$

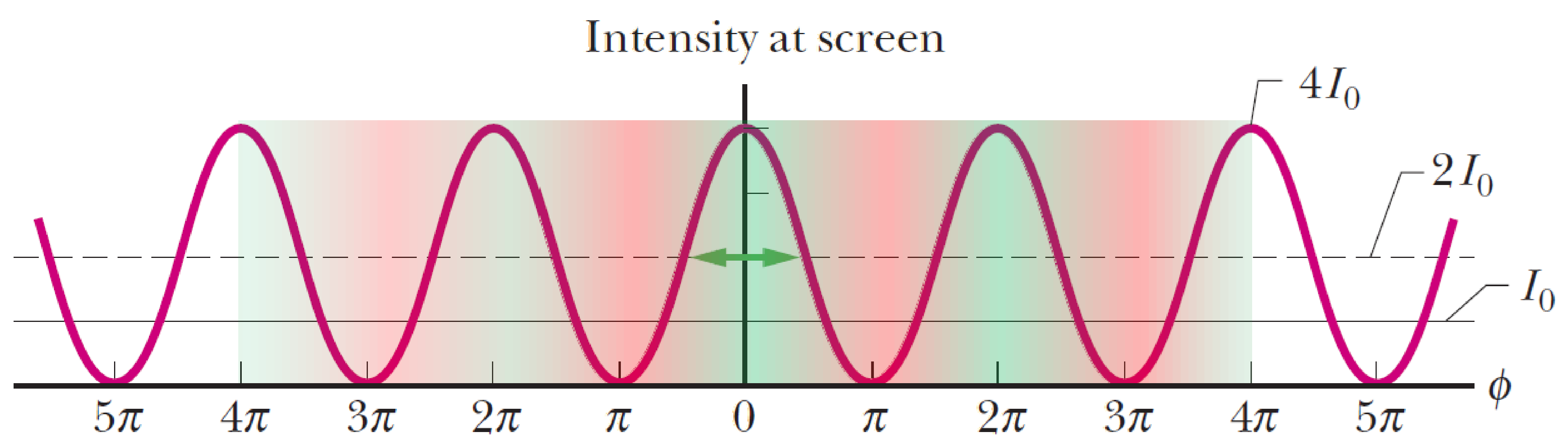
(Same results than on slide previously)

Intensity in double-slit experiment

We have: $I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$

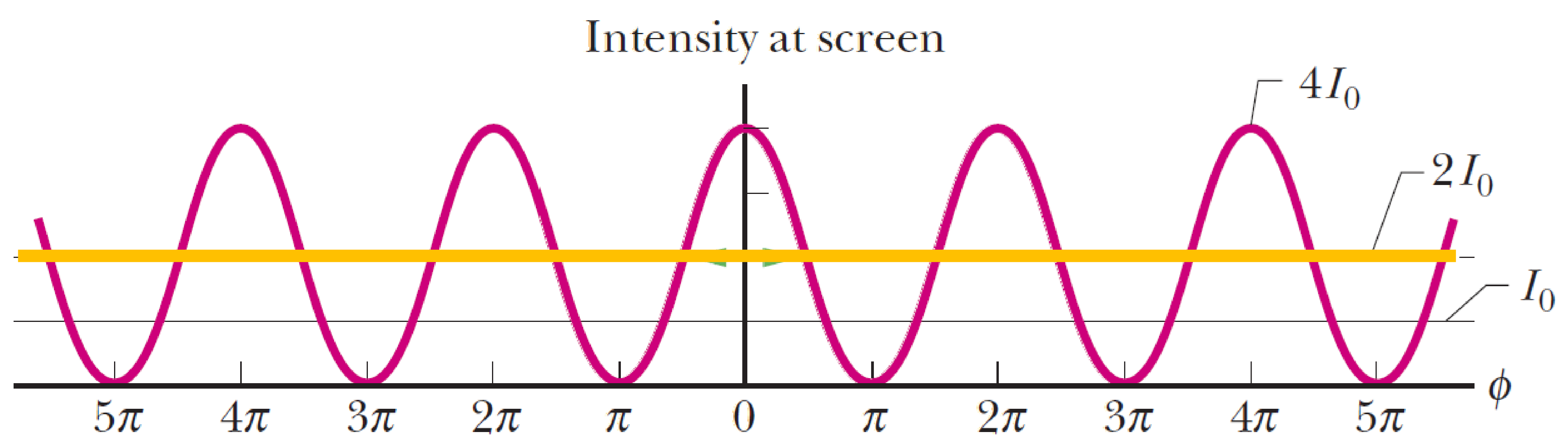
Max. I if $\phi/2 = m\pi$
$\rightarrow \phi = 2m\pi$ (m integer)

Min. I if $\phi/2 = (m + 1/2)\pi$
$\rightarrow \phi = (2m + 1)\,\pi$



Intensity in double-slit experiment

Note: **Interference do not create energy** but spatially redistributes it
The average intensity is still $2I_0$ → as if waves were incoherent

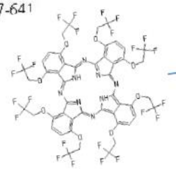


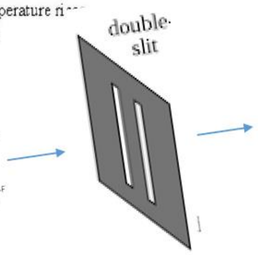
INTERFERENCE & DOUBLE-SLIT INTENSITY

For most physicists, the double slit experiment **with quantum particules** (not just light), is considered as the most beautiful experiment ever!

- Performed with a light source so faint that only one photon exists in the apparatus at any one time
  G I Taylor 1909 *Proceedings of the Cambridge Philosophical Society* **15** 114-115
- Performed with electrons
  C Jönsson 1961 *Zeitschrift für Physik* **161** 454-474,
  (translated 1974 *American Journal of Physics* **42** 4-11)
- Performed with single electrons
  A Tonomura *et al.* 1989 *American Journal of Physics* **57** 117-120
- Performed with neutrons
  A Zeilinger *et al.* 1988 *Reviews of Modern Physics* **60** 1067-1073
- Performed with He atoms
  O Carnal and J Mlynek 1991 *Physical Review Letters* **66** 2689-2692
- Performed with C60 molecules
  M Arndt *et al.* 1999 *Nature* **401** 680-682
- Performed with C70 molecules showing reduction in fringe visibility as temperature ri...
  and the molecules "give away" their position by emitting photons
  L. Hackermüller et al 2004 *Nature* **427** 711-714
- Performed with Na Bose-Einstein Condensates
  M R Andrews *et al.* 1997 *Science* **275** 637-641

- Perfomed with $C_{48}H_{26}F_{24}N_8O_8$





T. Juffmann et al., Real-time single-molecule imaging of quantum interference, Nat. Nanotechnology 7, 297-300, (2012).

> *« … a phenomenon which is impossible, absolutely impossible, to explain in any classical law, and wich has in it the heart of quantum mechanics. In reality it contains the only mystery. »*
>
> R.P. Feynman

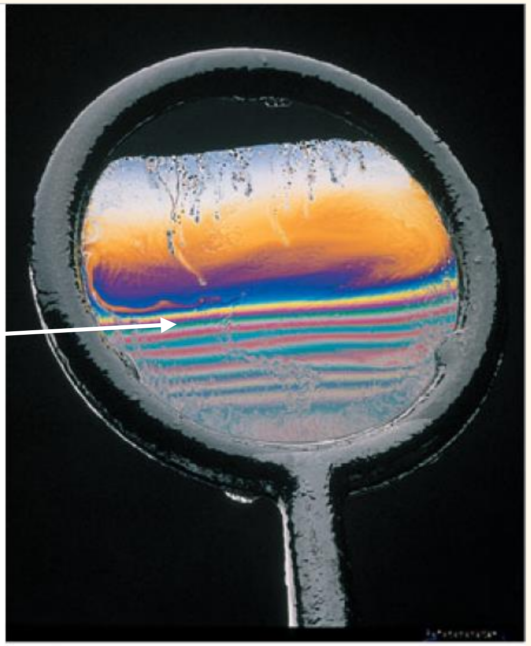
Observation of **fringes** on thin films (soap bubble, oil on water, …)

→ **Interference**

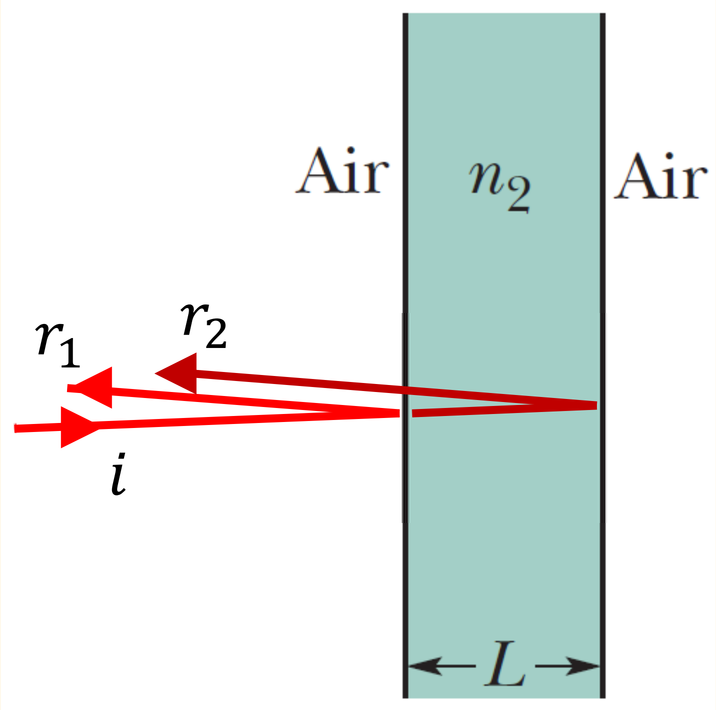
r_1: reflected

r_2: refracted – reflected – refracted

Phase difference between r_1 & r_2 when they reach the eye ?

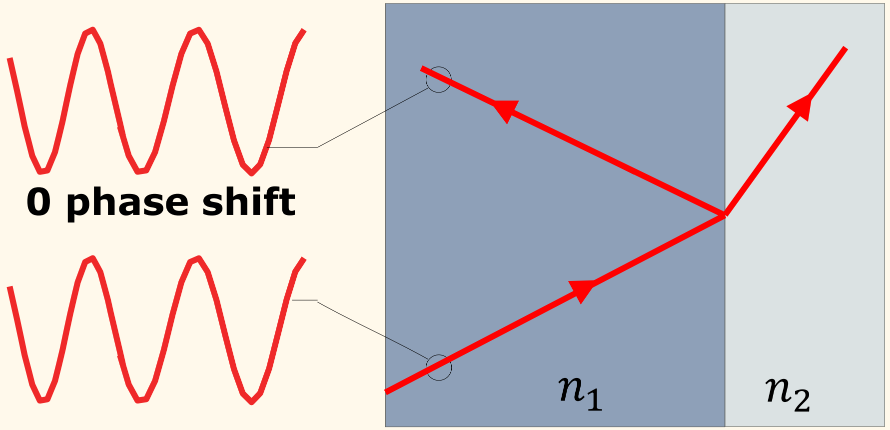


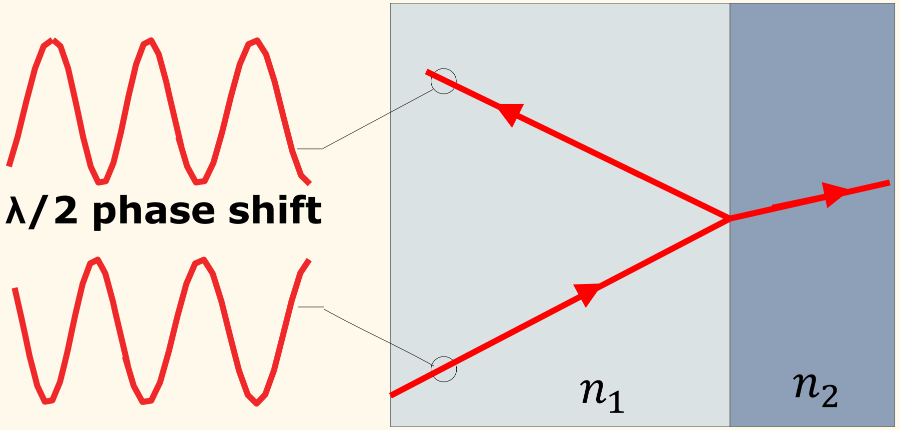
Richard Megna/Fundamental Photographs



Note: all angles between rays and normal are close to zero

ΔL between a & a′ = mλ





Phase difference due to different path length & different indexes

Reflection may also induce a phase shift

→ Depends of the **index** of the medium of which light is reflected

$n_1 > n_2$ → 0 phase shift

$n_1 < n_2$ → λ/2 phase shift

INTERFERENCE FROM THIN FILMS

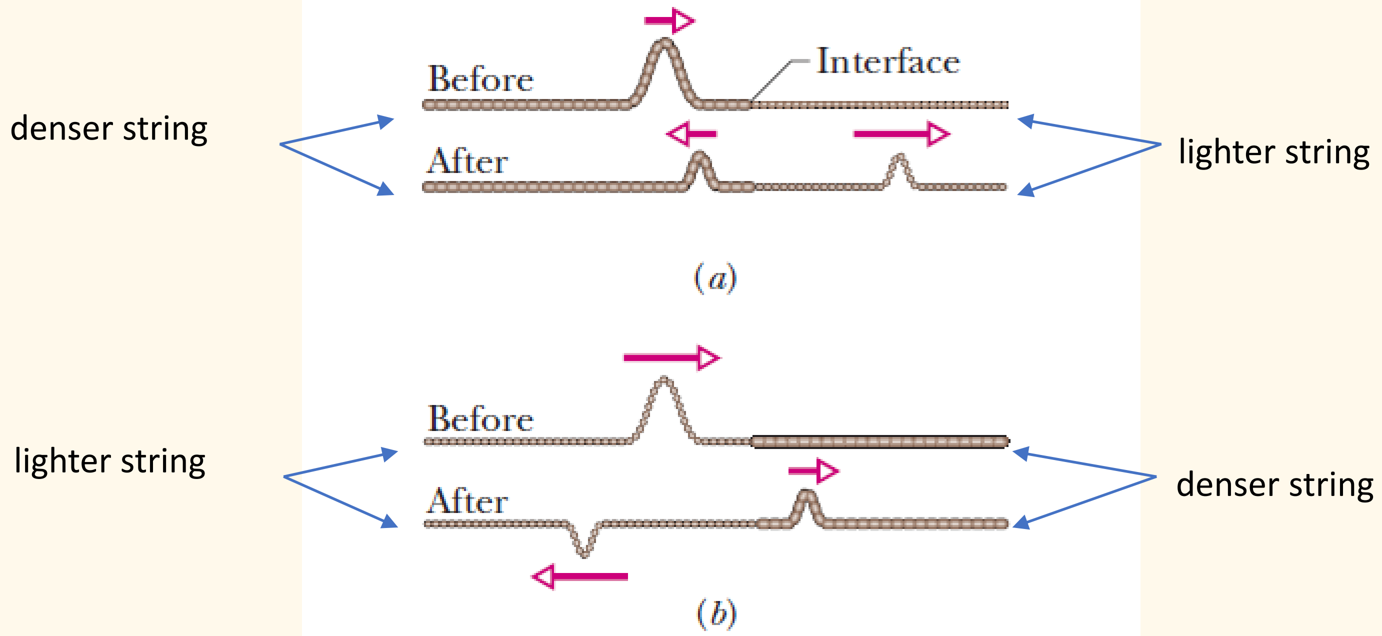


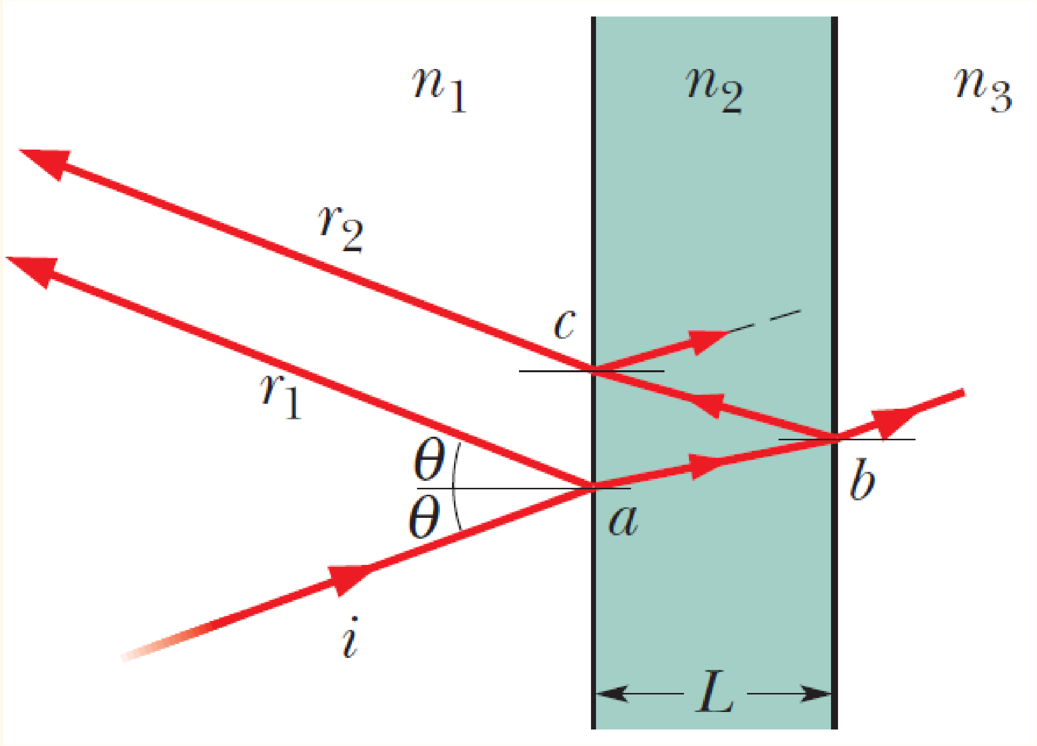
Figure 35-16 Phase changes when a pulse is reflected at the interface between two stretched strings of different linear densities. The wave speed is greater in the lighter string. (*a*) The incident pulse is in the denser string. (*b*) The incident pulse is in the lighter string. Only here is there a phase change, and only in the reflected wave.

Reflection	Reflection phase shift
Off lower index	0
Off higher index	0.5 wavelength

To understand **interference** in thin films contributions of

- **Reflection**
- **Path length**
- **Indexes**

Must be taken into account to calculate the **phase difference**



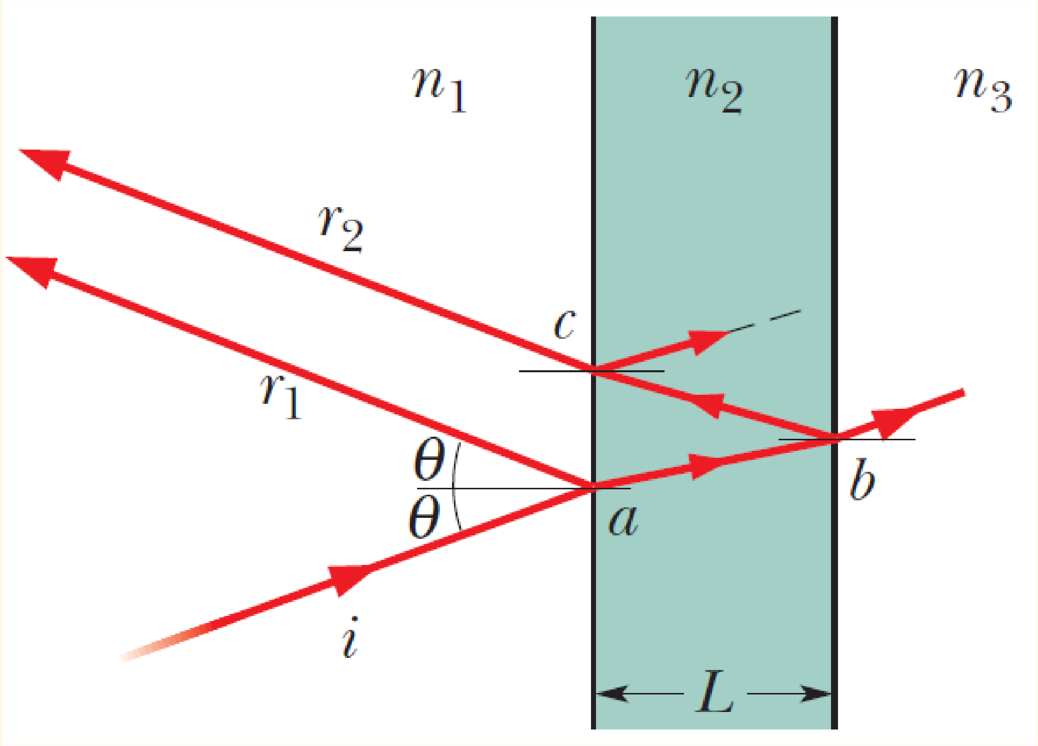
Note: all angles between rays and normal are close to zero and we assume $n_1 = n_3 = n_{air}$

INTERFERENCE FROM THIN FILMS

Reflection:

$n_2 > n_{air}$ → λ/2 phase shift for r_1

→ 0 phase shift for r_2



Path length & Indexes

→ r_2 travels ~ 2L in a medium of index n_2

We must consider

$$\frac{\Delta L}{\lambda_{n_2}} = n_2 \frac{\Delta L}{\lambda} = n_2 \frac{2L}{\lambda}$$

→ If equals to (m+1/2) → r_1 and r_2 are in phase
→ **Bright fringes**

If equals to m → r_1 and r_2 are out of phase
→ **Dark fringes**

INTERFERENCE FROM THIN FILMS

Notes:

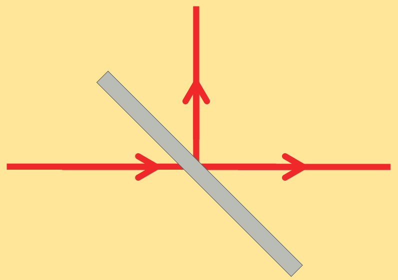
- The situation is not the same if n1 and n3 are not equals and/or > n2

- If $L < \lambda/10$, difference of path can be neglected
  → interference only due to reflection

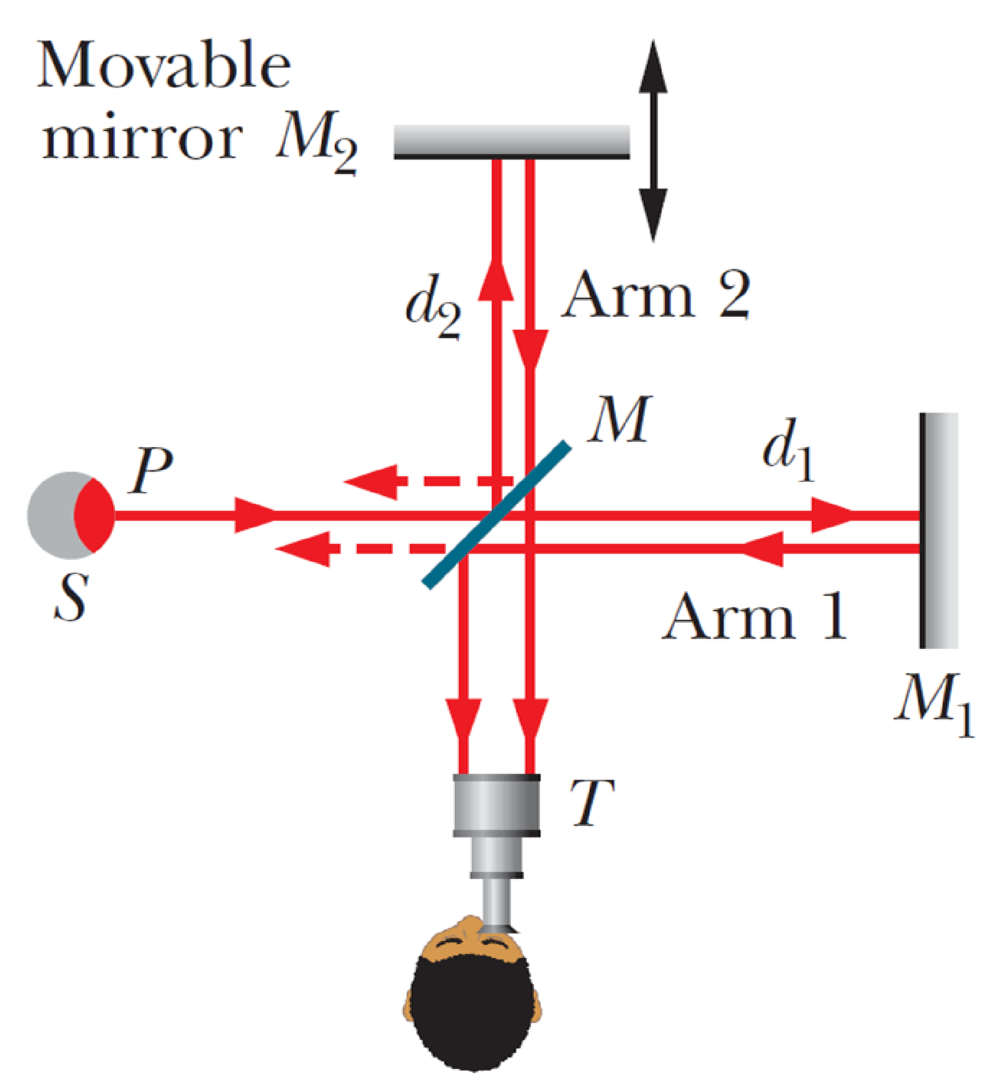
MICHELSON'S INTERFEROMETER

Interferometer → instrument that measure precisely difference of path by **interference**

M is a **Beam splitter**
= half transparent mirror



T telescope to observe fringes



Difference of path → $2d_2$ - $2d_1$

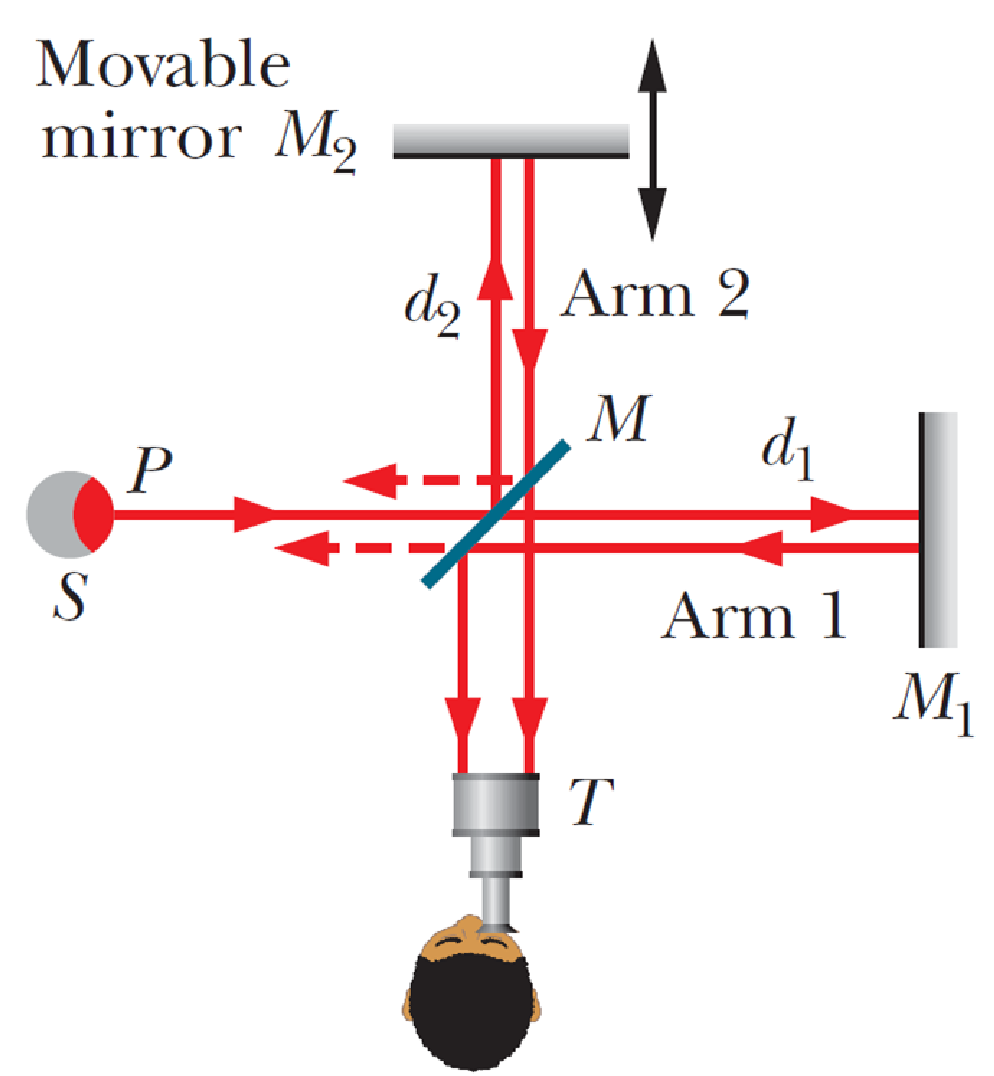
We assume that we see a bright (dark) fringe

→ If we move M_2 by λ/2 difference of path increases by λ

→ We observe the next bright (dark) fringe

→ If we move M_2 by λ/4 difference of path increases by λ/2

→ We observe the next dark (bright) fringe



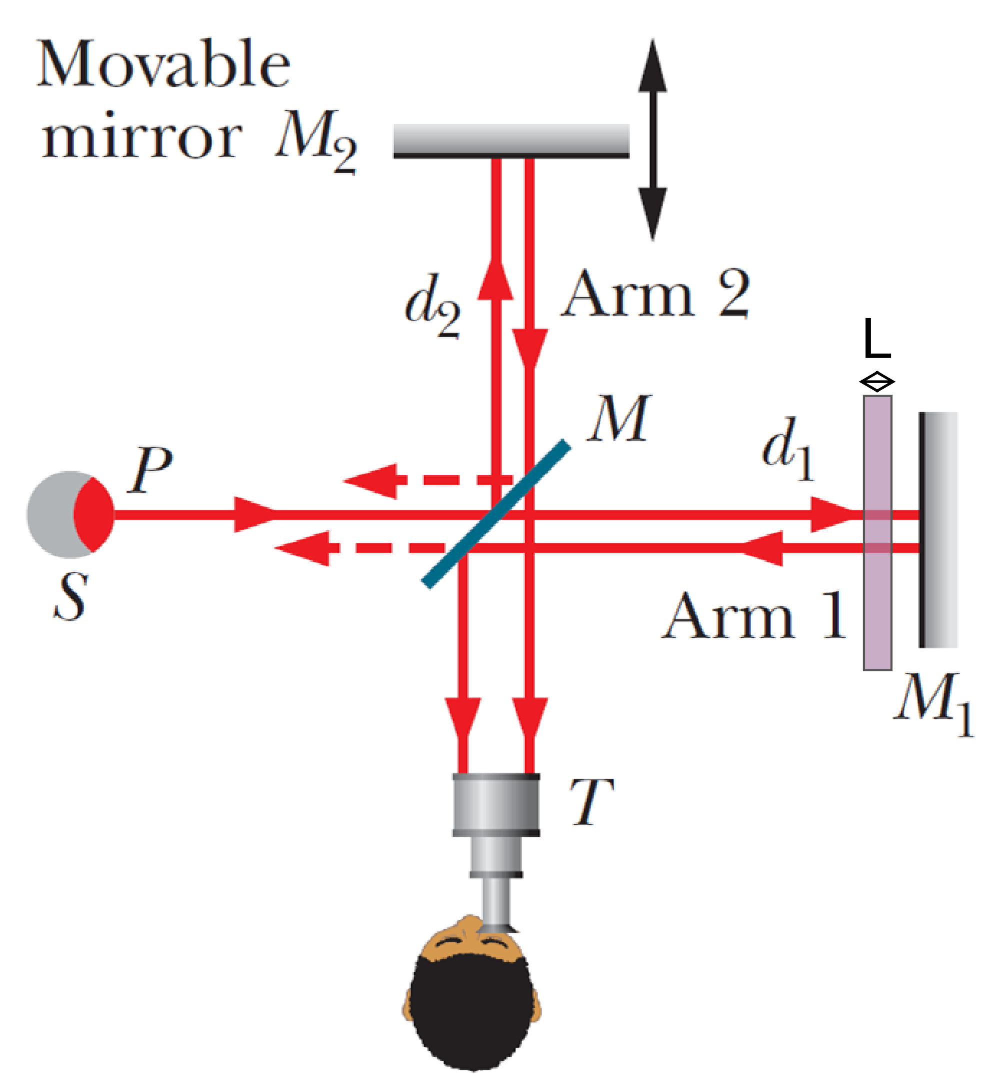
MICHELSON'S INTERFEROMETER

We put a sample of thickness L and index n in arm 1

Number of wavefronts in the sample traversed 2 times (N_m) and in the same region in air (N_a) before the sample is placed:

$$N_m = \frac{2L}{\lambda_n} = n\frac{2L}{\lambda} \qquad N_a = \frac{2L}{\lambda}$$

$$N_m - N_a = (n-1)\frac{2L}{\lambda}$$

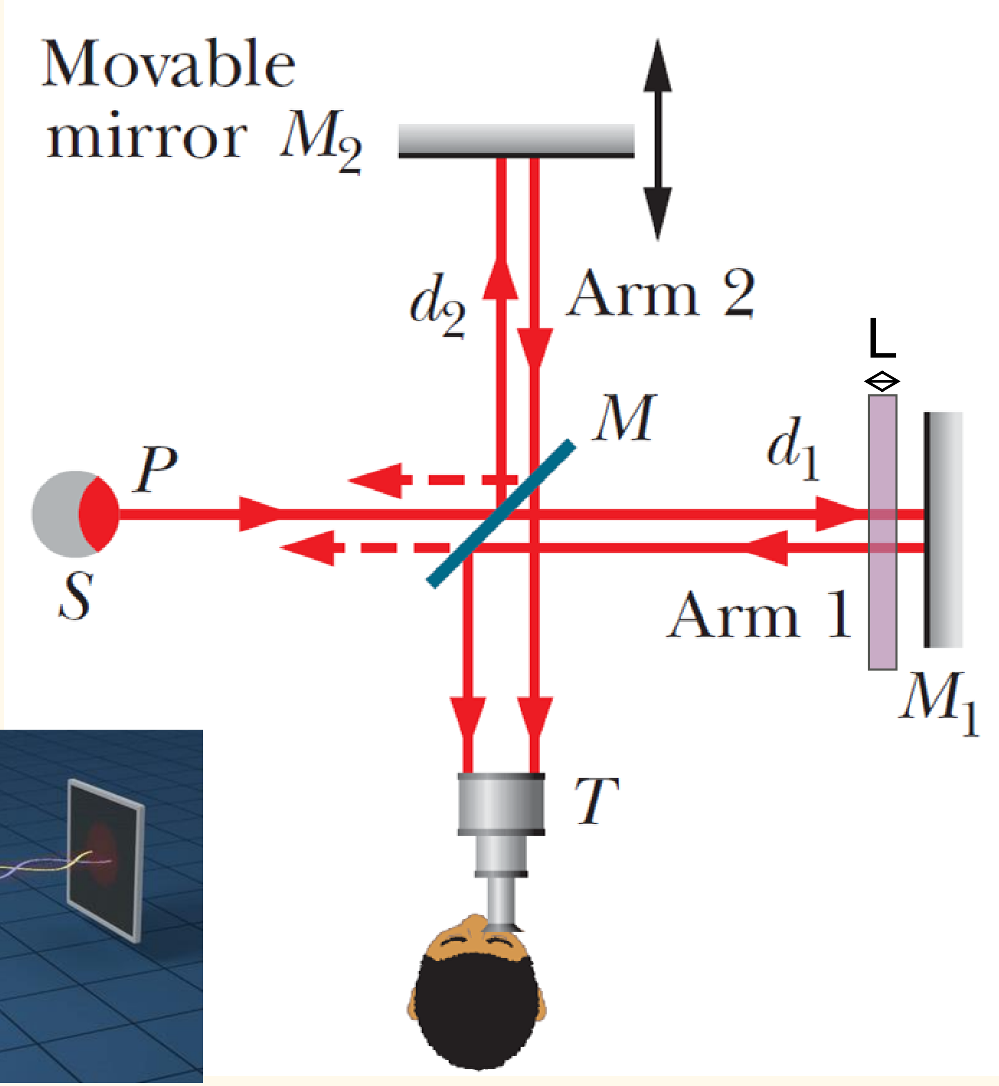


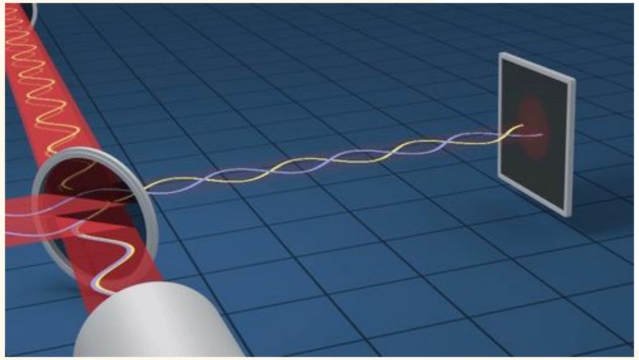
We put a sample of thickness L and index n in arm 1

$$N_m - N_a = (n - 1)\frac{2L}{\lambda}$$

If n is known, L can be determined from the shift of the interference pattern

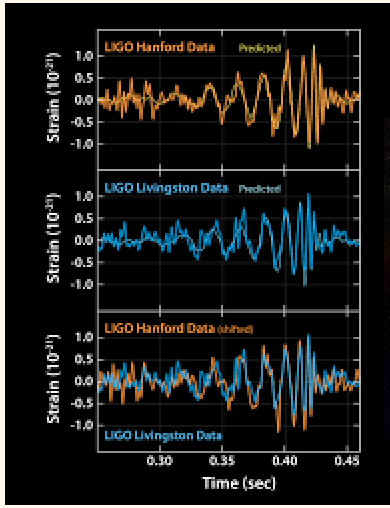
If L is known, n can be determined from the shift of the interference pattern

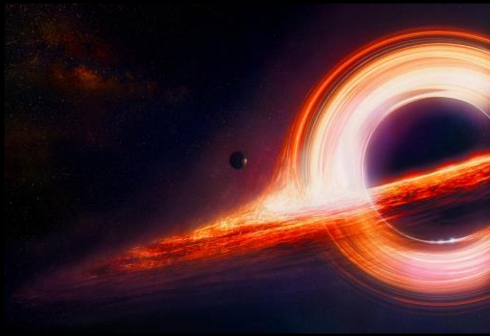




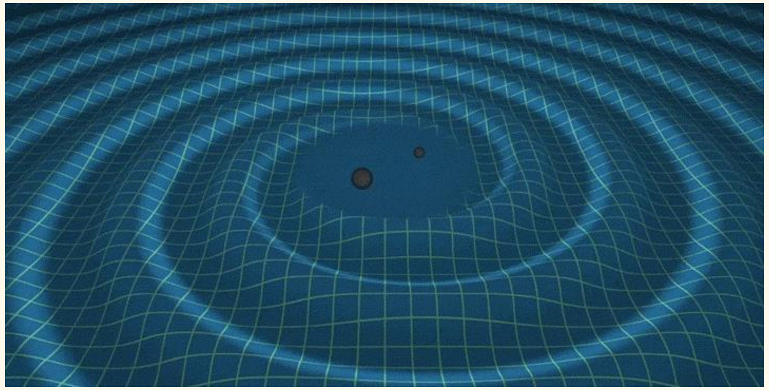
MICHELSON’S INTERFEROMETER

The 4km long arms of the LIGO experiment at Hanford. LIGO lab: www.ligo.caltech.edu





Discovery of gravitational waves from colliding black holes



KEY POINTS

Huygens' principle

Difference of phase caused by difference of path and/or indexes

Constructive and destructive interference

Double-slit experiments $\quad I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$

Reflection phase shifts

Interference of thin films

Principle of operation of the Michelson interferometer

READING ASSIGNMENT

Chapter 36 of the textbook